Thermal History

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Here I give a brief summary of the most important steps in the thermal evolution of the Universe. The student should try to compute the various parameters and check the similarities with other branches of Astrophysics.

After this we will deal with the coupling of matter and radiation and the formation of cosmic structures.

The cosmological epochs

- The present Universe
  - $T=t_0$ $z=0$ Estimate of the Cosmological Parameters and of the distribution of Matter.
- The epoch of recombination
  - Protons and electron combine to form Hydrogen
- The epoch of equivalence
  - The density of radiation equal the density of matter.
- The Nucleosynthesis
  - Deuterium and Helium
- The Planck Time
  - The Frontiers of physics

Recombination

$$Saha\ equation:\ \Omega_0 = 0.038\ H_0 = 72$$

$$\frac{N_2}{N_1} N_e = \left(\frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{\chi}{kT}};\ \chi_H = 13.6\ eV;\quad k = 8.62\ 10^{-5}\ eV\ deg^{-1}$$

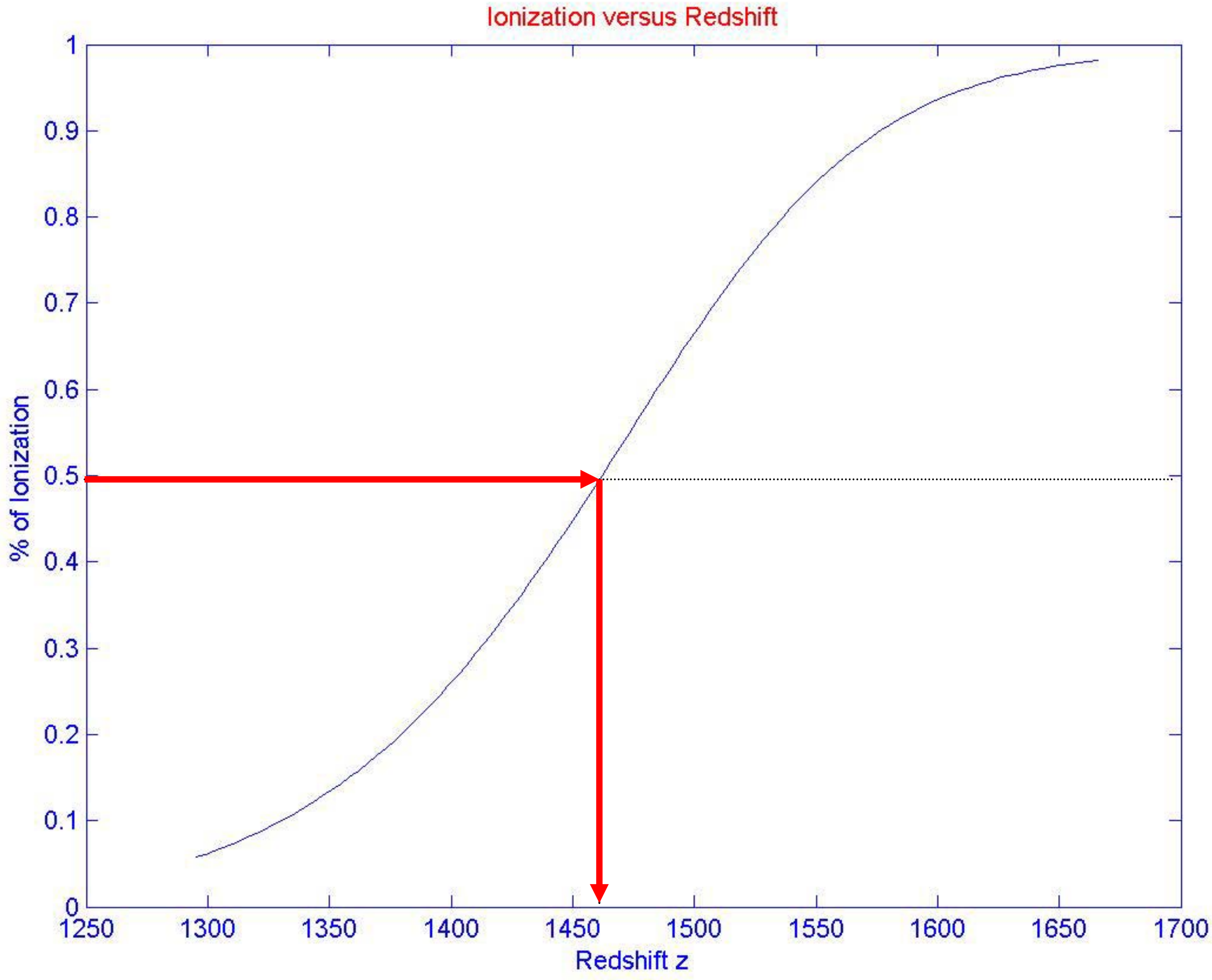
$$N_{Tot} = N_1 + N_2 \quad x = \frac{N_e}{N_{Tot}}\ and\ for\ Hydrogen\ N_2 = N_e$$

$$\frac{N_2}{N_1} N_e = \frac{N_e^{\ 2}}{N_1} = \frac{N_e^{\ 2}}{N_{Tot} - N_e} = \frac{x^2}{1-x} N_{Tot}$$

$$N_{Tot}(z) = N_0 (1+z)^3 = \frac{\rho_0}{m_p}(1+z)^3 = \frac{\rho_{0,c}\Omega_0}{m_p}(1+z)^3 = \frac{3H_0^2}{8\pi G}\frac{\Omega_0}{m_p}(1+z)^3$$

$$t_{recombination} \equiv t\left(x = \frac{N_e}{N_{Tot}} = 0.5 \right)$$

$z_{recombination}$



A Play Approach

- We consider a mixture of photons and particles (protons and electrons) and assume thermal equilibrium and photoionization as a function of Temperature (same as time and redshift).
- I follow the equations as discussed in a photoionization equilibrium and I use the coefficients as given in Osterbrock, see however also Cox Allen's Atrophyiscal Quantities.
- A more detailed approach using the parameters as a function of the Temperature will be done later on.
- The solution of the equilibrium equation must be done by numerical integration.
- The Recombination Temperature is defined as the Temperature for which we have: $N_e = N_p = N_{ho} = 0.50$
- $\Omega_{b,0}\, h^2 = 0.02 \quad H_0 = 72$

