Back to a (semi)-analytic view of the inner structure of Pulsar Wind Nebulae

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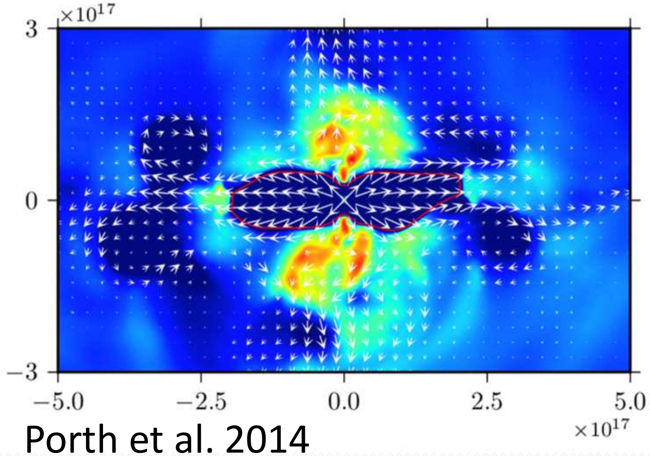
The two extremes of PWN modelling

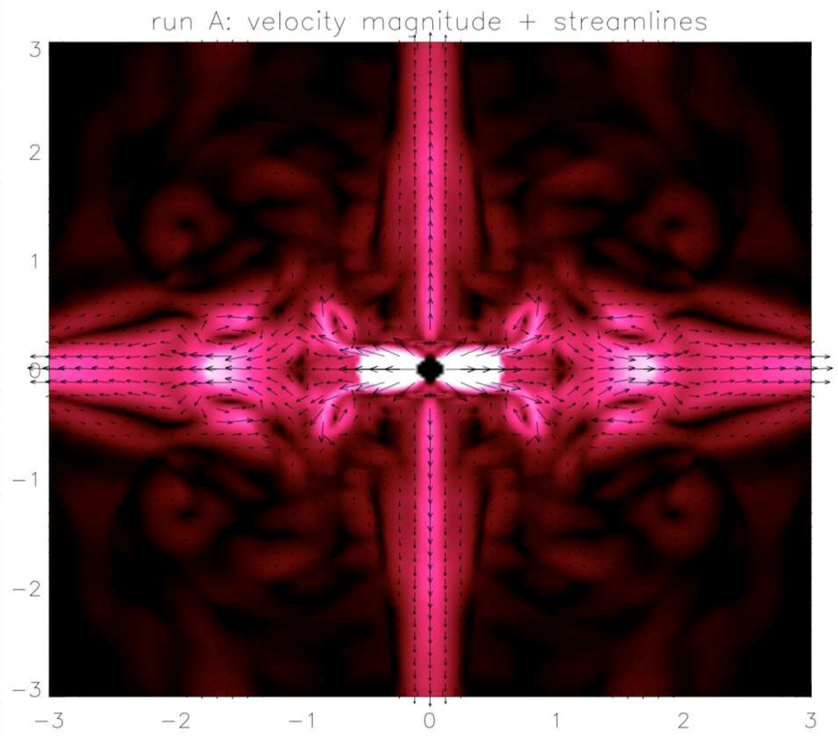
- Analytic one-zone models (0-D models)

  Evolution of the energy of magnetic fields and particles
  due to adiabatic and synchrotron losses (Pacini & Salvati 1973, …)

- Numerical simulations
  (2-D, or 3-D time dep. models)



Porth et al. 2014



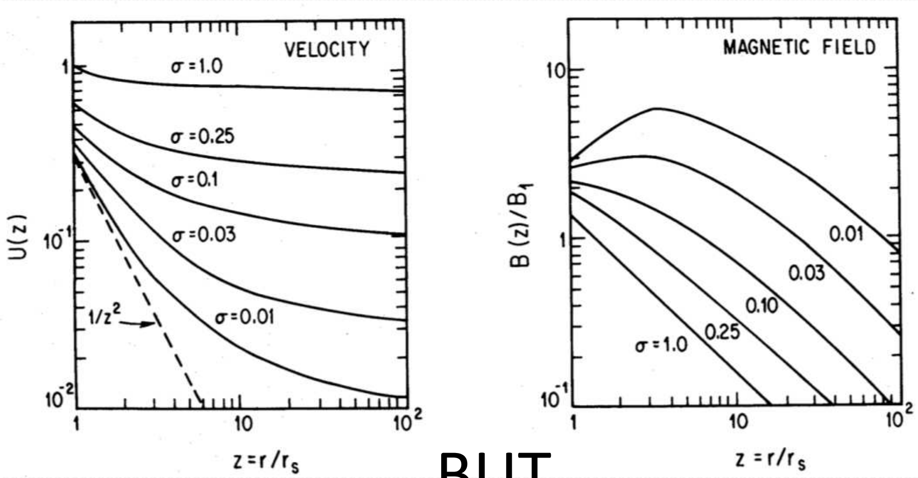
Del Zanna et al. 2006

Kennel & Coroniti approach (1984)

