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Reverberation by a relativistic accretion disk

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ABSTRACT
We calculate the response of a line emitted from a relativistic accretion disk to a continuum variation of the central illuminating source. This model might be relevant to the very broad and sometimes redshifted emission lines which have been observed at optical and X-ray energies in Active Galactic Nuclei and X-ray binaries. We consider separately the cases of a point-like and of an extended central source. Based on the first three moments of the line energy distribution (i.e. the line intensity, centroid energy and width) and the two-dimensional transfer function, we identify a number of characteristic features, which can help assessing the presence of a relativistic accretion disk and estimating its parameters. In particular, by combining an absolute time measurement of the maximum of line response function with a relative measurement in units of light crossing time the mass of the central object can be derived.

Key words: accretion – galaxies: Seyfert – galaxies: nuclei – X-rays: galaxies

1 INTRODUCTION

It has long been suspected that the innermost regions of accretion flows towards the central object in Active Galactic Nuclei (AGNs) might be characterised by a disk geometry. The blue-UV bump (Malkan & Sargent 1982) and/or soft X-ray excess (Arnaud et al. 1985) observed in a number of quasars and Seyfert galaxies, provide indirect evidence in favour of this interpretation. The size of the blue-UV emitting region is constrained by the measurement of the time delay which characterises the response at different wavelengths to variations of the ionising flux at the central source. From the monitoring of the Seyfert galaxy NGC 4151 (Clavel et al. 1990; Ulrich et al. 1991) a time delay of 4 ± 3 d between the UV-continuum variations and the response of two different lines has been estimated. A longer observing campaign for NGC 5548 (Clavel et al. 1991; Krolik et al. 1991) provided an upper limit of ~ 4 d to the delay between the optical-UV continuum variations at four different wavelengths. Reprocessing from accretion disk matter of the primary (hard) X-ray radiation can explain the absence of substantial delays between the optical and ultraviolet light curves (e.g. for NGC 5548 Molendi, Maraschi & Stella 1992).

The presence of a relativistic accretion disk can in principle be established on the basis of the characteristic double-horned line profile which arises from a flat Keplerian disk with a power-law emissivity dependence (Chen & Halpern 1989; Fabian et al. 1989). Broad asymmetric double-horned Balmer ($H\alpha$, $H\beta$) emission lines have been observed in several broad-line radio galaxies and radio loud quasars (Eracleous & Halpern 1994 and references therein). However double-horned line profiles are rare in AGNs and, even in these cases, some observations show features that are inconsistent with the simple and stationary relativistic accretion disk modelling (e.g. the red horn occasionally becomes brighter than the blue horn in 3C390.3, Zheng, Veilleux & Grandi 1991; see also Eracleous et al. 1995). From this discussion we can conclude that testing disk models based only on stationary profiles can be difficult or inconclusive.

The way matter "reverberates" the incident radiation from the central source provides an additional diagnostic that can be used to infer the geometry and the dynamics of the line emitting region (Blandford & McKee 1982; Welsh & Horne 1991; Pérez, Robinson & de la Fuente 1992). The information derived from the stationary line profile and from reverberation properties can therefore be compared for a consistency check. Based on the asymmetry and width of the stationary line profile, Mannucci, Salvati & Stanga (1992) estimate geometrical parameters and subsequently derive the radial dependence of the line emissivity law by adopting a deconvolution technique. The geometrical parameters and the emissivity law are then used to derive the expected response of the line intensity to continuum variations to be compared with the existing observations.