The Equations

$$N_{H_0}\int_{\nu_0}^{\infty} N_\nu a_\nu\left(H_0\right)d\nu = N_e N_p \alpha\left(H_0,T\right)\,;\, N_{H_0} = N_p$$

$$\int_{\nu_0}^{\infty} N_\nu a_\nu\left(H_0\right)d\nu = N_e N_p \alpha\left(H_0,T\right)\,;\, N_e = 0.5\ N_{Tot}\left(T\right) = 0.5\ N_0\,\frac{T^3}{T_0^3}$$

for T I use Radiation Temperature

$$N_0 = \Omega_{b,0}\,\frac{3H_0^2}{8\pi G m_p}\,;\, N_e = 0.5\,\frac{T^3}{T_0^3}\,\Omega_{b,0}\,\frac{3H_0^2}{8\pi G m_p}$$

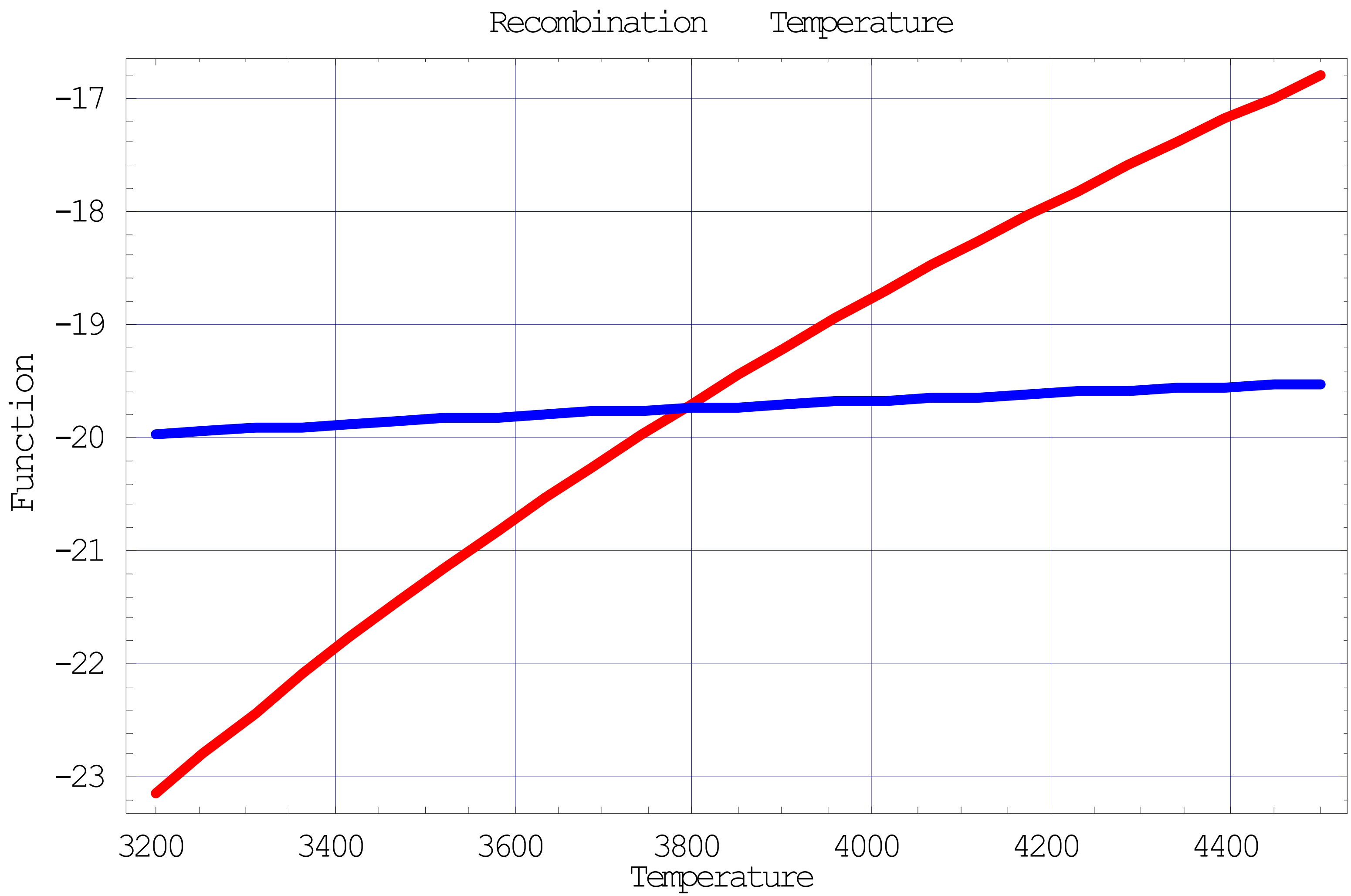
$$\int_{\nu_0}^{\infty} N_\nu a_\nu\left(H_0\right)d\nu = 0.5\,\frac{T^3}{T_0^3}\,\Omega_{b,0}\,\frac{3H_0^2}{8\pi G m_p}\,\alpha\left(H_0,T\right)$$

$$for\ a_\nu\left(H_0\right) = 6.3\ 10^{-18}\left(\frac{\nu_0}{\nu}\right)^3$$

$$I\ Part \equiv \int_{\nu_0}^{\infty} N_\nu a_\nu\left(H_0\right)d\nu = \int_{\nu_0}^{\infty}\frac{4\pi}{h\nu}\frac{2h\nu^3/c^2}{e^{\frac{h\nu}{kT}}-1}6.3\ 10^{-18}\left(\frac{\nu_0}{\nu}\right)^3 d\nu =$$

$$= \frac{2*4\pi*6.3\ 10^{-18}*\nu_0^3}{c^2}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu\left(e^{\frac{h\nu}{kT}}-1\right)} = B*\int_{\nu_0}^{\infty}\frac{d\nu}{\nu\left(e^{\frac{h\nu}{kT}}-1\right)}$$

$$II\ Part \equiv 0.5\frac{T^3}{T_0^3}\Omega_{b,0}\frac{3H_0^2}{8\pi G m_p}\alpha\left(H_0,T\right) = C*T^3$$