"IN MEDIO STAT DEFECTUM"

- Physically meaningful assumptions (Def. of TS, highly relativistic MHD flow, …) → PWN structure

σ = magnetization



BUT

- Spherical symmetry does not allow matching the data

ANY ROOM FOR MORE DETAILED ANALYTIC MODELS ?

Why (semi-)analytic models?

- Philosophical view: what does it mean “understanding”

  Detailed numerical models are anyway “numerical experiments”

- More practical view: expected to…
  - allow (simplified) modelling with a smaller computing time
  - allow drawing analytical relations between quantities
- Possible weaknesses:
  - more simplified physics and structure

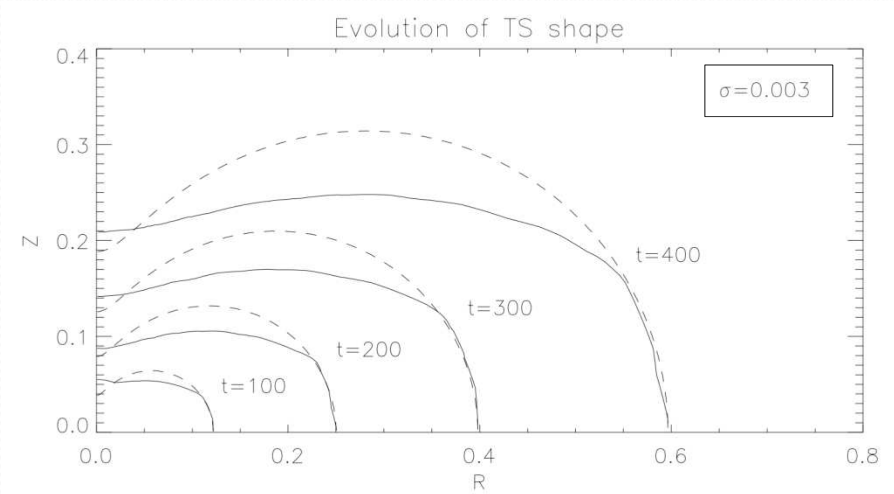
    (e.g. only laminar, steady state flows)
  - limited to specific spatial regions
  - some input required from results of numerical models
- Possible use:
  - as a tool, complementary to numerical simulations

IS IT WORTH TRYING ?

Reference numerical model

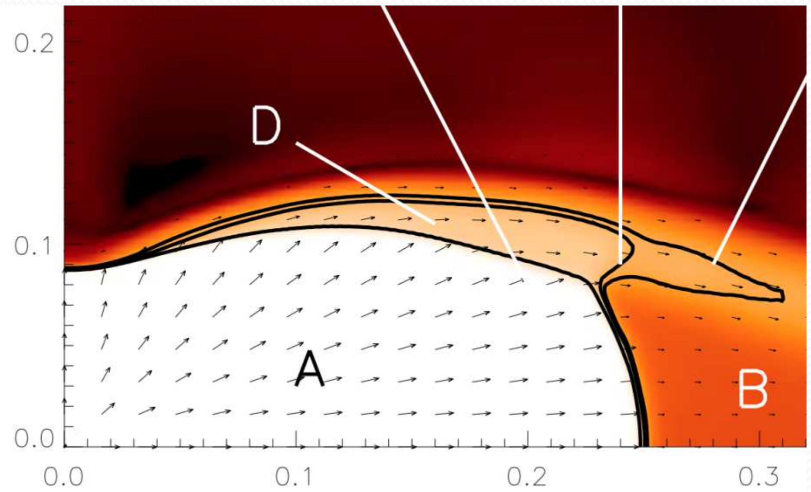
(Del Zanna et al. 2004)

- Low magnetization (σ=0.003)



Time evolution of the TS shape

Flow structure around the TS



HERE ONLY FORM FACTORS

Basics of our model (part 1)