Beside Balmer emission lines a promising feature to test the innermost emission line regions in AGNs and X-ray binaries is likely represented by iron Kα lines (with rest energy of $6.4-6.9$ keV). In a number of cases these lines are probably broader than ~ 1 keV (FWHM) and/or redshifted to centroid energies as low as ~ 6.0 keV (e.g. Barr, White &

Page 1985; Day et al. 1990). Detailed line profiles are not yet available as most studies have been carried out with low resolution X-ray instrumentation ($E/\Delta E \lesssim 10$) and/or poor statistic (Smale et al. 1993). The X-ray CCD detectors on board ASCA ($E/\Delta E \sim 50$) are beginning to provide more detailed observations of iron K-shell lines (Fabian et al. 1994). In the most likely interpretation these broadened and shifted lines are produced by bulk plasma motion in the vicinity of the accreting collapsed object, possibly in a disk-like geometry (Kallman & White 1989; Fabian et al. 1989; George & Fabian 1991; Matt, Perola & Piro 1991). The reprocessing of hard radiation by cold matter in the form of an accretion disk gives rise to iron Kα lines, the centroid energy, widths and equivalent widths (EWs) of which are in agreement with the observed values (Nandra & Pounds 1994). Therefore iron K-shell lines might provide a sensitive diagnostic of the innermost regions ($r \sim 10 - 100\, r_g$) of accretion flow towards collapsed objects, where relativistic effects are expected to be important. Detailed modelling of the physical conditions of a relativistic accretion disk shows that more complex profiles than a simple double-horn are expected in the case of high accretion rates (Matt, Fabian & Ross 1993).

If reprocessing of hard X-ray radiation from a central source is responsible for the production of iron Kα lines in the innermost region of accretion flows towards collapsed objects, then reverberation studies relating line to continuum variations can provide crucial information on the geometry and dynamics of the line emitting region (Fabian et al. 1989). As shown by Stella (1990), sufficiently short variations at the central source should give rise to two characteristic sharp features in the profile that drift from the line wings towards the centre. Such features provide in principle a powerful diagnostic of the disk parameters and the mass of the central object. In the absence of a detailed knowledge of the line profile, the response of the line moments, in particular the intensity, I_l, centroid energy, E_c, and width, W, can nevertheless yield a wealth of useful information. Along these lines Stella & Campana (1991) and Campana & Stella (1993) discussed a method for constraining the disk parameters and the mass of the central object by using the stationary line width and centroid energy of iron Kα lines and the cross-correlation delays between the continuum and the line intensity variations. Matt & Perola (1992) first considered the detailed response of the iron Kα line centroid energy, intensity and width to continuum variations from a point-like source above the disk plane. This geometry was introduced in order to approximate an extended source and maximise the flux intercepted by the disk.

In this paper we further investigate the response of a line from a relativistic disk to variations at the central source, by using the transfer function formalism (see Section 2). In Section 3 we derive the line intensity, centroid energy and width for a flat, thin accretion disk around a Schwarzschild black hole, taking into account relativistic effects. We adopt two limiting geometries for the central illuminating source: a point-like source lying in the disk plane and a more realistic model involving a bulge-like source straddling the inner edge of the accretion disk. For the first case we calculate also the two-dimensional transfer function characterising the disk response. Based on line moments, we identify and discuss several features that can be used as indicators of geometry and scale length in a fashion similar to the work of Matt & Perola (1992). Our results are discussed in Section 4.

2 REVERBERATION

The response of a matter distribution to radiation from a central source can be described with the formalism of transfer functions which relate the line output to the intensity continuum input (Blandford & McKee 1982). As customary, we shall assume that the line emissivity is proportional to the incident continuum flux.

At a given time t, the observer sees the line profile built up by contributions of different portions of the line emitting region, each characterised by an energy shift $1+z = E_0/E$, where E_0 is the line rest energy. Each portion is also characterised by a given light propagation delay with respect to the observer, which regulates its response to variations at the central source. The response can, in general, be described through the two-dimensional transfer function (2D-TF) $\psi(E,\, t)$, defined by:

$$I_l(E,\, t) = \psi(E,\, t) * I_c(t) = \int_0^\infty \psi(E,\, t-t')\, I_c(t')\, dt' \qquad (1)$$

with $I_l(E,\, t)$ the line intensity per unit energy seen by the observer at a time t and $I_c(t)$ the continuum intensity; "$*$" denotes a convolution. The 2D-TF is the time-energy Green's function of the system.

