POLARIZATION OF NEUTRON STAR SURFACE EMISSION – WHAT DO WE ACTUALLY MEASURE?

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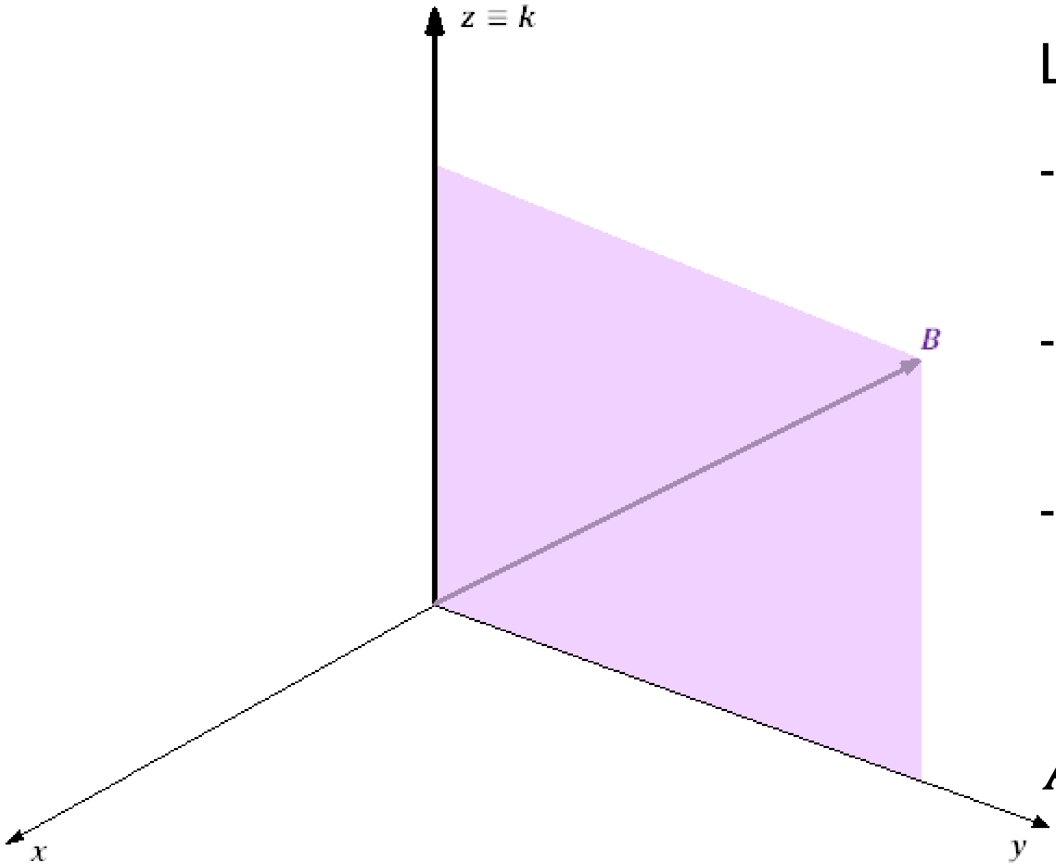
Outline

- ❑ Geometrical effects
- ❑ Vacuum polarization effects
- ❑ Results
- ❑ Conclusions

Geometrical effects

Photon polarization modes

- Due to strong magnetic fields, photons emitted by the NS surface are expected to be linearly polarized in the X- or O-modes



Local frame (x, y, z):

- $\hat{z}$ axis aligned with the photon propagation direction $\hat{k}$
- $\hat{x}$ axis perpendicular to the k-B plane
- $\hat{y} = \hat{k} \times \hat{x}$

$$\boldsymbol{E} = \boldsymbol{A}(z) e^{i(k_0 z - \omega t)}$$

$$\boldsymbol{A} = \left(A_x, A_y\right) = \left(a_x e^{-i\varphi_x}, a_y e^{-i\varphi_y}\right)$$

Photon polarization modes

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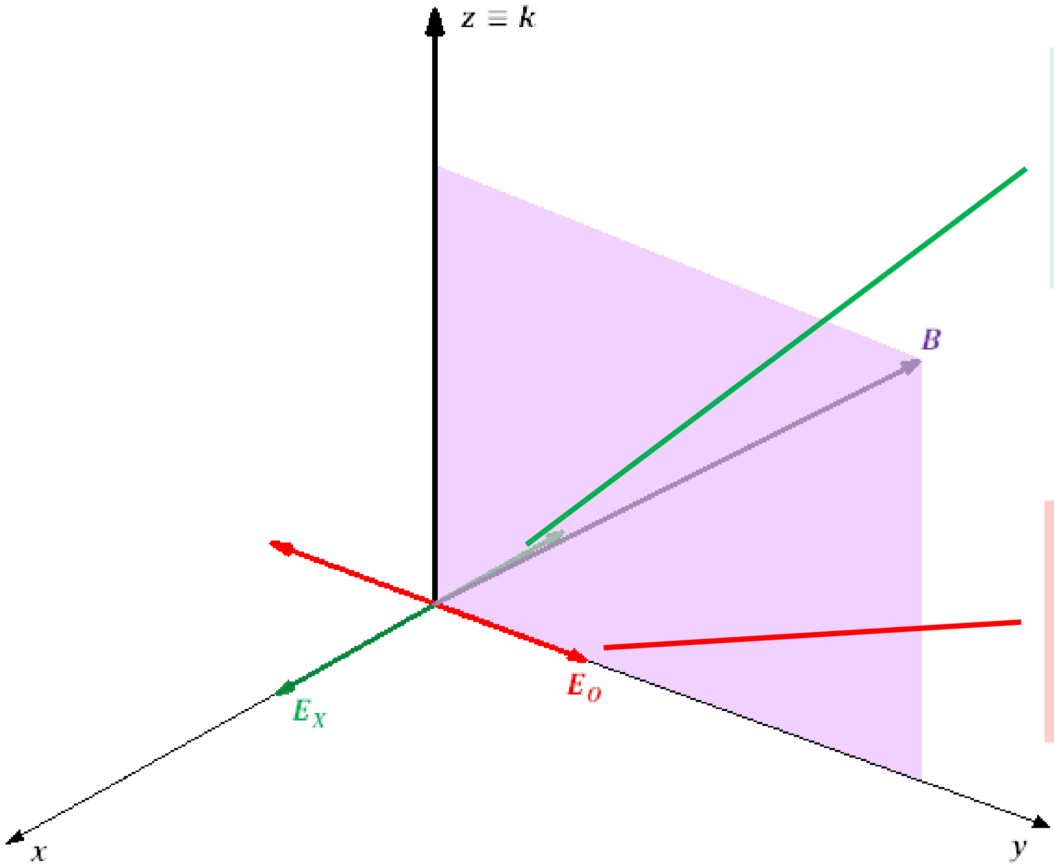