- Start from a given shape of the TS (← a numerical model)
- Use jump equations to derive immediately downstream quantities (e.g. Komissarov & Lyutikov 2011)

$\rho\, u_{\parallel} = \mathrm{const};$	Conservation of mass
$(w + b^2/4\pi)\gamma u_{\parallel} = \mathrm{const};$	Conservation of energy
$(w + b^2/4\pi)u_{\parallel}^2 + p + b^2/8\pi = \mathrm{const};$	Conservation of momentum parallel to V_{sh}
$(w + b^2/4\pi)u_{\parallel}u_{\perp} = \mathrm{const};$	Conservation of momentum perpendicular to V_{sh}
$b\, u_{\parallel} = \mathrm{const},$	Conservation of magnetic flux (*b* is TOROIDAL)

  - ρ and b in the comoving frame; $w = \rho c^2 + 4p$ Enthalpy; (γ, u) = 4-velocity

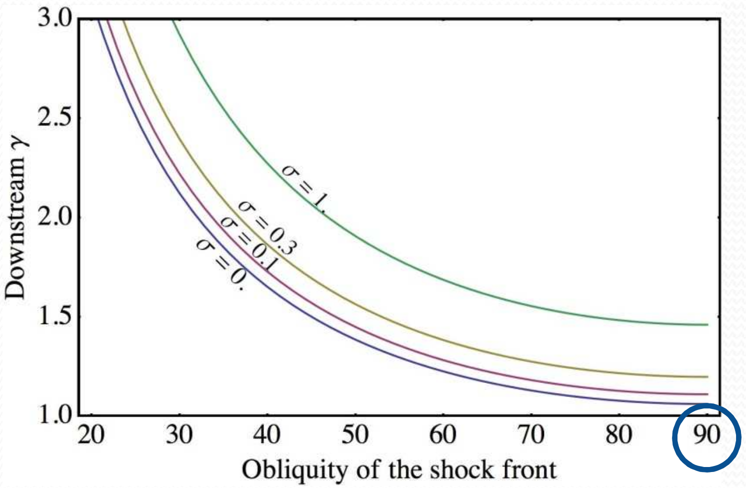
- With respect to K&C, treatment not limited to u parallel to V_{sh} (oblique shocks)

Effects of the shock obliquity

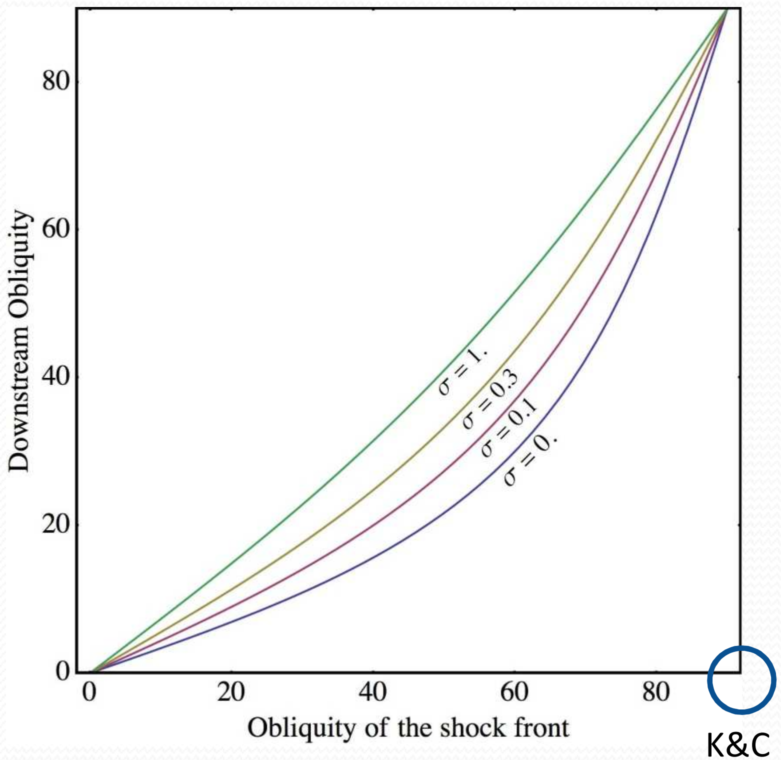
Incidence angle: 0° = grazing; 90° perp. to the shock front

For oblique shocks:

- Higher downstream γ
- Downstream direction further away from the shock front normal



