Apparent magnitude

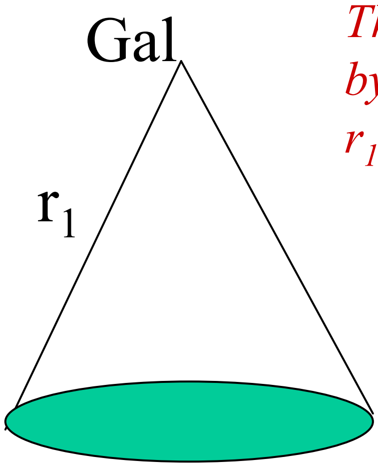
Normalized Intensity

$$I(\lambda)\,d\lambda = \frac{Energy(\lambda)}{L}\,d\lambda$$

Total Energy emitted at t_1

$$E(\lambda)\,d\lambda \equiv dL = L\,I(\lambda)\,d\lambda$$

$$or \quad J(\nu)\,d\nu = I(\lambda)\,d\lambda \text{ with } J(\nu) = \tfrac{1}{c}\lambda^2 I(\lambda)$$



The emission at time t_1 (constant) at position r_1 (constant) – or by the time it reaches us it is distributed over a sphere of radius r_1 so that the line element is

$$ds^2 = -r^2 a^2(t)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

The line element on the surface of a Euclidean sphere of Radius r a(t).

The Area therefore is: $4\pi r_1^{\,2}\, a^2(t_o)$ since it is the surface of a sphere
But how much light?

$$I\ observe: \quad \lambda_0 \quad \lambda_0 + \Delta\lambda_0 \quad \Im(\lambda_0)\Delta\lambda_0$$

$$At\ departure, t_{em}, \frac{\lambda_0}{1+z}\ and\ \frac{\lambda_0 + \Delta\lambda_0}{1+z}$$

$$And\ the\ Energy\ emitted\ at\ t_{em}:$$

$$L\, I\left(\frac{\lambda_0}{1+z}\right) \frac{\Delta\lambda_0}{1+z} \Delta t_{em}$$

$$Energy\ per\ photon: \quad h\frac{c(1+z)}{\lambda_0}$$

$$Number\ of\ photons: L\,I(\frac{\lambda_0}{1+z})\ \frac{\Delta\lambda_0}{1+z}\Delta t_{em}\frac{\lambda_0}{hc(1+z)}$$

$$L\,I(\frac{\lambda_0}{1+z})\ \frac{1}{(1+z)^2}\frac{\lambda_0}{hc}\Delta\lambda_0\Delta t_{em}$$

The photons go over the area $4\pi r_1^2 a^2(t)$ and are collected over a time $\Delta\ t_o$ per unit area. Their Energy is $h\ c/\lambda_o$ so that finally (multiplying the Number of photons by their energy) we have:

$$\Im(\lambda_0)\Delta\lambda_0 = L\,I(\frac{\lambda_0}{1+z})\ \frac{1}{(1+z)^2}\frac{1}{4\pi r_1^2 a^2(t)}\Delta\lambda_0\Delta t_{em}\frac{1}{\Delta t_0}$$

$$and\ as\ we\ have\ shown\ that: \Delta t_{em}\frac{1}{\Delta t_0} = \frac{1}{(1+z)}$$

$$\Im(\lambda_0) = L\,I\left(\frac{\lambda_0}{1+z}\right)\ \frac{1}{(1+z)^3}\frac{1}{4\pi r_1^2 a^2(t)}$$

Bolometric Luminosity and Bolometric Flux:

$$\Im(\nu_0) = \frac{L\, J\left(\nu_0 \bullet (1+z)\right)}{(1+z)\, 4\pi r_1^2 a^2(t_0)}$$

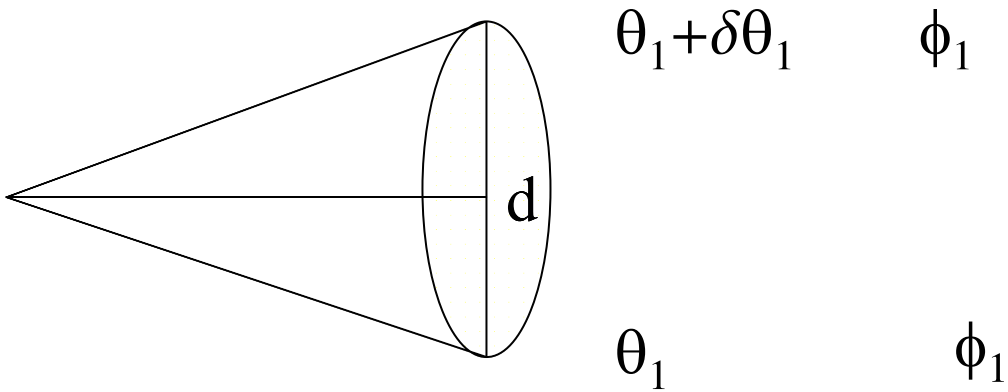
$$\Im_{bol} = L_{em} \int_0^\infty I\left(\frac{\lambda_0}{1+z}\right) \frac{1}{(1+z)^3} \frac{1}{4\pi r_1^2 a^2(t)}\, d\lambda =$$

$$= L_{em} \frac{1}{(1+z)^3} \frac{1}{4\pi r_1^2 a^2(t)} \int_0^\infty I(\frac{\lambda_0}{1+z}) d(\frac{\lambda_0}{1+z}) * (1+z)$$

$$\Im_{bol} = \frac{L}{(1+z)^2\, 4\pi r_1^2 a^2(t_0)}$$

Angular size

- We have seen we are capable of measuring mag nitudes only if we know $r_1\, a(t_0)$.



I choose a system of coordinates so that I measure the angle between two points in the coordinate θ. Since I am talking about a proper separation in the RW metric I have: $\Delta t=0$, $\Delta r=0$, $\Delta \phi=0$ and $\Delta\theta\neq 0$. That is I can write:

$$ds^2 = -r_1^2\, a^2(t_1)\, \Delta\theta^2 = -d^2$$

Continue

On the other hand I measure at the time t_0 and I will have:

$$d = r_1 a(t_1)(\Delta\theta_1)^2$$

$$\Delta\theta_1 = \frac{d}{r_1 a(t_1)} = \frac{d(1+z)}{r_1\, a(t_0)}$$

We need a Model to estimate r_1 $a(t_0)$. Note also that while r_1 increases with z (I am looking further away) $a(t_0)$ decreases.