Recombination Temperature
Function
−17
−18
−19
−20
−21
−22
−23
3200
3400
3600
3800
4000
4200
4400
Temperature

The Agreement is excellent

Time of equivalence

$$\rho_r(t) = \frac{\varepsilon_r(t)}{c^2} \propto a^{-4}(t) \quad \rho_m(t) \propto a^{-3}(t) \quad t_{eq} \Rightarrow \rho_r(t_{eq}) = \rho_m(t_{eq})$$

$$\rho_m(t_{eq}) = \rho_{m,0} \frac{a^{-3}(t_0)}{a^{-3}(t_{eq})} = \rho_r(t_{eq}) = \rho_{r,0} \frac{a^{-4}(t_0)}{a^{-4}(t_{eq})}$$

$$\frac{\rho_{m,0}}{\rho_{r,0}} = \frac{a(t_0)}{a(t_{eq})} = 1 + z_{eq}$$

$$\rho_{m,0} = \rho_{c,0}\Omega_{m,0} = \frac{3H_0^2}{8\pi G} 0.3 = 2.91\ 10^{-30}$$

$$\rho_{r,0} = \frac{\sigma T^4}{c^2} = 4.46\ 10^{-34}\ g\ cm^{-3}\ ;\ \Omega_{r,0} = \frac{\rho_{r,0}}{\frac{3H_0^2}{8\pi G}} = 4.6\ 10^{-5}$$

$$1 + z_{eq} = \frac{2.91\ 10^{-30}}{4.46\ 10^{-34}} = 6540$$

The need of Nucleosynthesis

- I assume that the Luminosity of the Galaxy has been the same over the Hubble time and due to the conversion of H into He.
- To get the observed Luminosity I need only to convert 1% of the nucleons and that is in disagreement with the observed Helium abundance which is of about 25%.
- The time approximation is rough but reasonable because most of the time elapsed between the galaxy formation and the present time [see the relation t=t(z)].
- To assume galaxies 100 time more luminous would be somewhat in contradiction with the observed mean Luminosity of a galaxy.
- Obviously the following estimate is extremely coarse and could be easily done in more details.

$$L_G\left(in\ eV\ over\ Hubble\ time\right)=\frac{\left(L^*\equiv\right)2.31\ 10^{10}\,2\ 10^{33}\,1.36\ 10^{10}\,3.15\ 10^{7}}{1.6\ 10^{-12}}=1.24\ 10^{73}$$

or in a different way

$$\mathfrak{M}=2\ 10^{11}\ ;\ \frac{\mathfrak{M}}{L}=10\ ;\ \frac{L}{\mathfrak{M}}=0.1\ ;\ \frac{L_\odot}{\mathfrak{M}_\odot}=2\ erg\ s^{-1}gr^{-1}$$

$$L(in\ eV\ )=0.2\ \mathfrak{M}\ HubbleTime=\frac{0.2\ 2\ 10^{11}\,2\ 10^{33}\,1.36\ 10^{10}\,3.15\ 10^{7}}{1.6\ 10^{-12}}=2.14\ 10^{73}$$

$$\#\ nucleons=\frac{\mathfrak{M}}{mp}=\frac{2\ 10^{11}\,2\ 10^{33}}{1.6\ 10^{-24}}=2.5\ 10^{68}$$

$$Em\ per\ Nucleon=\frac{L}{\#\ nucleons}\approx\frac{2\ 10^{73}}{2.5\ 10^{68}}=0.8\ 10^{5}\,eV=0.08\ MeV$$

$$The\ reaction\ H\rightarrow He\ produces\ 6\ MeV\ so\ that$$

$$I\ need\ only\ \frac{0.08}{6}=1.3\%\ nucleons\ to\ react$$

Temperature and Cosmic Time

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = 0 \Rightarrow \frac{dr}{dt} = \sqrt{\frac{2GM}{r}}$$

$$\int_0^r dr\, r^{\frac{1}{2}} = \int_0^t \left(2GM\right)^{\frac{1}{2}} dt \Rightarrow t = \frac{1}{\sqrt{6\,\pi\, G\,\rho}} \quad accurately\,(slide\ 16)$$

$$t = \left\{\frac{3}{32\,\pi\, G\,\rho_r}\right\}^{\frac{1}{2}} = \left\{\frac{3\,c^2}{32\,\pi\, G\,\sigma T^4}\right\}^{\frac{1}{2}}$$

$$T = \left\{\frac{3\,c^2}{32\,\pi\, G\,\sigma}\right\}^{\frac{1}{4}} t^{-\frac{1}{2}}$$

$$for\ t = 1\ s \quad T = 1.52\ 10^{10}\ {}^\circ K \quad Nuclear\ Reaction\ are\ possible$$

Gamow 1948

The reasoning by Gamow was rather simple. To form heavy elements I must start from elementary particles and in particular I should be able to form Deuterium from protons and neutrons according to the reaction:

$$n + p => d + \gamma$$

A reaction that need a Temperature of about 10^9 degrees Kelvin. If I have the γ photons at higher Temperature ($T>10^9$) these dissociate the Deuterium as soon as it forms. The Temperature is therefore very critical if I want to accumulate Deuterium as a first step in the formation of heavy elements.