K&C

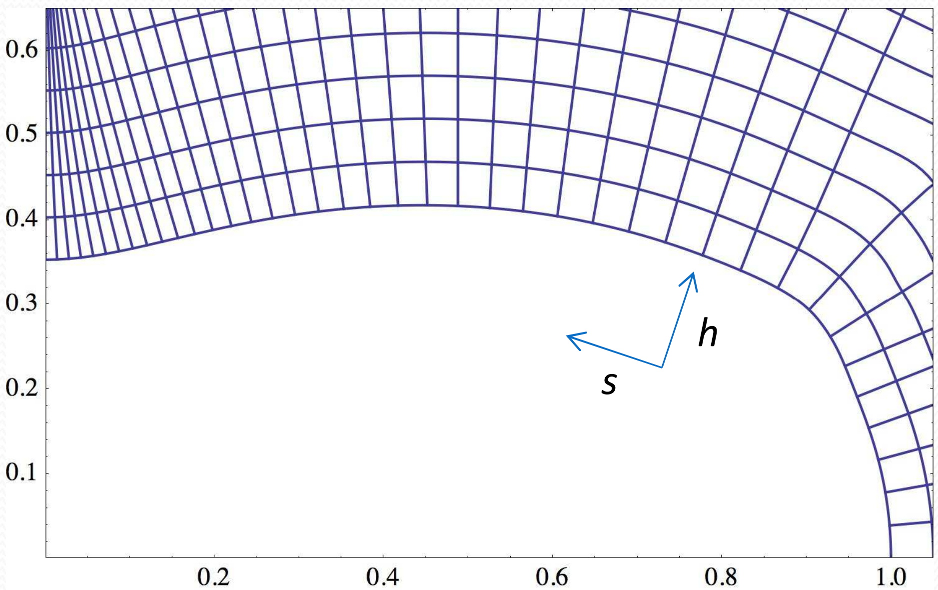


K&C

Basics of our model (part 2)

- Relativistic MHD eqs (steady state) in the downstream
- Curvilinear, orthogonal coordinate system, such that:

at the TS *h* perpendicular to the TS surface
s tangential to the TS surface



- 1st – order expansion of the main MHD quantities in *h*:

  P, u_h, u_s, (bu_h), (bu_s), ρ (but limit of zero inertia)

- Non-linear treatment of some other derived quantities:

  $\gamma^2=1+u_h^2+u_s^2$; $b^2=((bu_h)^2+(bu_s)^2)/(u_h^2+u_s^2)$

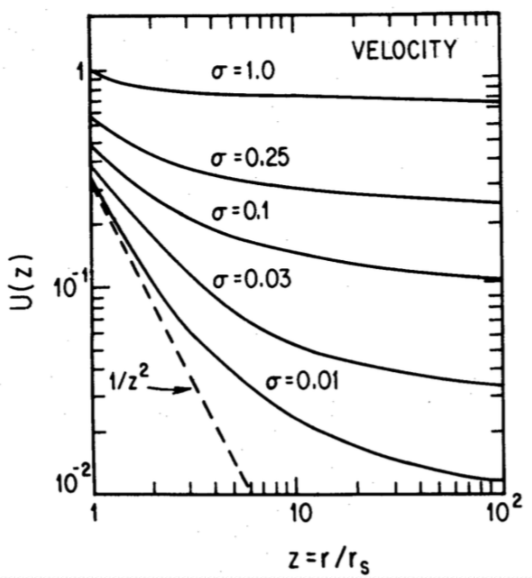
- Also differential operators to the 1st – order in *h*