The deconvolution of Eq. 1 based on Fourier transforms requires high signal-to-noise data and a nearby equispaced grid of observation times. Two-dimensional maps of non-relativistic accretion disks have been reconstructed through observations of emission lines in cataclysmic variables, by using the technique developed by Horne & Marsh (Horne 1985; Marsh & Horne 1988; Marsh & Horne 1990). These authors apply a maximum entropy deconvolution technique to overcome practical limitations due to relatively high noise and incomplete data sets (Skilling & Bryan 1984). Observations are often too short and noisy to reconstruct the 2D-TF, so it is useful to introduce the moments:

$$I_l^{(n)}(t) = \int_{-\infty}^\infty E^n\, I_l(E,\, t)\, dE \qquad \text{and}$$

$$\psi^{(n)}(t) = \int_{-\infty}^\infty E^n\, \psi(E,\, t)\, dE$$

In particular, the one-dimensional transfer function (1D-TF) $\psi(t) \equiv \psi^{(0)}(t)$ is:

$$I_l(t) = \psi(t) * I_c(t) = \int_0^\infty \psi(t-t')\, I_c(t')\, dt' \qquad (2)$$

The 1D-TF represents the line intensity $I_l(t) \equiv I_l{}^{(0)}(t)$ produced in response to a $\delta-$function continuum variation at $t=0$, i.e. it is the time Green's function of the system.

Higher line moments can be derived in a similar manner; the line centroid energy for a δ−function continuum variation, normalised to the rest line energy, is $E_c(t) = \psi^{(1)}(t)/\psi(t)$. For the line width we use $W(t) = 2.35\sqrt{[\psi^{(2)}(t) - E_c^2(t)]/\psi(t)}$, where a normalisation of 2.35 is taken to reproduce the FWHM in the case of a Gaussian profile.

Given $I_l(t)$ and $I_c(t)$ the 1D-TF can be derived from the deconvolution of Eq. 2. Blandford & McKee (1982) apply this "reverberation mapping technique" to a variety of geometry and velocity distributions of astrophysical relevance. The 1D-TF for the $H\alpha$ line in NGC 4151 relative to the optical continuum has been evaluated in this way (Maoz et al. 1991). Maximum entropy deconvolution techniques have been used to derive the 1D-TF for several emission lines (e.g. $Ly\alpha$, C III] or $H\beta$) in NGC 5548 relative to the optical and UV continuum (Horne, Welsh & Peterson 1991), as well as the 1D-TF for $H\beta$ line in Mrk 590 relative to the optical continuum (Peterson 1993).

3 RELATIVISTIC ACCRETION DISK

We assume a flat, geometrically thin disk, orbiting a Schwarzschild black hole. The line emitting disk extends from r_i to r_o (measured in units of the gravitational radius $r_g = G\,M/c^2$) and is observed at an inclination i. To calculate the energy shift of a photon emitted by an infinitesimal surface element of the disk at r_{em} and ϕ_{em} (ϕ is the azimuthal angle from the line of nodes with $0^\circ < \phi < 180^\circ$ on the side nearer to the observer) we use the fully relativistic treatment described in the appendix of Fabian et al. (1989). The energy shift $1+z$ is obtained by the product of two terms: one term depends only on the emission radius and represents the strong field equivalent of the gravitational and transverse redshifts; this term becomes dominant at small radii. Besides the radius, the other term depends on the relative orientation of the velocity of the disk matter and on the direction of emission of photons reaching the observer and it can produce either a blueshift or a redshift. This term corresponds to the Doppler shift. The intensity and the profile of a line arising from the reprocessing of the central source radiation, depends also on geometry of the central source. In the following we consider two limiting cases: a point-like central source lying in the disk plane and a spherical source straddling the disk inner edge.

3.1 Point-like limit for the central source

We consider here a central source which is point-like, emits isotropically and lies in the disk plane. Even if the EW of lines originating from thin disk reprocessing of central source radiation is formally zero, this geometry is usually adopted to discuss lines arising from disk-like broad-line regions (e.g. Welsh & Horne 1991; Pérez et al. 1992), the dimensions of which are much larger than the dimensions of the central source.