Electric field for X-mode photons oscillates along $\hat{x}$

Electric field for O-mode photons oscillates along $\hat{y}$

Stokes parameters

- A convenient way to describe polarized radiation is through the Stokes parameters (that are additive):

$$\mathcal{I} = A_x A_x^* + A_y A_y^* = a_x^2 + a_y^2$$
$$\mathcal{Q} = A_x A_x^* - A_y A_y^* = a_x^2 - a_y^2$$
$$\mathcal{U} = A_x A_y^* + A_y A_x^* = 2 a_x a_y \cos(\varphi_x - \varphi_y)$$
$$\mathcal{V} = i\left(A_x A_y^* - A_y A_x^*\right) = 2 a_x a_y \sin(\varphi_x - \varphi_y)$$

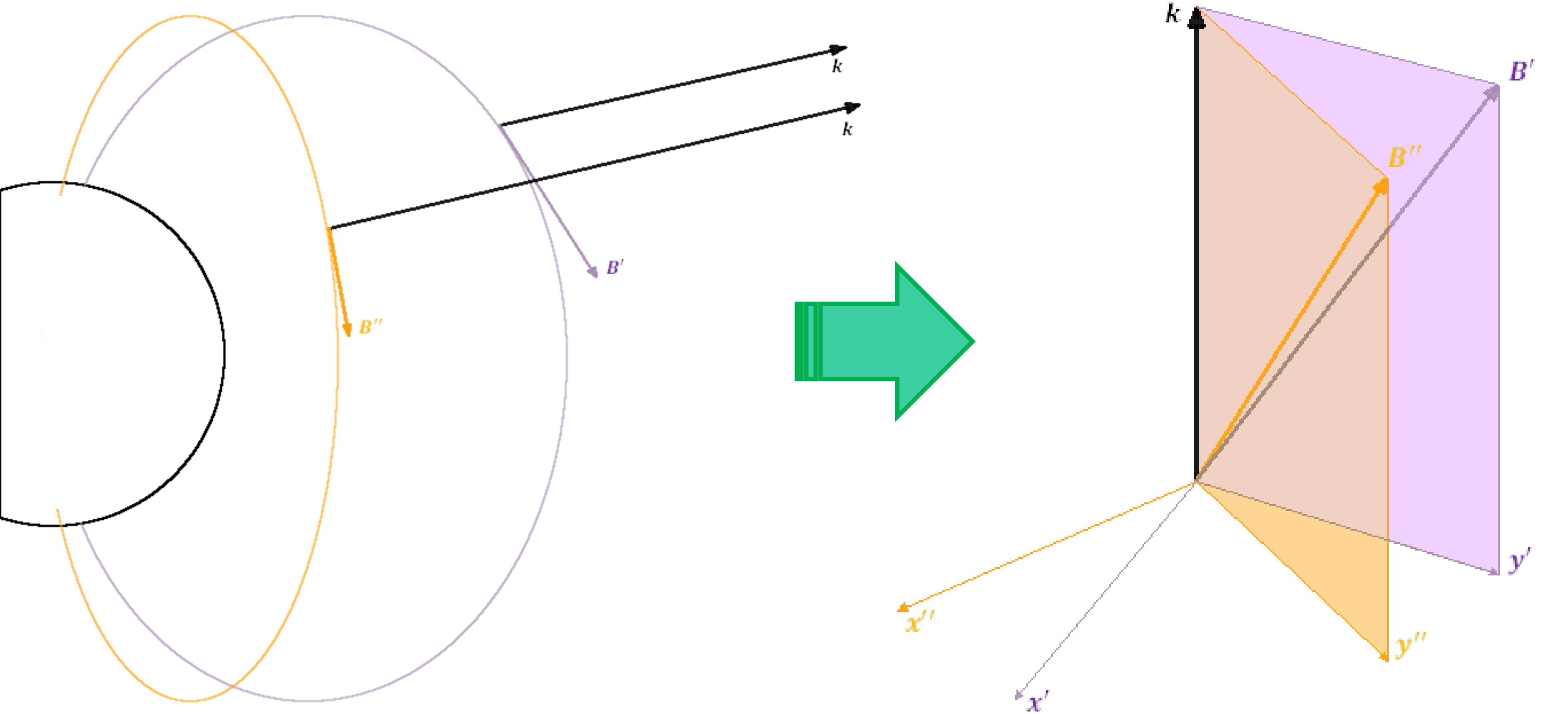
- Normalizing to the total intensity:

$$\begin{pmatrix} \bar{\mathcal{Q}} \\ \bar{\mathcal{U}} \\ \bar{\mathcal{V}} \end{pmatrix}_X = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \bar{\mathcal{Q}} \\ \bar{\mathcal{U}} \\ \bar{\mathcal{V}} \end{pmatrix}_O = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Stokes parameters rotation

- The local frame (x, y, z) is in general different for each photon

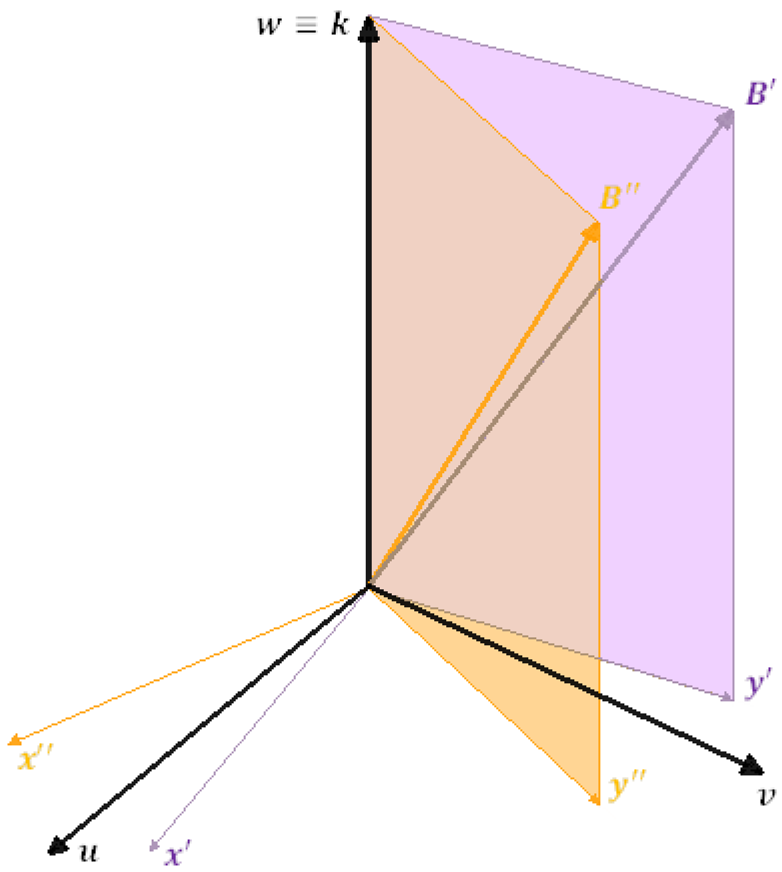
Stokes parameters rotation



Stokes parameters rotation

Polarimeter frame (u, v, w):

- $\hat{w}$ axis aligned with the photon propagation direction $\hat{k}$ (LOS)
- $\hat{u}$ and $\hat{v}$ axes arbitrarily chosen in the plane perpendicular to $\hat{w}$