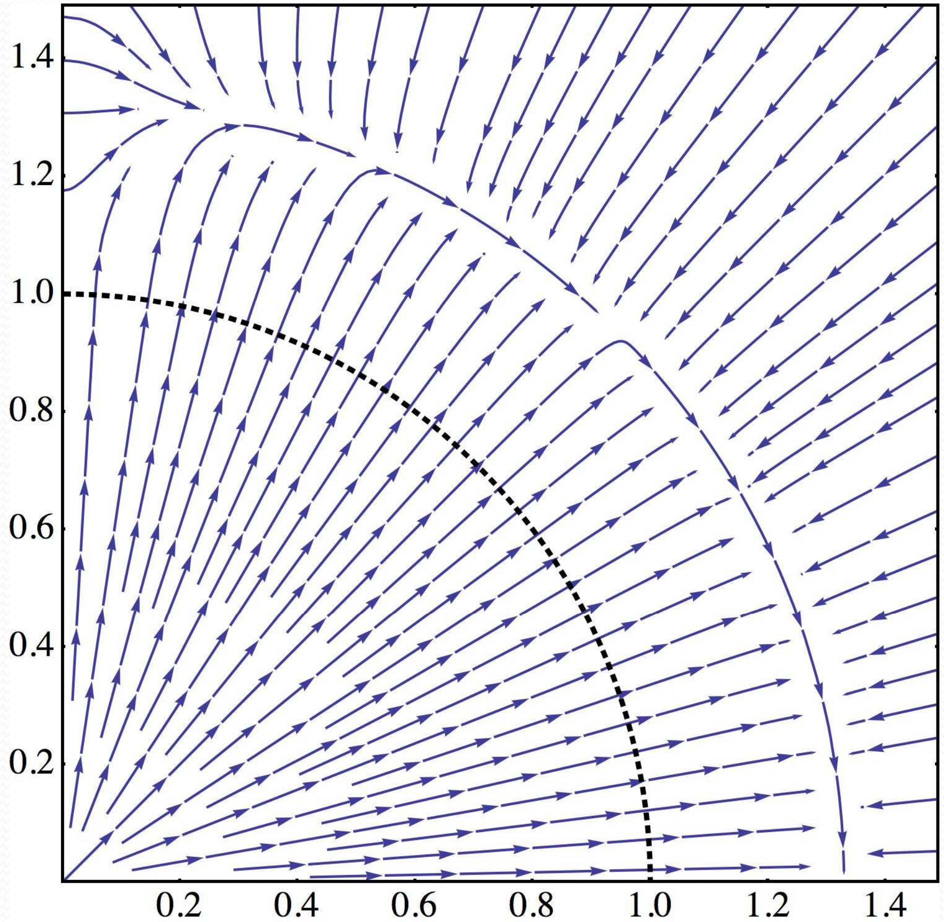
Set of differential Eqs. → System of linear algebraic equations

RESULTS VALID ONLY "CLOSE" TO THE TS
BUT "HOW CLOSE"?

Test case: spherical symmetry (K&C)

- Spherical TS
- Isotropic pulsar wind
- Inversion in u_h

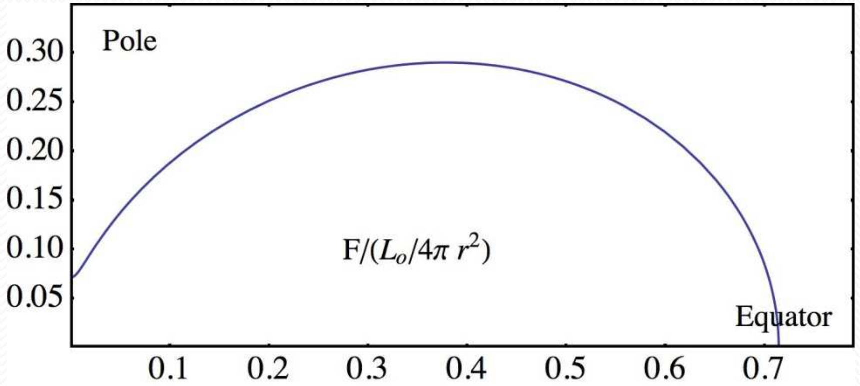




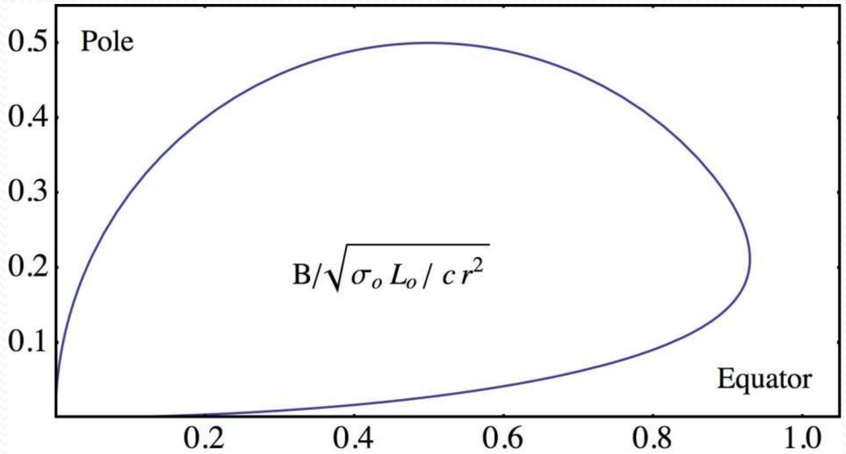
2-D case, with wind anisotropy

(as from Olmi et al. 2014)