The Density ρ is critical as well. The density must allow a reasonable number of reaction. However it must not be too high since at the end I also have a constraint due to the amount of Hydrogen I observe and indeed I need protons to form hydrogen.

Finally since the Temperature is a function of time the time of reaction in order the reaction to occur must be smaller than the time of expansion of the Universe.

Gamow

During the time t I have :

Number of encounters = $n\sigma vt$

n = Density at the time t

σ = cross section

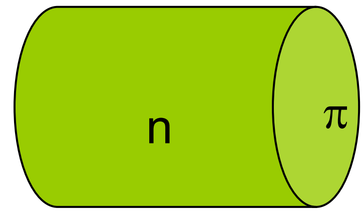
v = thermal velocity of particles

Or I have one enclounter in

$1/n\sigma vt$ seconds and if the temperature is correct I will have the reaction. As we have said therefore the reaction time must by fast, in other words I must have:

$1/ n\sigma v\, t < t_{exp}$ or $n\sigma v\, t_{exp} > 1$.

Here we obviously have a radiation dominated Universe and Ω =1 (E=0 in the simple minded Newtonian model. Let play quickly:



$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} * \frac{H_0^2}{H_0^2}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho = \rho_0 \frac{a_0^4}{a^4}}{3} \frac{H_0^2}{H_0^2}$$

$$\left(\frac{\dot a}{a_0}\right)^2 = \Omega_0 (=1) H_0^2 \frac{a_0^2}{a^2}$$

$$a\,da = H_0 a_0^2\,dt \quad \textit{integrating}$$

$$\frac{1}{2}a^2 = H_0 a_0^2 t; \quad \Rightarrow \frac{1}{2}a_0^2 = H_0 a_0^2 t_0; \quad t_0 = \frac{1}{2H_0}; \quad \frac{a}{a_0} = \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\rho_R = \rho_{R,0}\frac{a_0^4}{a^4} = \rho_{R,c}\Omega_R(=1)\left[\left(\frac{t}{t_0}\right)^{\frac{1}{2}}\right]^4 = \frac{3H_0^2}{8\pi G}\left(\frac{t}{t_0}\right)^2 = \frac{3H_0^2}{8\pi G}\left(\frac{t}{2H_0}\right)^2$$

$$\rho_R = \frac{3}{32\pi G t^2} = \frac{a' T^4}{c^2}; \quad t = \left(\frac{3}{32\pi G \rho_R}\right)^{\frac{1}{2}} = \left(\frac{3c^2}{32\pi G a'(=7.56\ 10^{-15})T^4}\right)^{\frac{1}{2}} = (\mathit{for}\ T = 10^9) = 230.5\ s$$

$$\frac{\rho_R}{\rho_{R,0}} = \frac{a_0^4}{a^4} = \frac{T^4}{T_0^4}$$

σ Given by Physics
v derived from Temp.
t from Cosmology
n(t) to be determined

From slide 15: n σ v t = 1
$n(t) = 10^{18}$ nucleons cm^{-3}

Look at this

$$H_0 = 65\ km/s/Mpc$$

$$\left.\begin{array}{l} Radiation\ \dfrac{a(t)}{a(t_e)} = \left(\dfrac{t}{t_e}\right)^{\frac{1}{2}} \Rightarrow \quad t_e = \dfrac{1}{2\,H_e} \\ \\ Dust\ \dfrac{a(t)}{a(t_0)} = \left(\dfrac{t}{t_0}\right)^{\frac{2}{3}} \Rightarrow \quad t_0 = \dfrac{2}{3}\dfrac{1}{H_0} \end{array}\right\} \quad t_0 = 4.75\ 10^{17};$$

$$t_e = (slide\ 10) = \left(\frac{a(t_e)}{a(t_0)}\right)^{\frac{3}{2}} t_0 = \left(\frac{1}{1+z_e}\right)^{1.5} 4.75\ 10^{17} = 8.98\ 10^{11}$$

$$T(t_0) = T(230.5)\frac{a(t_{Deuterium})}{a(t_0)} = \frac{a(t_{Deuterium})}{a(t_e)}\frac{a(t_e)}{a(t_0)} = \left(\frac{t_{Deuterium}}{t_e}\right)^{\frac{1}{2}}\left(\frac{t_e}{t_0}\right)^{\frac{2}{3}} =$$

$$= \left(\frac{230.5}{8.98\ 10^{11}}\right)^{\frac{1}{2}}\left(\frac{8.98\ 10^{11}}{4.75\ 10^{17}}\right)^{\frac{2}{3}} = 2.5\ Kelvin$$

And

$$n(t_0) = n(230.5)\left(\frac{a(t)}{a(t_0)}\right)^3 =$$

$$= 10^{18}\left(\frac{a(t_{Deuterium})}{a(t_e)}\frac{a(t_e)}{a(t_0)}\right)^3 = 10^{18}\left(\left(\frac{t_{Deuterium}}{t_e}\right)^{\frac{1}{2}}\left(\frac{t_e}{t_0}\right)^{\frac{2}{3}}\right)^3 =$$

$$= \left(\left(\frac{230.5}{8.98\ 10^{11}}\right)^{\frac{1}{2}}\left(\frac{8.98\ 10^{11}}{4.75\ 10^{17}}\right)^{\frac{2}{3}}\right)^3 = 1.47\ 10^{-8}\ cm^{-3} \Leftrightarrow * m_H$$

$$1.47\ 10^{-8}\ cm^{-3}\ 1.67\ 10^{-24}\ g = 2.4\ 10^{-32}\ g / cm^3\ Barions$$

Nowadays **:**

$$\Omega_B h^2 \approx 0.02 \qquad \rho_B = \Omega_B \frac{3H_0^2}{8\pi G} =$$

$$= \frac{3\Omega_B h^2\, 100}{8\pi G} = \frac{3\ 0.02\,(0.65)^2\,\left(100\ 10^5 / (3.09\ 10 \wedge 24)\right)^2}{8\ \pi\ 6.67\ 10^{-8}} = 1.58\ 10^{-31}\ g\ cm^{-3}$$

See Gamow and MWB

Following the brief description of Gamow reasoning I introduce here the Microwave background and the discussion on the discovery.