We assume that the line emitting disk matter responds instantaneously to the photons from the central source and that the line photons freely propagate away from the disk. The specific line intensity of the disk is approximated by a δ−function, i.e. $I_{E_{em}} = \epsilon(r,\phi)\,\delta(E_{em} - E_0)$, where E_0 is the rest energy of the line and $\epsilon(r,\phi)$ the surface line emissivity of the disk; phenomena affecting the local line shape, such as line thermal broadening, blending or Compton scattering are therefore neglected. In stationary conditions, the radial dependence of the line emissivity can be derived self-consistently through numerical integration once a geometry for the central source and the disk is prescribed (Chen & Halpern 1989; George & Fabian 1991; Matt et al. 1991); in the limit of a point-like central source (i.e. small respect to the inner radius of the disk) lying in the disk plane, the line emissivity varies as r^{-3} (Mardaljevic, Raine & Walsh 1988).

Flux variations at the central source induce variations in the line emissivity with the delays introduced by light travel time effects. Neglecting general relativistic corrections, the flux emitted at $r_0 = 0$ and $\tau_0 = 0$ will be echoed, after a time delay τ, by the disk region defined by:

$$\tau - \tau_0 = \frac{r - r_0}{c}\,(1 - \sin i \sin\phi) \qquad (3)$$

Therefore, at any given time the echo front describes an ellipse with eccentricity $\sin i$ and the central source in the most distant focus. Moreover, the angular dependence of the echo front introduces a ϕ dependence of the line surface emissivity.

Some relevant delays can be identified, which characterise the propagation of the echo front on the disk: $\tau_1 = r_i/c\,(1 - \sin i)$, corresponding to the echo front entering the line emitting portion of the disk in the direction closest to the observer (i.e. $r = r_i$ and $\phi = 90^\circ$); $\tau_2 = r_i/c$, the delay at which the echo front reaches the line of nodes (i.e. $\phi = 0^\circ$ and $\phi = 180^\circ$) for $r = r_i$; and $\tau_3 = r_i/c\,(1 + \sin i)$, the delay at which the entire echo front is included in the line emitting region (i.e. $r = r_i$ and $\phi = 270^\circ$).

To calculate numerically the TFs and the line moments we adopt a short, rectangular impulse from the central source ($\Delta t = r_i/50\,c \ll r_i/c$), in order to approximate a δ−function variation*. With these assumptions, we compute the line profile produced in response by a disk for a range of times and for selected values of the inner radius r_i and inclination i. In order to investigate the strong and weak gravitational field regimes we adopt $r_i = 6\,r_g$ (the radius of the marginally stable orbit for a Schwarzschild black hole) and $r_i = 50\,r_g$, respectively. For the outer radius we use $r_o = 10^4 r_g$.

3.1.1 *Line moments evolution*

The line intensity response, $I_l(t)$, is shown in Fig. 1. For low inclinations ($i < 30^\circ$), the line intensity shows a single peak centred around a time delay of $\sim 7 - 9\,r_g/c$ for $r_i = 6\,r_g$ (Fig. 1a) and $\sim 60 - 80\,r_g/c$ for $r_i = 50\,r_g$ (Fig. 1b). For higher inclinations an additional peak forms, which reaches its maximum for time delays shorter than r_i/c. For $r_i = 50\,r_g$ the peak at short delays becomes dominant for $i \gtrsim 50^\circ$ (note that for a classical disk this happens for $i \gtrsim 40^\circ$). On the contrary for $r_i = 6\,r_g$ the peak at longer delays (τ_3) always dominates, because the gravitational and transverse

* Note however that, from a physical point of view, the shortest global variation at the central source, in absence of other effects, should be $\gtrsim 2\,r_g/c$. In this context we are interested in the TFs and therefore we adopt a shorter variation.

redshifts decrease the line intensity for small radii. We note that in the case of a point source above the disk plane the relative importance of the first peak is much reduced due to the different geometry echo front (Matt & Perola 1992).