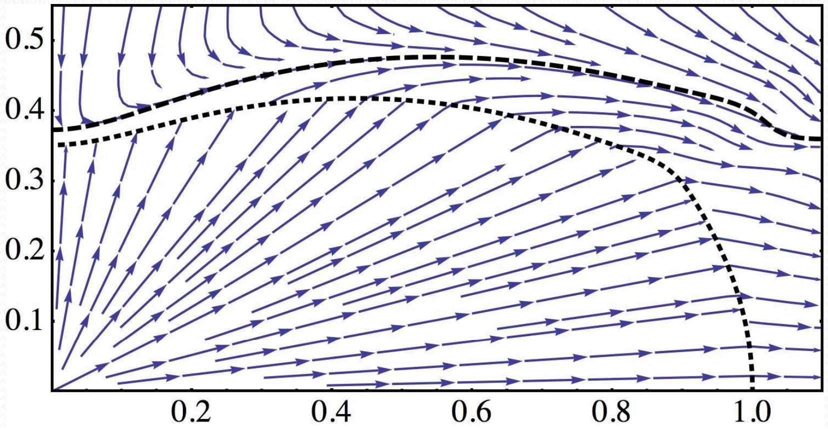
- The total energy flux

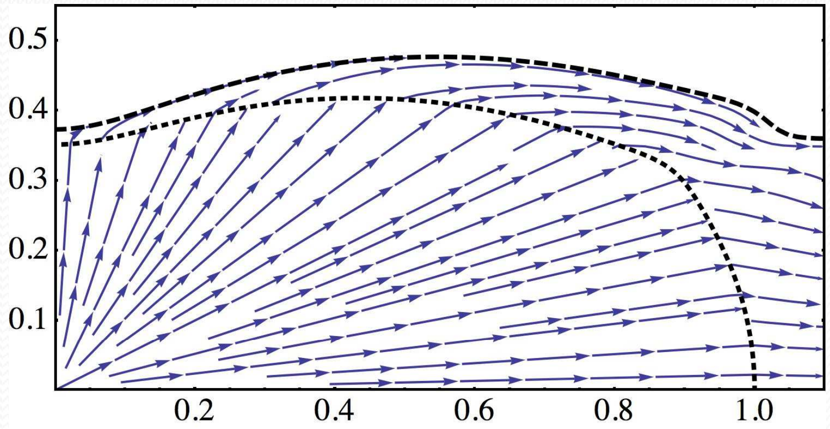


- The magnetic field

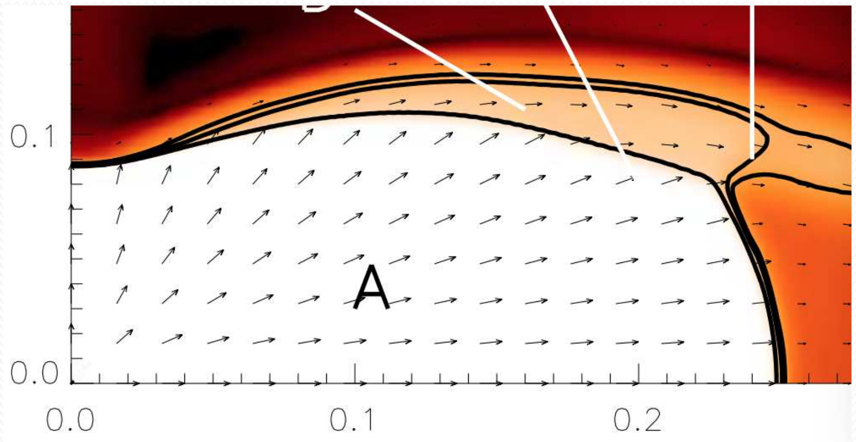


The flow pattern