The discovery by Penzias and Wilson and related history.

The details of the distribution with the point measured by Penzias and Wilson.

The demonstration that a Blackbody remains a Blackbody during the expansion.

The explanation of the observed dipole as the motion of the observer respect the background touching also upon the Mach principle.

Some example and computation before going to the next nucleosynthesis slides.

The analysis of the WMAP observations and the related fluctuations will be eventually discussed later also in relation with the density perturbation and the formation of galaxies and large scale structure. The Horizon and The Power spectrum and Clusters of galaxies.

The main reactions

$$for\ T \succ 10^{10} \quad T \gg \frac{m_e c^2}{k} \sim 6\ 10^9$$

$$p + e^- \rightleftarrows n + \nu$$

$$p + \bar{\nu} \rightleftarrows e^+ + n$$

$$e^+ + e^- \rightleftarrows \gamma \ and\ when\ T\ decreases\ e^+ + e^- \rightarrow \gamma\ only$$

That is at some point after the temperature decreases under a critical value I will not produce pairs from radiation but I still will produce radiation by annihilation of positrons electrons pairs.

That is at this lower temperature the reaction above, proton + electron and neutron + positron do not occur any more and the number of protons and neutron remain frozen.

Boltzman Equation

$$m_p c^2 = 938.2592\ MeV\ ;\ m_n c^2 = 939.5527\ MeV\ ;\ \Delta = 1.2935\ MeV$$

$$\frac{n_{neutrons}}{n_{protons}} = e^{-\frac{(m_n - m_p)c^2}{kT}} = e^{-\frac{1.294\ 10^6\ 1.6\ 10^{-12}\ (to\ erg)}{1.38\ 10^{-16}\ 10^{10}}} = 0.22 \sim \frac{1}{5} \begin{cases} 1\ Neutron \\ every \\ 5\ Protons \end{cases}$$

$$Neutron\ could\ \beta\ decay\ n \rightarrow p + e^- + \bar{\nu}_e$$

$$However\ it\ takes\ 15\ minutes,\ too\ long\ !!$$

- At this point we have protons and neutrons which could react to form deuterium and start the formation of light elements. The temperature must be high enough to get the reaction but not too high otherwise the particles would pass by too fast and the nuclear force have no time to react.
- The equation are always Boltzman equilibrium equations.

$$n_i = g_i \frac{(2\pi m_i kT)^{\frac{3}{2}}}{h^3} e^{\frac{-m_i c^2 + \mu_i}{kT}} \; ; g_n = g_p = \frac{2 g_d}{3} = 2 \; ; \mu_n + \mu_p = \mu_d$$

$$X_i = \frac{n_i}{n_{Tot}}$$

$$n_d = g_d \frac{(2\pi m_d kT)^{\frac{3}{2}}}{h^3} \frac{(2\pi)^3}{(2\pi)^3} e^{\frac{-m_d c^2 + \mu_d}{kT}} = g_d \frac{\left(m_d k \frac{T}{2\pi}\right)^{\frac{3}{2}}}{\hbar^3} e^{\frac{-m_d c^2 + \mu_d}{kT}} =$$

$$3 \frac{\left(m_d k \frac{T}{2\pi}\right)^{\frac{3}{2}}}{\hbar^3} e^{\frac{-(m_n + m_p - m_n - m_p + m_d) c^2 + \mu_n + \mu_p}{kT}} = 3 \frac{\left(m_d k \frac{T}{2\pi}\right)^{\frac{3}{2}}}{\hbar^3} e^{\frac{-(m_n + m_p) c^2 + B_d + \mu_n + \mu_p}{kT}}$$

$$X_n = \frac{n_n}{n_{Tot}} = \frac{1}{n_{Tot}} g_n \frac{\left(m_n k \frac{T}{2\pi}\right)^{\frac{3}{2}}}{\hbar^3} e^{\frac{-m_n c^2 + \mu_n}{kT}}$$