Concerning the evolution of the line centroid energy, shortly after the echo front has entered the disk ($\tau > \tau_1$), a redshift is produced by gravitational and transverse shifts, which is more pronounced for smaller inner radii (see Fig. 2). For low values of i ($\lesssim 40^\circ$) the line centroid remains redshifted also for larger time delays, due to the small projected velocities. On the contrary, for higher values of i a blueshift appears for time delays of order τ_2, due to the increasing importance of Doppler boosting. The blueshift reaches its maximum shortly after than the time when the echo front goes beyond the line of nodes (τ_2). In a weak field approximation, it can be shown that the largest shifts in the echo front are produced for:

$$\sin\phi_M = \frac{1}{3}\Big(\frac{1}{\sin i} - \sqrt{\big(\frac{1}{\sin^2 i} + 3\big)}\Big) \qquad (4)$$

i.e. regions $\lesssim 20^\circ$ beyond the line of nodes (Stella 1990). The corresponding time τ_2', obtained by substituting $\sin\phi_M$ in Eq. 3, predicts quite accurately the maxima of the line centroid energy for $r_i = 50\,r_g$ (Fig. 2b); due to more pronounced photon deflection part of the accuracy is lost in the strong field case ($r_i = 6\,r_g$; Fig. 2a). As the echo front moves further outwards ($\tau \geq \tau_3$), the line centroid energy approaches unity for any inclination, due to the decreasing disk velocities and gravitational field.

In the weak field case ($r_i = 50\,r_g$), the line width rises after τ_1, reaches a maximum at τ_2' when the highest redshifts and blueshifts are also produced, and decays showing a flattening after τ_3, when the entire echo front is within the line emitting disk (Fig. 3b).

The strong gravitational field case presents instead with a significant difference (Fig. 3a): the largest widths are achieved close to the delay for which the entire echo front enters the disk (i.e. τ_3). This happens for any inclination. General relativistic effects are responsible for the maximum around τ_3; due to the strong light bending in the innermost disk regions ($\sim 6\,r_g$), photons reaching the observer from $\phi \sim 270^\circ$ are emitted locally at relatively small angles from the perpendicular to the disk. Surface projection and gravitational lensing effects therefore increase the flux from these regions, which are also characterised by a range of shifts (see also Matt, Perola & Stella 1993; Rouchi & Blandford 1994). For inclinations $i \gtrsim 60^\circ$, the relative importance of the regions which, according to Eq. 4, produce the highest shifts, increases and a local maximum is produced for delays $\sim \tau_2'$. We note that the evolution of the line centroid energy and line width are remarkably similar to the ones derived by Matt & Perola (1992) adopting a point-like central source above the disk plane.

3.1.2 2D-Transfer Function

By definition, the two-dimensional transfer function, $\psi(E,\,t)$, contains all the information on the evolution of the line profile and higher line moments and provides in principle the most powerful diagnostic of the line emitting disk region. Welsh & Horne (1991) and Pérez et al. (1992) calculated 2D-TFs for different geometries in the non-relativistic regime. In particular, they calculate the 2D-TF for a Keplerian disk in the non-relativistic regime and for a point-like central source lying in the disk plane: in the time delay-energy plane the 2D-TF has a symmetric bell shape (see Fig. 5 in Welsh & Horne 1991 and Fig. 6 in Pérez et al. 1992). The 1D-TF as well as higher line moments can be easily recovered from the 2D-TF by carrying out the relevant integration. (Integration of the 2D-TF over the time delay axis provides the stationary line profile).

Here we generalise these calculations in order to include general relativistic effects by using the approximations of the disk model described in Sect. 3. As expected the relativistic 2D-TF is strongly affected by relativistic effects for small delays and gradually approaches the classical limit as the echo front expands. Fig. 4a shows the 2D-TF for $i = 10^\circ$ and $r_i = 6\,r_g$. Because of the low disk inclination the projected velocities are small and the line photons are strongly redshifted: these effects produce a narrow 2D-TF at small delays. For higher delays regions with larger velocities are reached by the echo front and the 2D-TF rises monotonically approaching a profile centered around unity. It is apparent that the centroid of the profile grows monotonically for increasing delays (see also Fig. 2a), while the line width does not change appreciably (see also Fig. 3a).