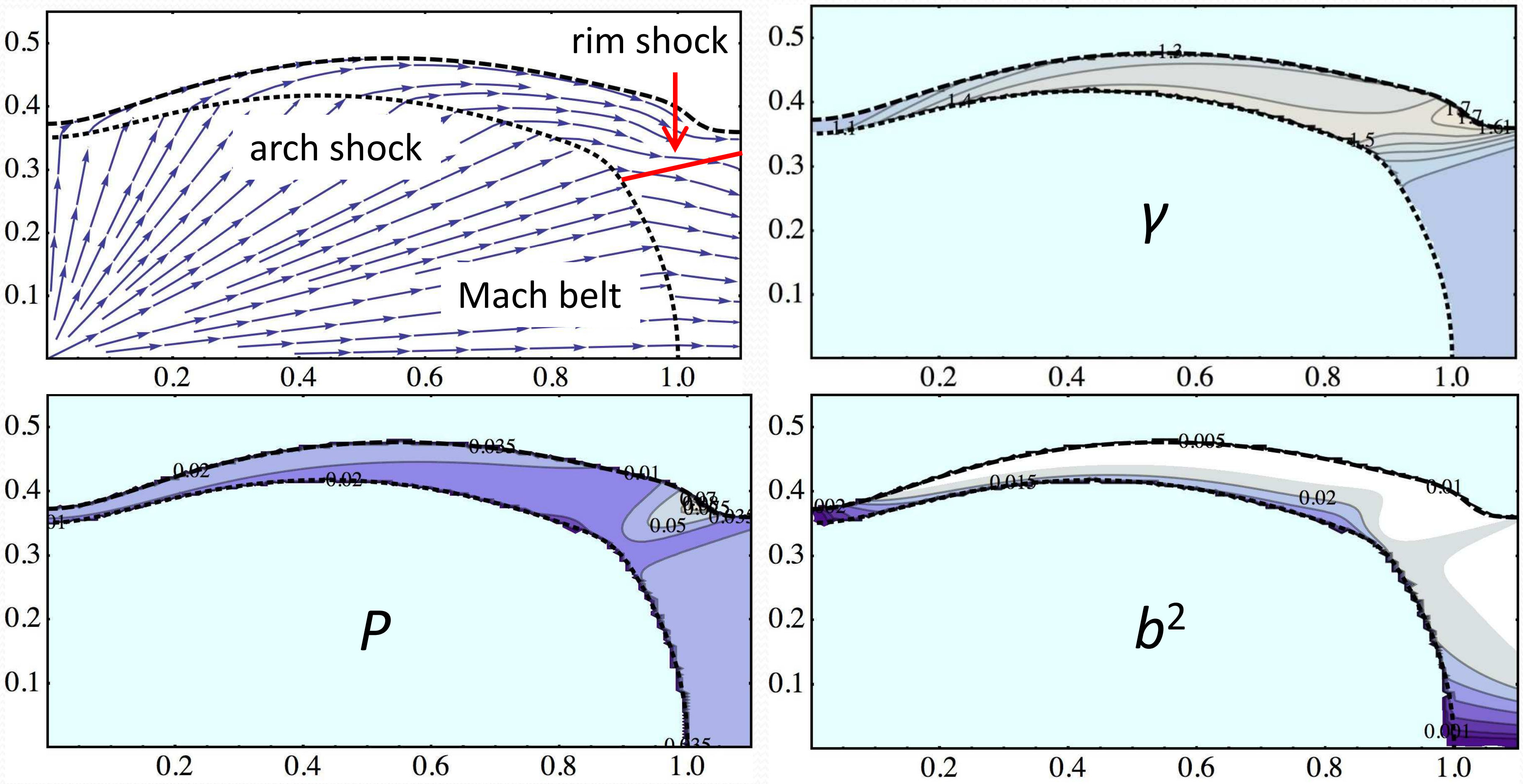




- Physical+unphysical domain
- Boundary surface
- Agreement (qualitatively at least) with the numerical model



- More reliable in the polar zone (smaller distance from TS)