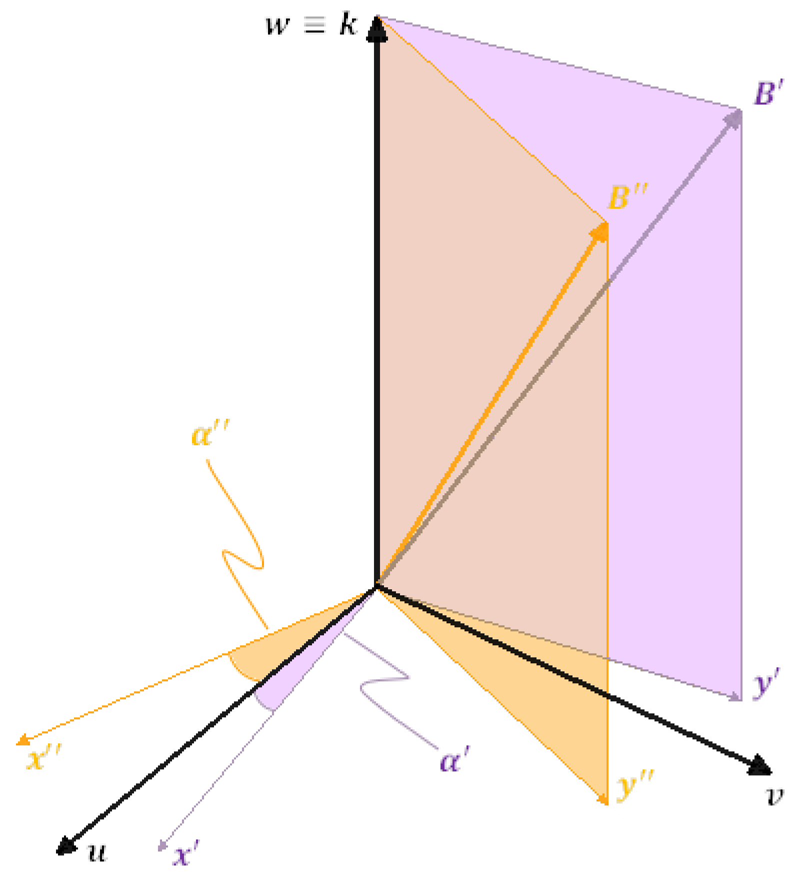


Stokes parameters rotation

Polarimeter frame (u, v, w):

- $\hat{w}$ axis aligned with the photon propagation direction $\hat{k}$ (LOS)
- $\hat{u}$ and $\hat{v}$ axes arbitrarily chosen in the plane perpendicular to $\hat{w}$
- the local frames (x', y', z') and (x'', y'', z'') have to be rotated by the angles

$$\alpha' = \hat{u} \cdot \hat{x}'$$
$$\alpha'' = \hat{u} \cdot \hat{x}''$$



Stokes parameters rotation

- The local frame (x, y, z) is in general different for each photon

- Under a rotation by an angle α_i the Stokes parameters transform as:

$$I_i = \bar{\mathcal{I}}_i \qquad Q_i = \bar{\mathcal{Q}}_i \cos(2\alpha_i) + \cancel{\bar{\mathcal{U}}_i \sin(2\alpha_i)}$$

$$V_i = \bar{\mathcal{V}}_i \qquad U_i = \cancel{\bar{\mathcal{U}}_i \cos(2\alpha_i)} - \bar{\mathcal{Q}}_i \sin(2\alpha_i)$$

- Hence the Stokes parameters associated to the whole radiation are given by:

$$Q = \Sigma_i^{NX} \cos(2\alpha_i) - \Sigma_i^{No} \cos(2\alpha_i) \qquad U = \Sigma_i^{No} \sin(2\alpha_i) - \Sigma_i^{NX} \sin(2\alpha_i)$$

Polarization observables

- The polarization properties of NS thermal emission can be described by polarization fraction and polarization angle

$$\Pi_{\mathrm{L}} = \frac{\sqrt{Q^2 + U^2}}{I}$$

$$\chi_{\mathrm{p}} = \frac{1}{2}\arctan\left(\frac{U}{Q}\right)$$

At the surface

$$\Pi_{\mathrm{L}}^{0} = \frac{|N_X - N_O|}{N}$$

Polarization observables

- The polarization properties of NS thermal emission can be described by polarization fraction and polarization angle

- By substituting the expression of Q and U for the whole radiation one obtains

$$\Pi_{\mathrm{L}} = \frac{1}{N}\Big[N + 2\,\Sigma_i^{N_X}\Sigma_{k>i}^{N_X}\cos\,(2\alpha_i - 2\alpha_k) + 2\,\Sigma_j^{N_O}\Sigma_{r>j}^{N_O}\cos\left(2\alpha_j - 2\alpha_r\right)$$

$$-2\,\Sigma_i^{N_X}\Sigma_j^{N_O}\cos\left(2\alpha_i - 2\alpha_j\right)\Big]^{1/2}$$

$$\chi_{\mathrm{p}} = \frac{1}{2}\arctan\left[-\frac{\Sigma_i^{N_X}\sin(2\alpha_i) - \Sigma_j^{N_O}\sin(2\alpha_j)}{\Sigma_i^{N_X}\cos(2\alpha_i) - \Sigma_j^{N_O}\cos(2\alpha_j)}\right]$$

Polarization observables

- The polarization properties of NS thermal emission can be described by polarization fraction and polarization angle
- By substituting the expression of Q and U for the whole radiation one obtains
- Only in the case $\alpha_i = \alpha_j \;\; \forall\, i, j$ the observed Π_{L} and χ_{p} coincide with the intrinsic ones

$$\Pi_{\mathrm{L}} = \frac{1}{N}\left[N + N_X(N_X - 1) + N_O(N_O - 1) - 2N_X N_O\right]^{1/2} = \frac{|N_X - N_O|}{N}$$

$$\chi_{\mathrm{p}} = \frac{1}{2}\arctan\left[-\frac{(N_X - N_O)\sin(2\alpha)}{(N_X - N_O)\cos(2\alpha)}\right] = -\alpha$$

Vacuum polarization effects

Photon polarization in strong magnetic fields

- According to QED photons can temporarily convert into virtual $e^{\pm}$ pairs, modifying the $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ tensors of the vacuum