For $i = 45^\circ$ (Fig. 4b) the projected velocities are higher and produce a broadening of the 2D-TF with respect to the low inclination case. The maximum of the 1D-TF corresponds to the edge of the internal ellipse that can be clearly seen in Fig. 4b, where a high value of the 2D-TF is achieved for blueshifted energies. The high intensity of the 2D-TF in the inner edge of the ellipse affects also the centroid energy, which achieves its maximum slightly before τ_3. The line width maximum is reached around τ_3 as a result of the contributions from the red part of the 2D-TF.

For $i = 80^\circ$ (Fig. 4c) an even stronger blue component of the 2D-TF develops, corresponding to the highest value of E_c and of the line width for $\tau \sim \tau_3$ (note that the maximum of E_c is for times slightly smaller than τ_3). This effect derives from gravitational light bending and surface projection effects.

In the weak field limit ($r_i = 50\,r_g$) the 2D-TF is still asymmetric (i.e. it is redshifted for every inclination for low inclinations and shows the characteristic enhancement in the blue horn near τ_3), but with a much smaller amplitude.

3.2 Bulge-like source

Several lines of evidence support the view that the dimensions of the central illuminating region in AGNs are not negligible in comparison with the emitting disk regions. Chen, Halpern & Filippenko (1989) have shown that reprocessing in a standard geometrically thin disk does not reproduce the properties of the $H\alpha$ and $H\beta$ lines in Arp 102B. Molendi et al. (1992) have shown that a modified self-irradiating disk, in which the central region is blown up into a bulge-like shape, can account for the optical and UV variability of NGC 5548. Finally, the strength of the fluorescence iron Kα lines in many AGNs (EW $\sim 100 - 200$ eV), requires that the accreting material subtends larger solid angles than the thin disk model allows (George & Fabian 1991; Matt et al.

1991). A possible solution to these problems is represented by a standard disk, the innermost regions of which have become geometrically thick possibly as a consequence of radiation-pressure related instabilities (e.g. Shapiro, Lightman & Eardley 1976; Wandel & Liang 1991). In order to investigate the response of the disk line emissivity to variations from an extended central source, we consider the limiting case of an optically thin spherical source of radius r_c that extends up to the disk inner radius (i.e. $r_c = r_i$). The emissivity law for this geometry, $\epsilon(r) = r^{q(r)}$, is calculated following the method outlined in Chen & Halpern (1989) (note that photons from the central source are assumed to propagate along straight lines). The value of $q(r)$ is < -3 for small radii and approaches $q \sim -3$ for larger radii ($r \gtrsim 3\,r_i$, see Fig. 5b in Chen & Halpern 1989). The fraction of radiation emitted by the central source which is intercepted by the disk is $\sim 25\%$ (Chen & Halpern 1989), about half than in the case of a point-like source above the disk plane as discussed by Matt & Perola (1992).

To calculate the line response a prescription for the fastest possible variation of the extended central source is also needed: we assume that a point in the volume of the central source varies its emission simultaneously with that of any other point. The intensity of the continuum variation that reaches a point in the disk depends on its distance, due to the different solid angle under which the central sphere is seen. In particular, the continuum intensity that reaches at a given time delay $\bar{\tau}$ (from the turning on of the central source) a point located at $(r,\, \phi)$ must be proportional to the area of the intersection between the central source and the sphere centred at $(r,\, \phi)$ of radius $\bar{\tau}\, c$. With this geometrical prescription we derive the intensity that an observer at infinity sees at a time delay $\tau = \bar{\tau}\,(1 - \sin\phi\,\sin i)$ (see Eq. 3), reaching the considered point:

$$I_c(r,\, \tau) = \frac{I_c^0\, \tau}{{r_c}^2}\left(\tau - \frac{\tau^2 + r^2 - {r_c}^2}{2\,r}\right) \qquad (5)$$

with I_c^0 the value of the maximum intensity.