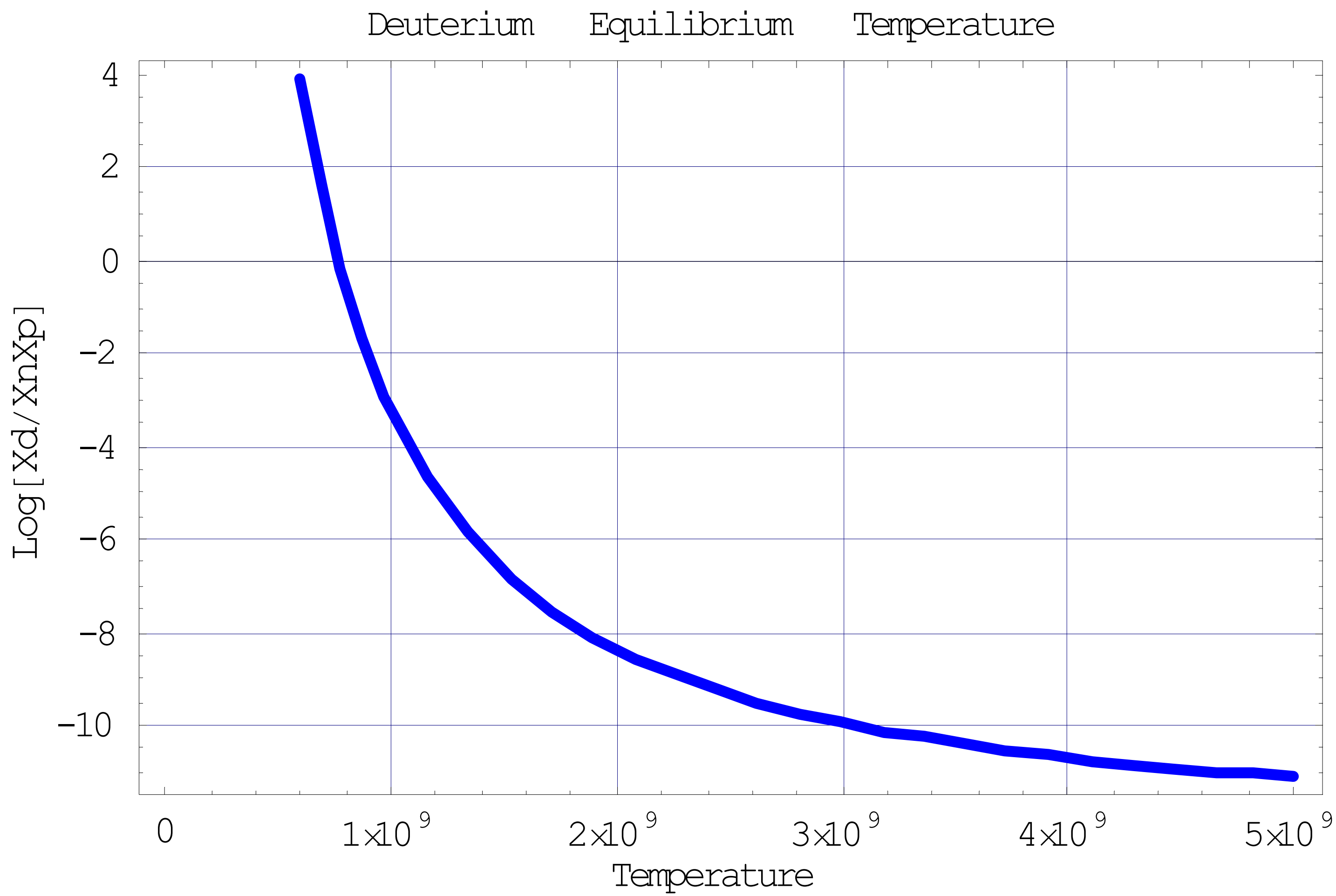
$$X_p = \frac{n_p}{n_{Tot}} = \frac{1}{n_{Tot}} g_p \frac{\left(m_p k \frac{T}{2\pi}\right)^{\frac{3}{2}}}{\hbar^3} e^{\frac{-m_p c^2 + \mu_p}{kT}}$$

$$\frac{X_d}{X_n X_p} = n_{Tot} \frac{g_d}{g_p g_n} \left(\frac{m_d}{m_n\, m_p}\right)^{\frac{3}{2}} \hbar^3 \left(\frac{k\,T}{2\pi}\right)^{-\frac{3}{2}} e^{\frac{B_d}{k\,T}}$$

Plot as a function of T

Comment

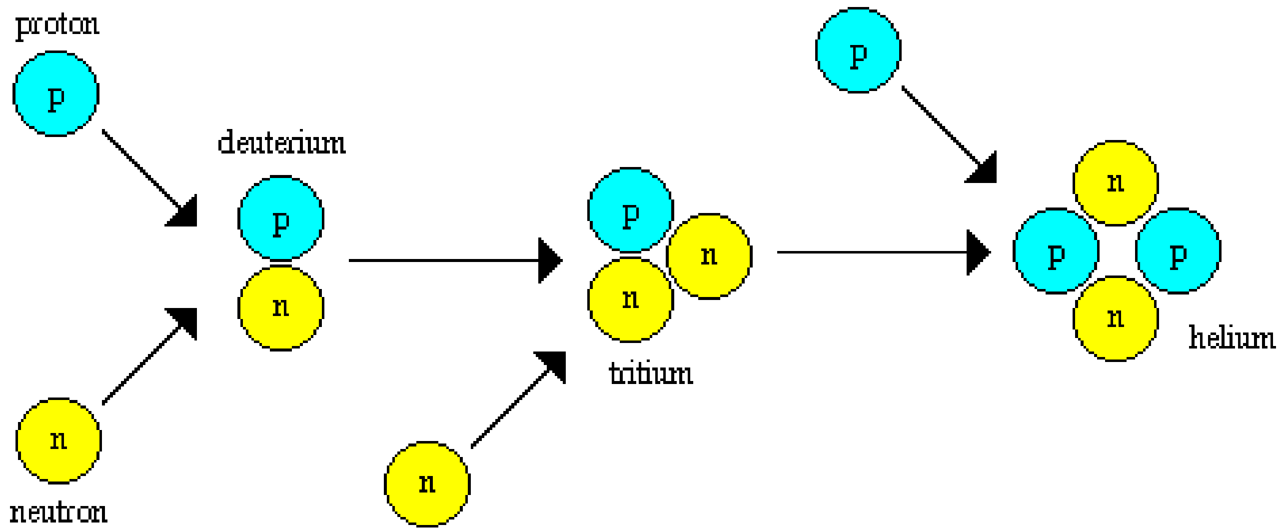
- As it will be clear from the following Figure in the temperature range 1 – 2 10^9 the configuration moves sharply toward an high Deuterium abundance, from free neutrons to deuterons.
- Now we should compute the probability of reaction to estimate whether it is really true that most of the free neutrons are cooked up into deuterium.
- Xd changes only weakly with $\Omega_B\, h^2$
- For T > 5 10^9 Xd is very small since the high Temperature would favor photo-dissociation of the Deuterium.