- The evolution of the polarization modes for photon propagating in vacuo is governed by the following system of differential equations (Heyl & Shaviv, 2002; Fernández & Davis, 2011)

$$\frac{d\bar{Q}}{dz} = -\frac{k_0\delta}{2}(2P\bar{V})$$

$$\frac{d\bar{U}}{dz} = -\frac{k_0\delta}{2}(N-M)\bar{V}$$

$$\frac{d\bar{V}}{dz} = \frac{k_0\delta}{2}[2P\bar{Q} + (N-M)\bar{V}]$$

$k_0 = \omega/c$

$\delta \propto B^2$

z coordinate along the LOS

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$$\frac{d\bar{V}}{dz} = \frac{k_0\delta}{2}[2P\bar{Q} + (N-M)\bar{V}]$$

$$\ell_A = \frac{2}{k_0\delta} \sim B^{-2}E^{-1}$$

$$\ell_B = \frac{B}{\left|\hat{k}\cdot\nabla B\right|} \sim r$$

Pho

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v ... um
- T
p ... of
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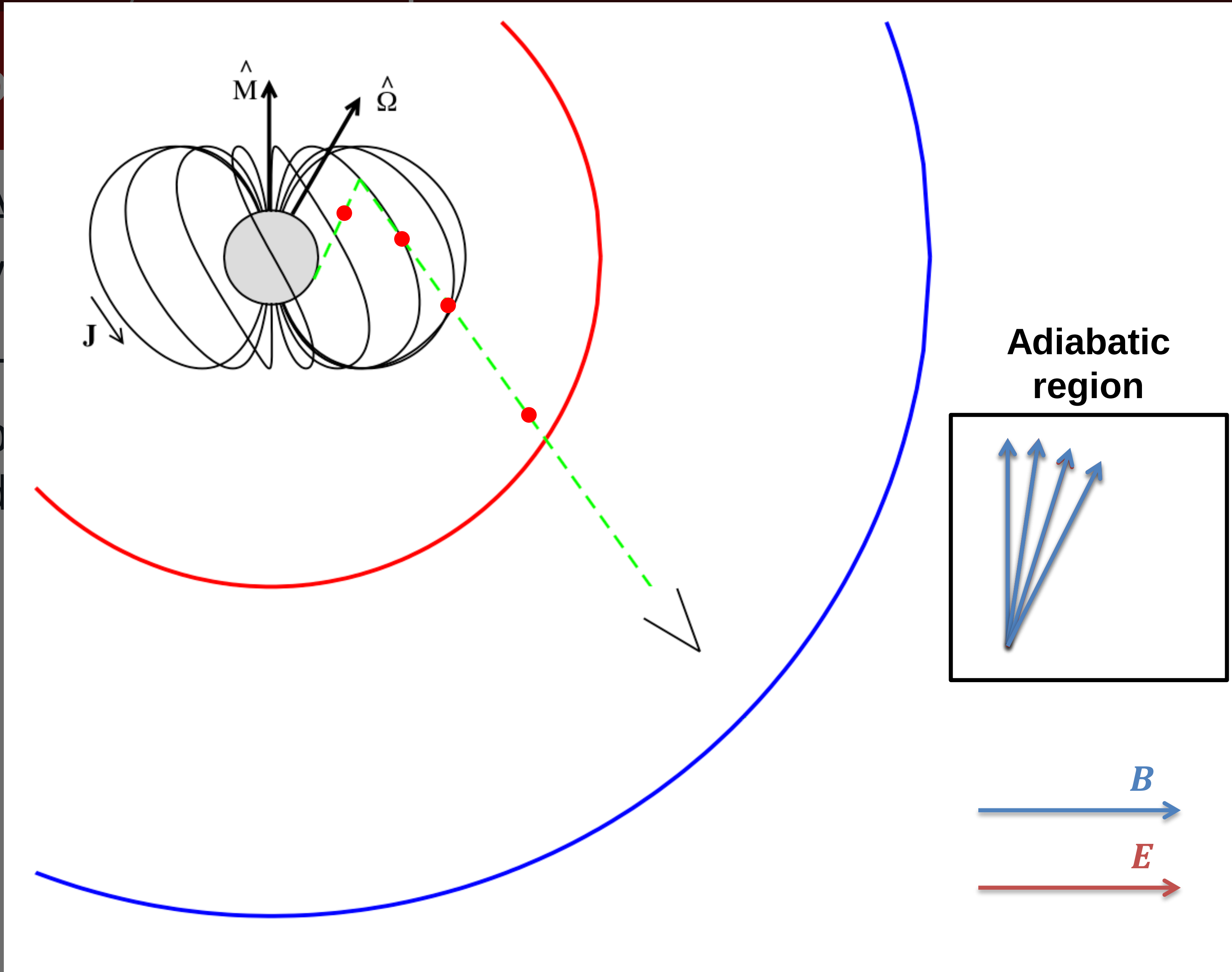

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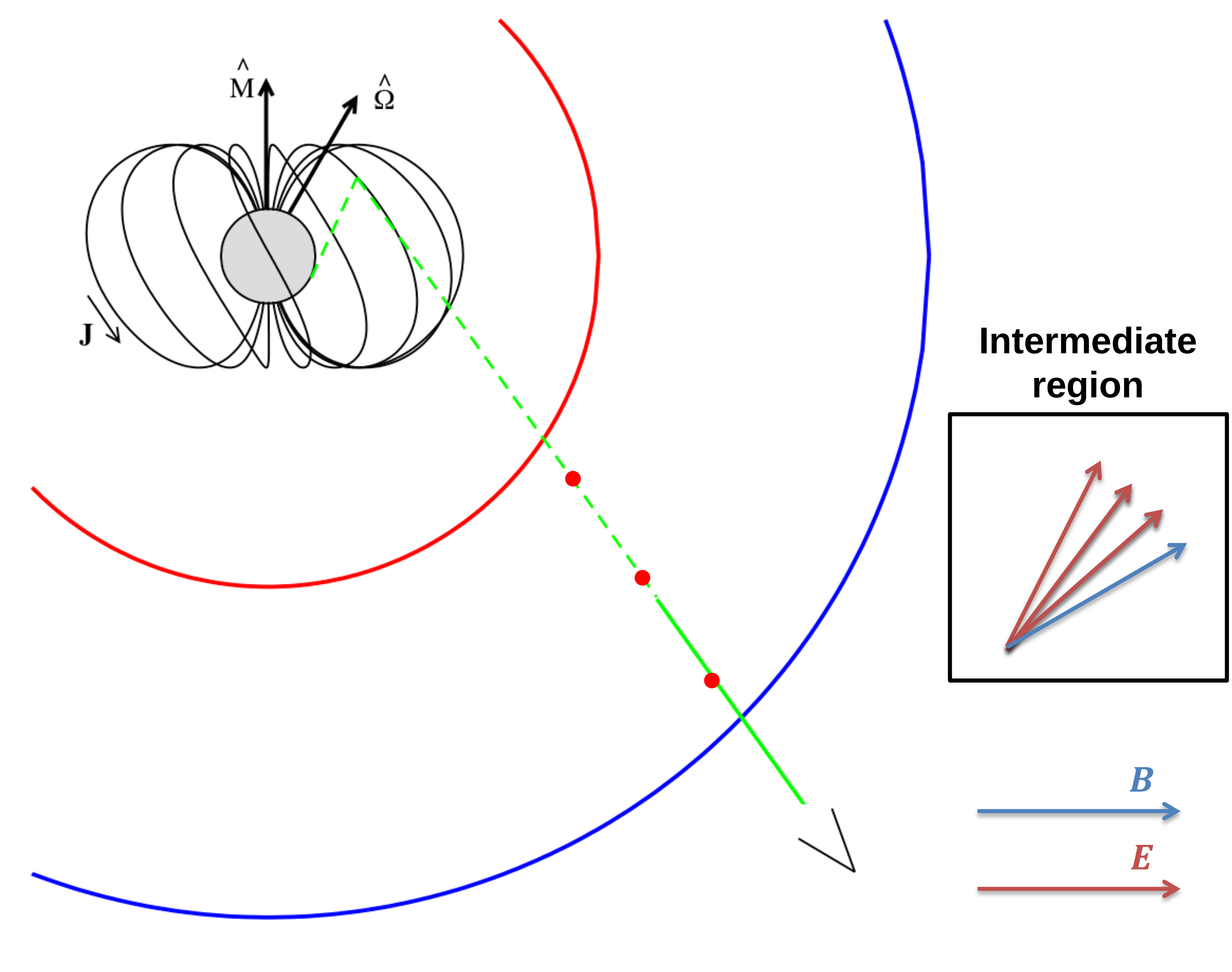