For this geometry, the echo front has a finite thickness. Its boundaries, neglecting photon bending, are $\tau_{in} = \frac{r-r_c}{c}\,(1 - \sin i \sin\phi)$ and $\tau_{out} = \frac{r+r_c}{c}\,(1 - \sin i \sin\phi)$, which are ellipses with the most distant focus in the source equator at $\phi = 270°$ and $\phi = 90°$ respectively. The evolution of the echo front is characterised by two times: the first corresponds to the inner part of the echo front entering the disk ($\tau_1^{ext} = 0$); the second to the outer part of the echo front being fully included in the line emitting part of the disk ($\tau_3^{ext} = 2\,r_i/c\,(1 + \sin i)$).

3.2.1 *Line moments evolution*

Strictly speaking, TFs cannot be calculated for this geometry, as, by definition, they give the response to a $\delta-$like continuum variation. We calculate instead the response of the line moments to the assumed (shortest) variation of the central source. The evolution of the line intensity and the higher line moments is computed for the same set of parameters as in the limit of a point-like source limit. Note that, in all cases, the line response is instantaneous because the disk is assumed to straddle the equator of the central source (i.e. $r_i = r_c$).

The line intensity increases, giving rise to a broad maximum for time delays of $2 - 3\,r_c/c$ depending on inclination (see Fig. 5). This broad peak is clearly suggestive of the convolution of the 1D-TF for the point-like source model (see Fig. 1) with the assumed variation of the extended source (note however that the radial dependence of the line emissivity is somewhat different in the two models).

Similarly, the line centroid energy does not show rapid variations (Fig. 6), but retains the general properties described for the case of a point-like central source: depending on inclination, it increases monotonically to unity or gives rise to a single broad maximum and then decays ($i \gtrsim 40°$).

The line width shows a rapid rise to a *plateau* followed by a slow decay (Fig. 7). This *plateau* is characterised by a broad peak which is less pronounced for low inclinations in the weak field (Fig. 7b). Note that in the strong field case the line width peaks at higher time delays for increasing inclinations (Fig. 7a), whereas the opposite trend is apparent in the weak field case (Fig. 7b). This phenomenon clearly represent the equivalent of the characteristic feature of Fig. 3a.

In general, due to the longer intensity variation at the central source, the evolution of the line moments in the case of an extended central source presents with considerably smoother features compared to that of a point-like central source and, in particular, all sharp features are washed out.

4 DISCUSSION

In this paper, by extending previous works, we have calculated the response of a relativistic accretion disk to a continuum variation from a central illuminating source. We have considered two limiting geometries: a point-like central source and an extended spherical source straddling the inner edge of the disk. Realistic geometries are likely to be in between these two extreme cases. We have identified a number of characteristic features, which could help assessing the presence of an accretion disk and evaluating its parameters. If the central illuminating source is point-like and lies in the disk plane, a disk geometry can be recognised through some features of the response function which are *qualitatively* the same for relativistic and non-relativistic disks.

For a relativistic accretion disk the strength of the gravitational field in the vicinity of the central object introduces substantial changes in the characteristics of the response functions. In particular, for a point-like central source the second peak of the 1D-TF dominates for every disk inclination (Fig. 1a). The evolution of the line centroid energy is fully dominated by relativistic effects, showing a monotonic rise to the line rest energy for low inclinations and a broad blueshifted maximum for high inclinations (Fig. 2a). The response of the line width is affected by gravitational lensing and surface projection effects; its peaks corresponds to the time in which the whole echo front is within the line emitting region of the disk (τ_3). For high inclinations these effects become very evident (Fig. 3a; see also Matt & Perola 1992; Matt, Perola & Stella 1993).

For a spherical central source extending to the innermost disk regions, the stationary line profile remains basically unchanged, while the sharp features of the line intensity response and of higher line moments are smoothed out by the longer duration of the central source variations. The intensity response function shows a broad maximum for any inclination and disk inner radius. The evolution of the line centroid energy and width resembles that for a point-like central source, even if the sharp features are smoothed out.