rim shock
arch shock
Mach belt
γ
P
b2

The observed synchrotron emission

- Synchrotron emissivity, in the flow reference frame

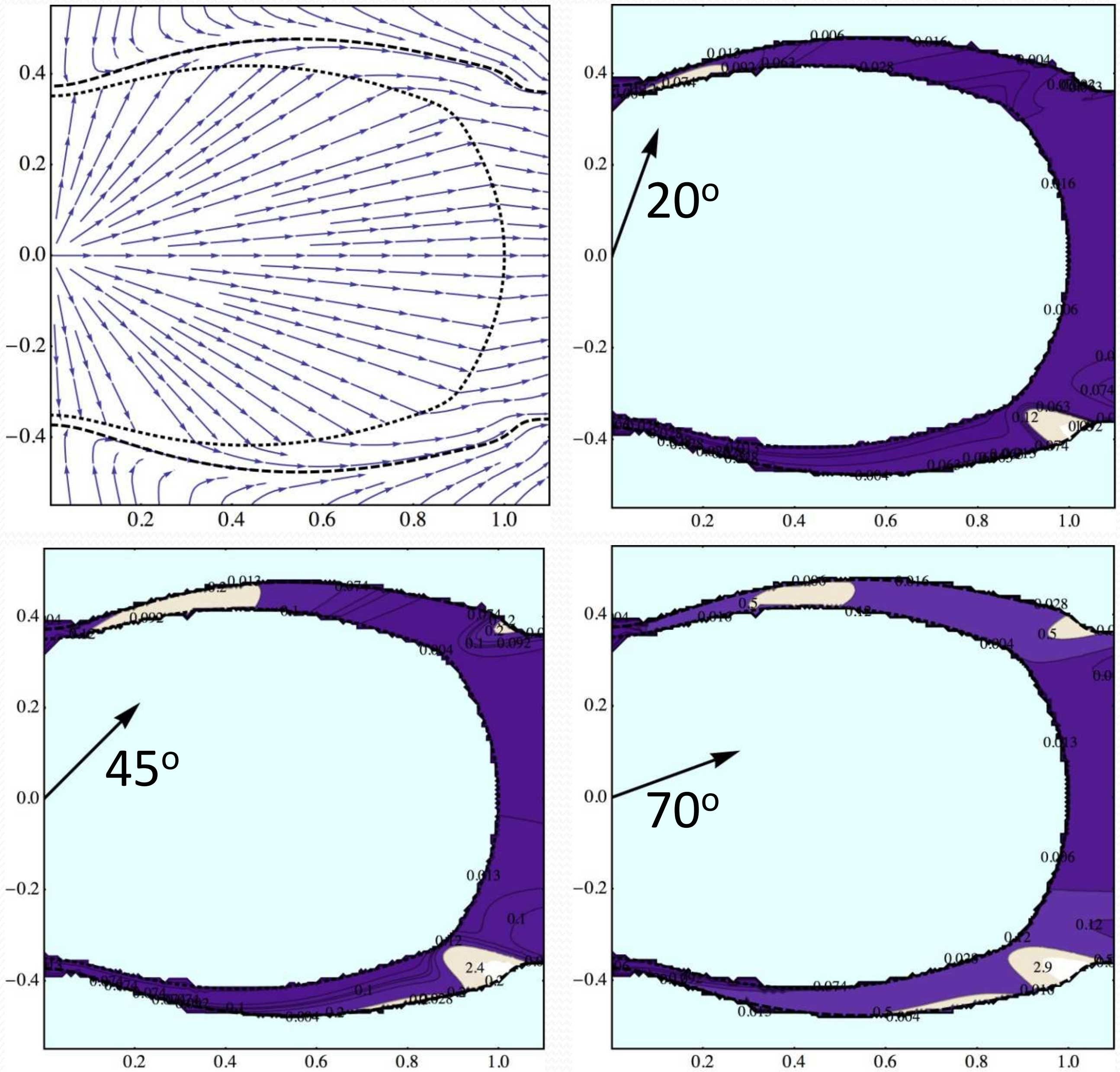
$$j'(\nu') \sim \frac{e^3}{mc^2}\epsilon_c f(\epsilon_c)|\boldsymbol{B}' \times \hat{n}'_z|; \nu' = \frac{3e}{4\pi mc}|\boldsymbol{B}' \times \hat{n}'_z|\epsilon_c^2$$

- Synchrotron emissivity, in the observer's reference frame

$$\nu = \mathcal{D}\nu'; j(\nu) = \mathcal{D}^2 j'(\nu'); \left|\boldsymbol{B}' \times \hat{n}'_z\right|^2 = b^2\left(1 - \mathcal{D}^2(\hat{n}_\phi \cdot \hat{n}_z)^2\right)$$

- (Adiabatic) evolution of the particle energy distribution, along a given fluid trajectory

$$f(\epsilon) = K\frac{p}{mc^2}\left(\epsilon_0^{\mathrm{cr}}\left(\frac{p}{p_0}\right)^{1/4}\right)^{2\alpha-1}\epsilon^{-(1+2\alpha)}$$



20°
45°
70°

CONCLUSIONS

- Limitations
  - The shape of the TS is an input parameter (so far)
  - The computed flow is reliable only nearby the TS (NO JET)
  - The model is steady state, while numerical models are not
  - The flows are laminar, no treatment of turbulence
- Advantages
  - Insight on the evolution in the innermost regions
  - Very high spatial resolution
  - Very short computing times