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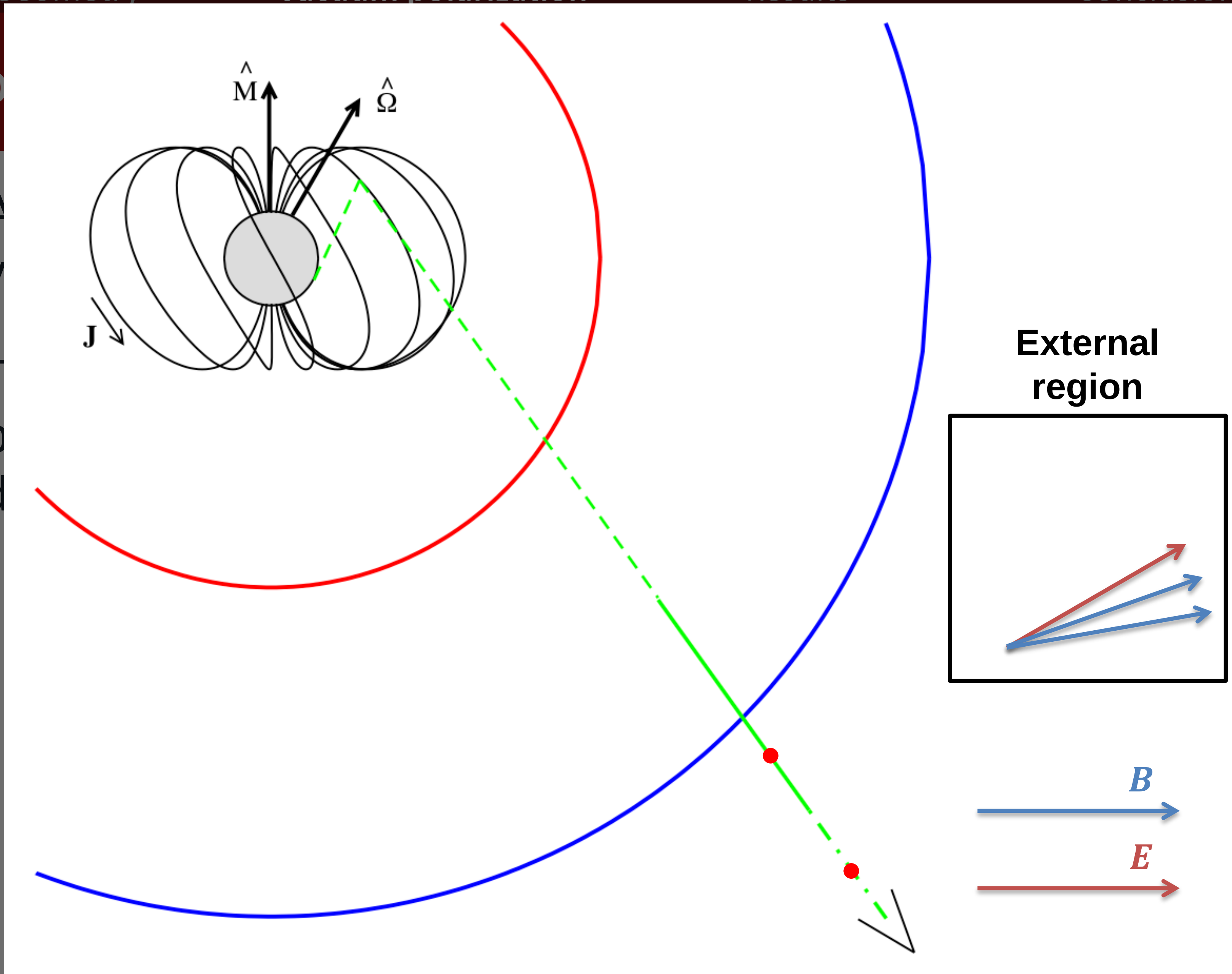


Simplified approach

- The integration of the full ode system (Heyl, Shaviv & Lloyd, 2003; Fernández & Davis, 2011; Taverna et al. 2014) is quite time consuming and it is not particularly suited to study the dependences of Π_{L} and χ_{p} on the various parameters
- We resort to a simpler, approximated treatment in which only the adiabatic region and the external one are included, divided by a sharp edge located at the adiabatic radius r_a

$$\ell_A = \ell_B \quad \Rightarrow \quad r_a \simeq 4.8 \left(\frac{B_p}{10^{11}\ \mathrm{G}} \right)^{2/5} \left(\frac{E}{1\ \mathrm{keV}} \right)^{1/5} R_{NS}$$