The response of the line moments can be especially useful to derive geometric information in those cases in which the energy resolution of the detector is insufficient to study the detailed line profile. In principle, the reverberation technique allows not only to infer disk geometry but also to estimate disk parameters, by studying the different behaviour of the line intensity response function and higher line moments or, alternatively, the 2D-TF. Moreover, in the case in which the presence of an accretion disk and its parameters are inferred based on the stationary line profile, the reverberation properties can be used as a consistency check (Mannucci et al. 1992).

In Campana & Stella (1993) we calculated the stationary values of the line centroid energy and width for a relativistic Keplerian accretion disk with an extended central source. Clearly the information of these stationary values is not sufficient to constrain univocally the disk parameters. To further constrain these parameters and derive an absolute scale length related to the light propagation time delays, different approaches can be adopted. One of them consists in selecting the values of the line centroid energy and width corresponding to the time delay at which the response of the line intensity reaches its maximum (τ_{max}). The dependence of $E_c(\tau_{max})$ and $W(\tau_{max})$ on the disk parameters r_i and i is stronger than the stationary values of E_c and W (compare Fig. 8 with Fig. 1 in Campana & Stella 1993). Alternatively the maximum and/or minimum values of the response of the line centroid energy and the maximum of the response of the line width can be considered. Note however that these values are somewhat dependent on the geometry assumed for the central source (compare Figs. 2 and 3 with Figs. 6 and 7). The latter prescription is similar to the one proposed by Matt & Perola (1992). These methods allow to constrain the disk parameters and derive the mass of the central object by combining the absolute time measurement of τ_{max} (or the delay for which the maximum of $E_c(t)$ and/or $W(t)$ is reached) with the relative measurement in units of r_g/c inferred from the line response or the characteristics of the stationary line profile.

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FIGURE CAPTIONS

Fig. 1. Line intensity response of a relativistic Keplerian accretion disk to a short rectangular flux variation produced at a point-like central source as a function of time delay in units of $r_g/c = G\,M/c^3 = 4.9 \times 10^{-6} M/\,M_\odot$ s. The first panel (a) refers to an inner radius $r_i = 6\,r_g$, while the second (b) to $r_i = 50\,r_g$; different curves correspond to different inclinations. All curves are normalised to a maximum value of 1. Here and in Figs. 2 and 3 the characteristic delays discussed in the text (τ_1, τ_2' and τ_3) are indicated with different symbols.

Fig. 2. Response of the line centroid energy (normalised to the rest energy) of a relativistic disk to a short rectangular flux variation at a point-like central source as a function of time delay. Inner radii and inclinations are as in Fig. 1.

Fig. 3. Response of the line width (normalised to the rest energy) of a relativistic disk to a short rectangular flux variation at a point-like central source as a function of the time delay. Inner radii and inclinations are as in Fig. 1.

Fig. 4. 2D-TF for a relativistic accretion disk orbiting a point-like central source. The disk inner radius is $r_i =$

$6\,r_g$. Panel (a) shows the 2D-TF for an inclination angle $i = 10^\circ$, panel (b) for $i = 45^\circ$ and panel (c) for $i = 80^\circ$.

Fig. 5. Line intensity response of a relativistic accretion disk to a short variation produced by an extended source (see text) as a function of time delay τ in units of r_g/c. Panel (a) refers to $r_i = r_c = 6\,r_g$, panel (b) to $r_i = r_c = 50\,r_g$. Different curves correspond to different inclinations.

Fig. 6. Response of the line centroid energy (normalised to the rest energy) of a relativistic disk to a short variation of an extended central source as a function of the time delay. Inner radii and inclinations are as in Fig. 5.

Fig. 7. Response of the line width (normalised to the rest energy) of a relativistic disk to a short variation of an extended central source as a function of the time delay. Inner radii and inclinations are as in Fig. 5.

Fig. 8. Normalised line centroid energy (panel a) and normalised line width (panel b) versus inclination for a time delay corresponding to the maximum of the line intensity response, for an extended central source. Different curves correspond to different inner radii ($r_i = 6, 8, 10, 12.5, 15, 17.5, 20, 25, 30, 40, 50, 100\ \ r_g$). Panel c gives the time delay (in units of r_g/c) corresponding to the maximum of the line intensity versus inclination.

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