- The rotation of Stokes parameters can be carried out at r_a

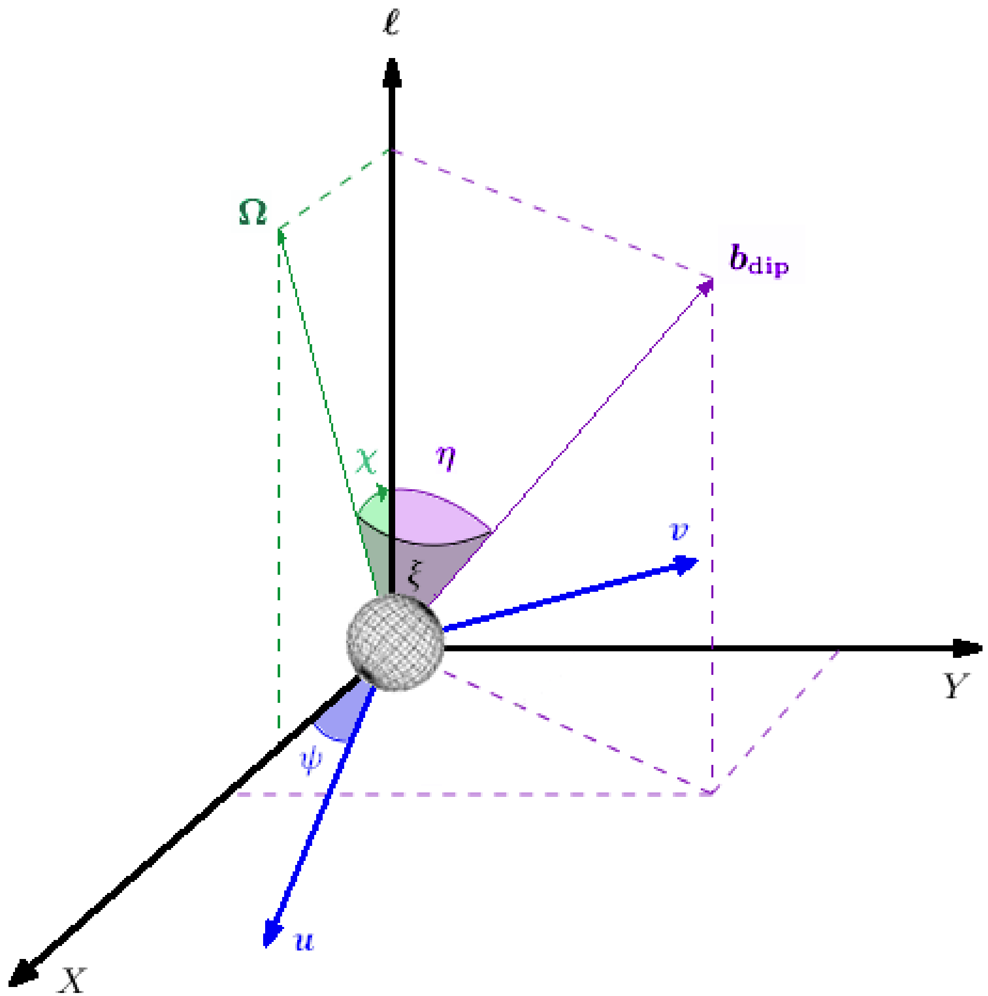
Results

Numerical implementation

- LOS reference frame

- angle χ between ℓ and $\mathbf{\Omega}$
- angle ξ between $\boldsymbol{b}_{\mathrm{dip}}$ and $\mathbf{\Omega}$
- angle ψ between $\boldsymbol{u}$ and $\boldsymbol{X}$

$$\alpha = \alpha(\chi, \xi, \psi, \Theta_{\mathrm{s}}, \Phi_{\mathrm{s}})$$



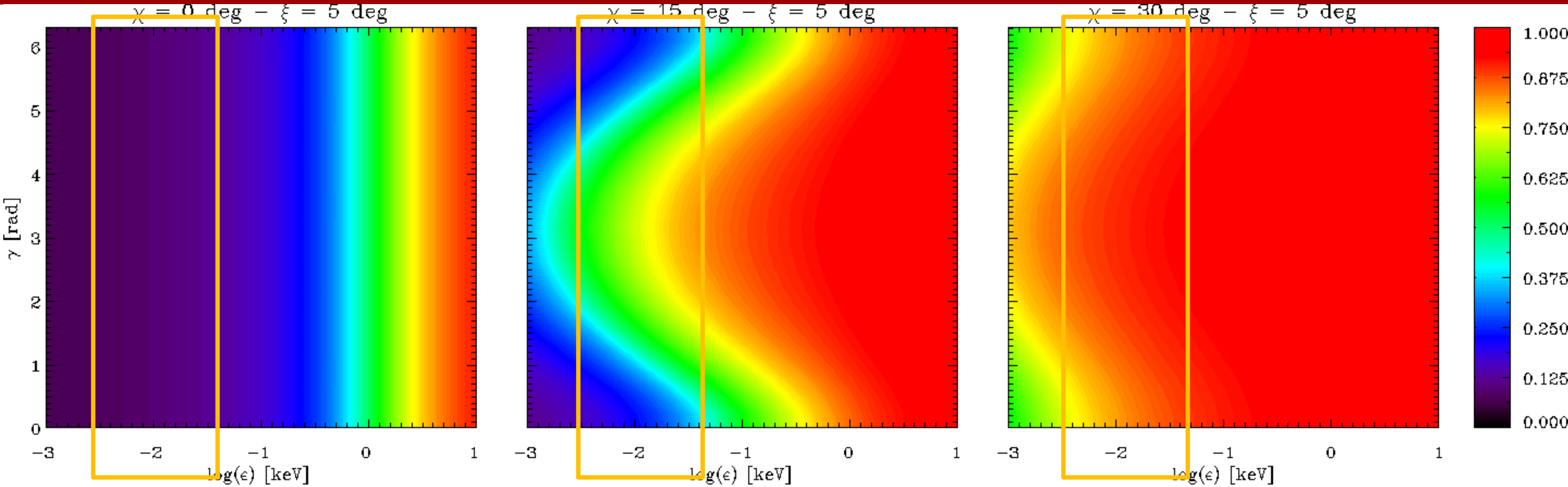
Basic assumptions

- The NS emits thermal radiation following an isotropic BB distribution

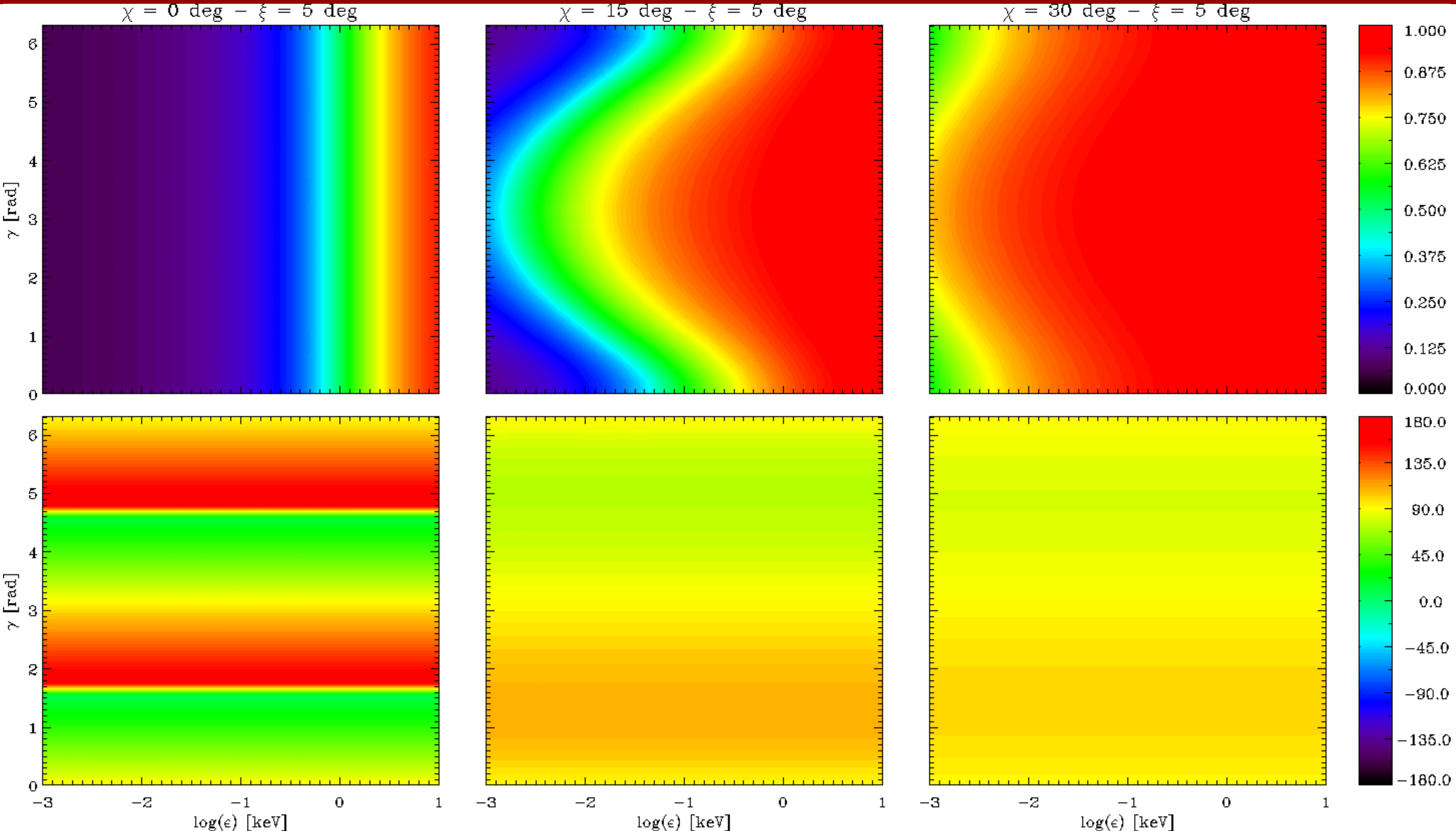
$$n = \frac{I}{E} = \frac{2}{h^2 c^2} \frac{E^2}{e^{E/kT} - 1}$$

- Seed photons are 100% polarized in the X-mode
- Dipolar and globally twisted magnetic fields
- General relativistic corrections are included
  - Magnetic dipole field corrections
  - Relativistic ray-bending

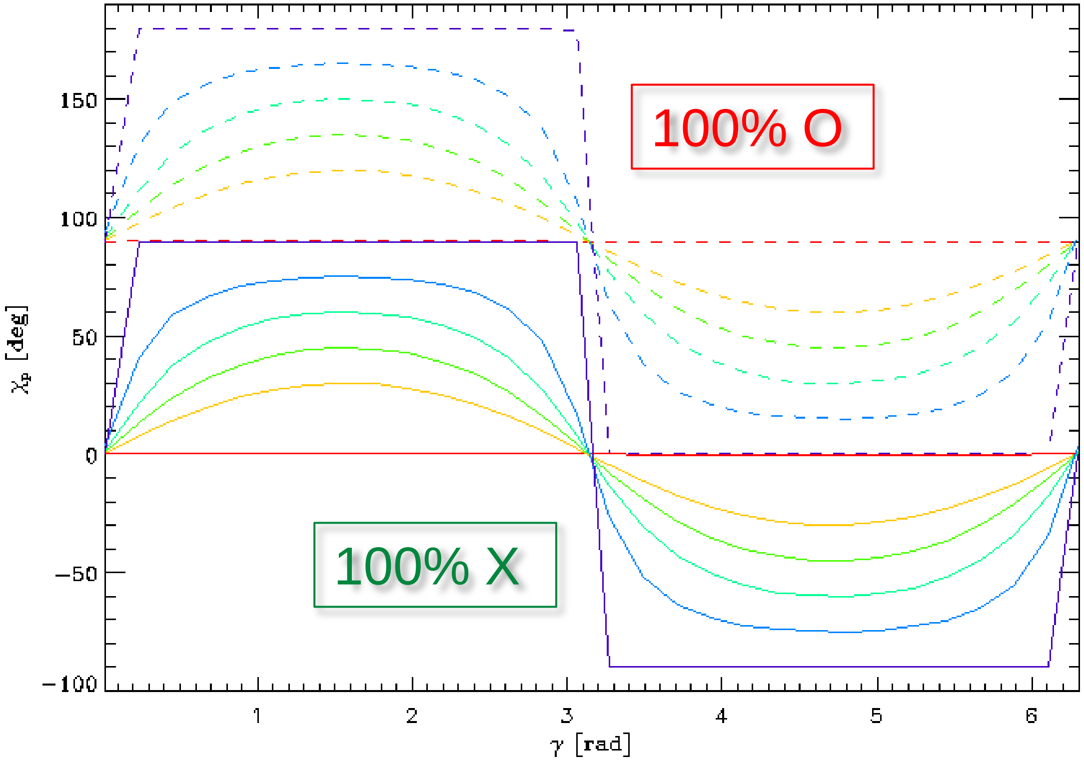
Simulations



Simulations



Polarization swing



$$\boldsymbol{\psi = 90°}$$

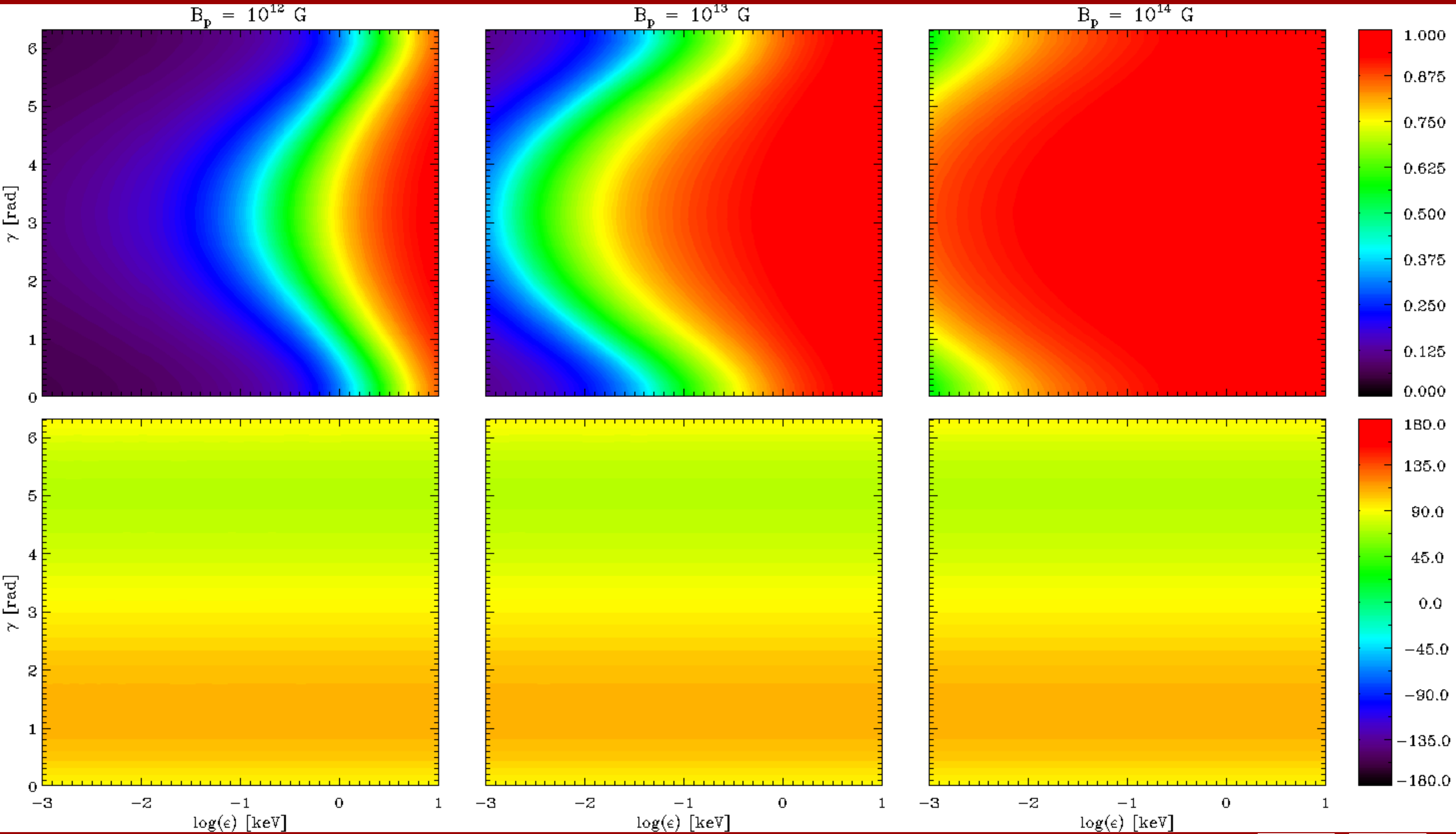
$\chi = 90°$

$\xi = 0, 30°, 45°, 60°, 75°, 90°$

- The polarization angle sweeps the entire range [0,180°] for values of χ, ξ such that the polar regions are always in view during the star rotation

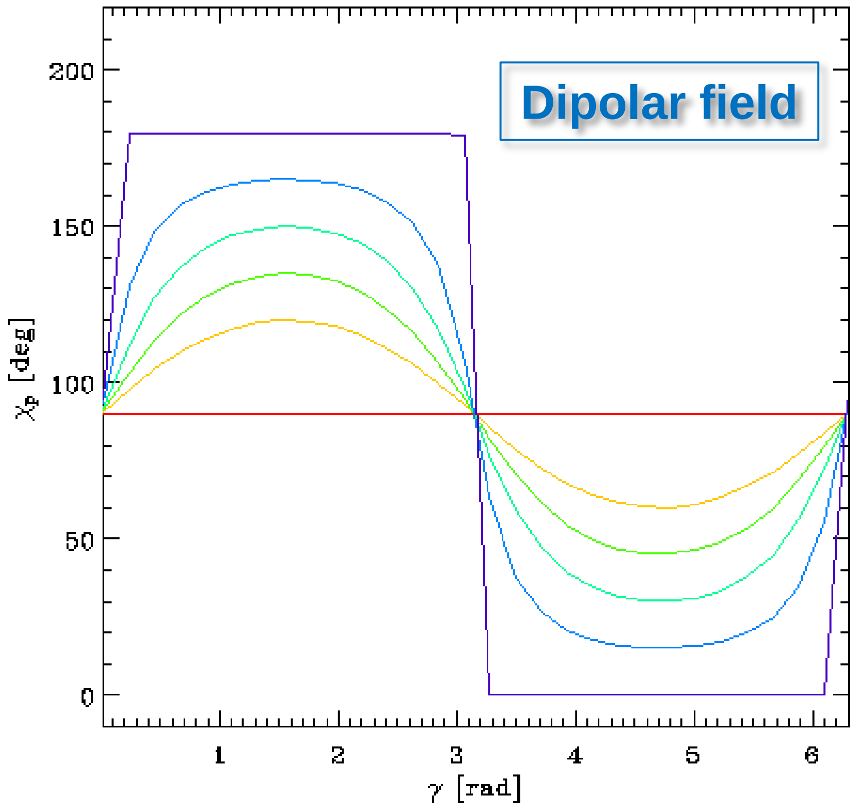
- Only an independent estimate of χ and ξ allows to understand if photons are polarized in the X- or in the O-mode

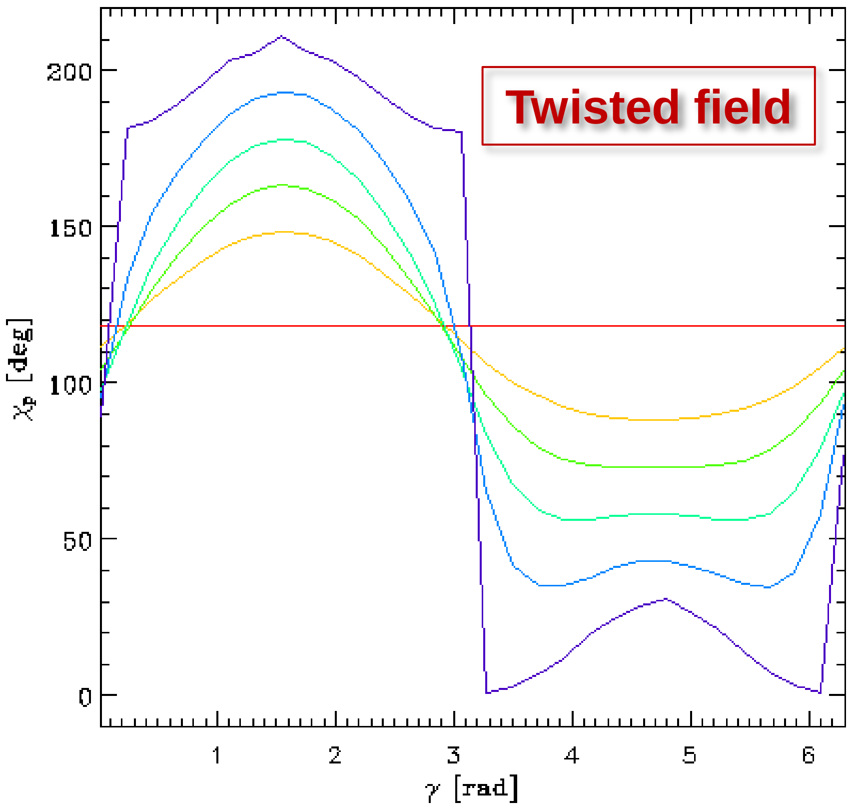
Dependence on the magnetic field strength



Dependence on the magnetic field topology

- The polarization angle distribution is much more sensitive to the geometry of the magnetic field





Conclusions

Conclusions

- Systematical study of the effects of Stokes parameters rotation and vacuum polarization on the observed polarization signals, on varying the values of typical parameters (χ, ξ, E, B, $\Delta\phi_{N-S}$)
- Our fast code allows to considerably save computational time, without loss of accuracy in re-obtaining the qualitative results, where available, of previous works (Heyl, Shaviv & Lloyd, 2003; Lai & Ho, 2003; van Adelsberg & Perna, 2003)
- The study of these effects turns out to be particularly crucial in relation to recent proposals of polarimetry missions (XIPE see E. Costa talk, IXPE)

Polarization of neutron stars surface emission:
a systematic analysis

R. Taverna, R. Turolla, D. Gonzalez Caniulef
S. Zane, F. Muleri, P. Soffitta, 2015,
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