Deuterium Equilibrium Temperature
4
2
0
-2
-4
-6
-8
-10
Log[Xd/XnXp]
0
1×10 9
2×10 9
3×10 9
4×10 9
5×10 9
Temperature

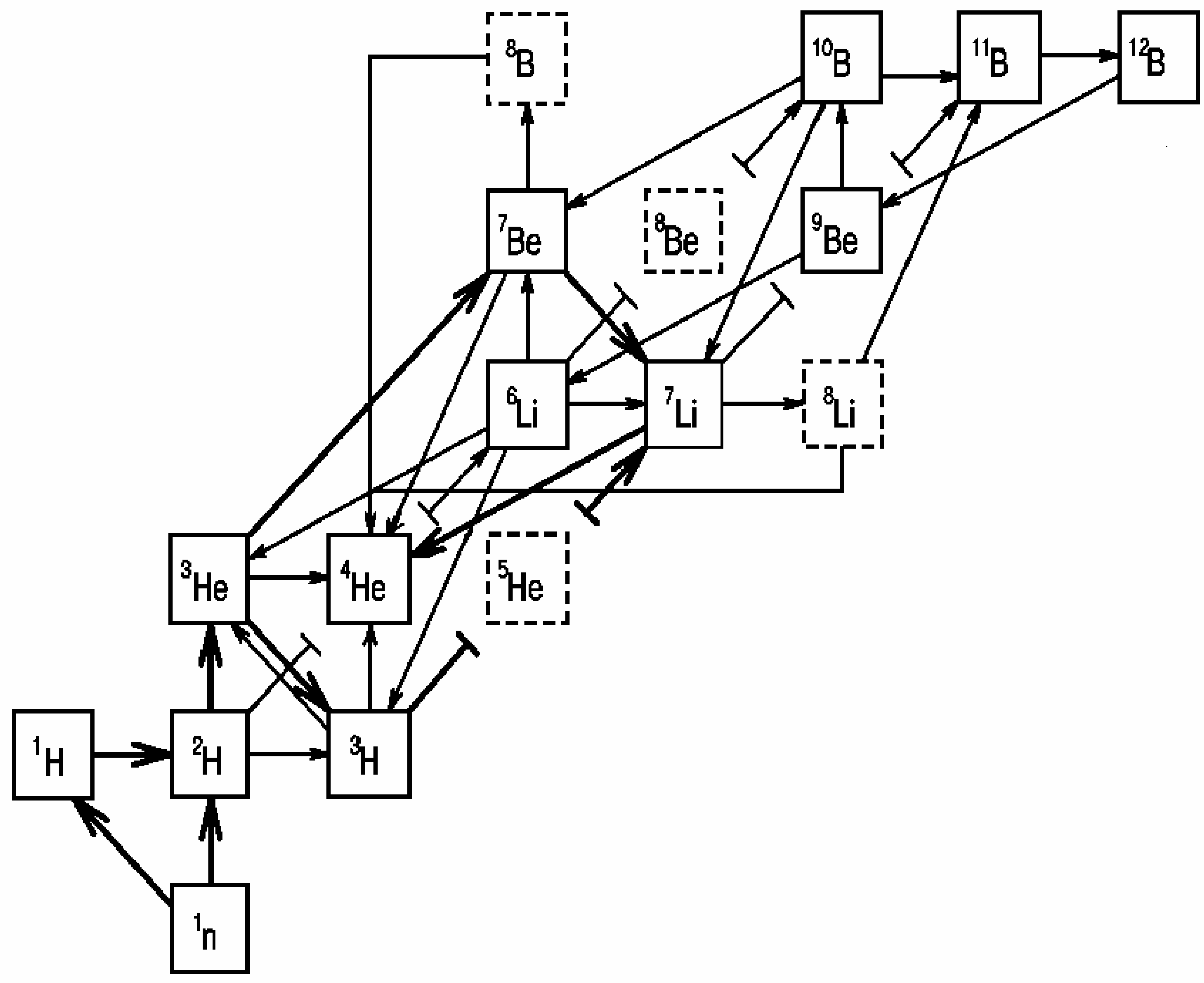
Helium

Nucleosynthesis

as the Universe cools, protons and neutrons can fuse to form heavier atomic nuclei



More



Compare to Observations

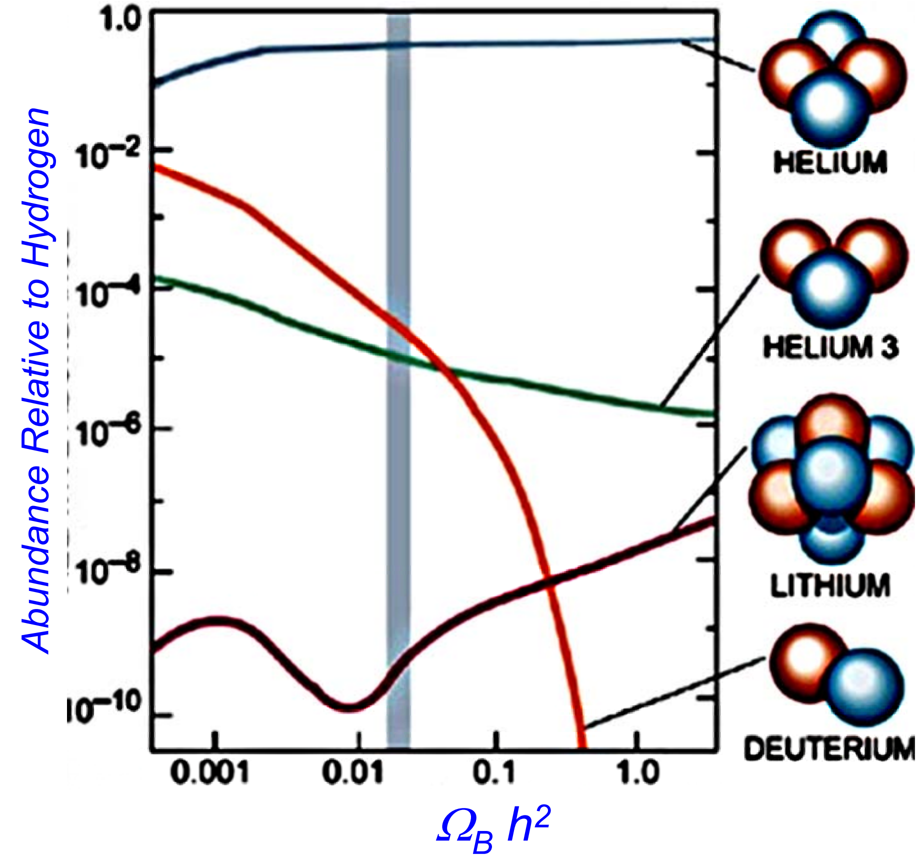
The current estimates are:

From the D/H Ratio in Quasars Abs. Lines:

$\Omega_B h^2 = 0.0214 \pm 0.002$

From the Power Spectrum of the CMB:

$\Omega_B h^2 = 0.0224 \pm 0.001$



After Deuterium

$$d + d \rightarrow He^3 + n \quad \Rightarrow p - p - n$$

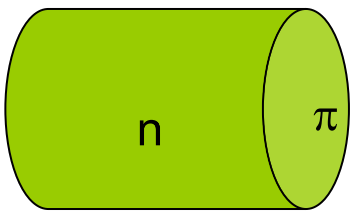
$$d + d \rightarrow H^3 + p \quad \Rightarrow p - n - n \quad Tritium$$

$$d + H^3 \rightarrow He^4 + n \Rightarrow p - p - n - n$$

$$d + He^3 \rightarrow He^4 + p \Rightarrow p - p - n - n$$

Probability of Reaction

- I assume also that at the time of these reaction each neutron collides and reacts with 1 proton. Indeed the Probability for that reaction at this Temperature is shown to be, even with a rough approximation, very high.



π r^2 cross section

V (t=1s)

Number of collision per second

= π r^2 v n

I assume an high probability of Collision so that each neutron Collides with a proton.

Probability Q is very high so that it Is reasonable to assume that all electrons React.

$$Q = \pi r^2 n\, v\, t \equiv \#\ of\ \ collision \equiv Probability\ of\ \ collision$$

$$T = 10^9;\Rightarrow t = 231\ s;\ r \sim 10^{-13};\ v = Sqrt\left[\frac{kT}{m_n}\right] = 2.9\ 10^8\ cm\ s^{-1};\ n = n_{crit}\Omega_{b,0}\left(\frac{T}{T_0}\right)^3$$

$$Q = 3.21\ 10^5 \gg 1\ \ (see\ however\ detailed\ computation)$$

Finally

$$\left.\begin{cases} 1\ neutron \rightarrow 5\ protons \\ 2\ neutrons \rightarrow 10\ protons \end{cases}\right\} \Rightarrow 1\ He\ every\ 10\ protons$$

$$\frac{n}{p} = 0.2 \qquad \frac{n}{n+p} = \frac{1}{1+\frac{p}{n}} = \frac{1}{6} = 0.17 \Rightarrow Accurate\ computation\ 0.12$$

$$\frac{m_{He}}{m_p} = 4 \qquad \frac{\mathfrak{M}_{Tot}}{m_p} = n_{Tot} \qquad n_{He} = \frac{1}{2} n_n$$

$$Y = \frac{\mathfrak{M}_{He}}{\mathfrak{M}_{Tot}} = \frac{n_{He} m_{He}}{\mathfrak{M}_{Tot}} * \frac{m_p}{m_p} = \frac{n_{He}\ 4}{n_{Tot}} = \frac{\frac{1}{2} n_n\ 4}{n_{Tot}} = 2\frac{n_n}{n+p} = 0.24$$

Neutrinos

- 1930 Wolfgang Pauli (1945 Nobel) assumes the existence of a third particle to save the principle of the conservation of Energy in the reactions below. Because of the extremely low mass Fermi called it neutrino.
- The neutrino is detected by Clyde Cowan and Fred Reines in 1955 using the reaction below and to them is assigned the Nobel Prize.
- The Muon neutrinos have been detected in 1962 by L. Lederman, M. Schwartz, and J. Steinberg. These received the Nobel Prize in 1988.
- We will show that the density of the neutrinos in the Cosmo is about the density of the photons.
- The temperature of the neutrinos is about 1.4 smaller than the temperature of the photons. And this is the consequence of the fact that by decreasing temperature I stop the creation of pairs from radiation and howver I keep annihilating positrons and electrons adding energy to the photon field.

$$
\begin{array}{cccc}
Leprons & Neutral & Mass & Temperature \\
e & \nu_e & \leq 15eV & 10^{9.7} \\
\mu & \nu_\mu & \leq .17MeV & 10^{12.1} \\
\tau & \nu_\tau & \leq 24MeV & 10^{13.3}
\end{array}
\left.\right\} \begin{array}{c}
Fermions \\
Massless\ ? \\
Move\ at\ speed\ of\ light \\
Follow\ geo\det ics
\end{array}
$$

Recent results

- It has been demonstrated by recent experiments [Super Kamiokande collaboration in Japan] that the neutrinos oscillate. For an early theoretical discussion see Pontecorvo paper.
- The experiment carried out for various arrival anles and distances travelled by the Neutrinos is in very good agreement with the prediction with neutrino oscillations and in disagreement with neutrinos without oscillations.
- The oscillations imply a mass so that finally it has been demonstrated that the neutrinos are massive particles.
- The mass is however very small. Indeed the average mass we can consider is of 0.05 eV.
- The small mass, as we will see later, is of no interest for the closure of the Universe.
- On the other hand it is an important element of the Universe and the total mass is of the order of the baryonic mass.
- www => neutrino.kek.jp // hep.bu.edu/~superk

$$\text{The distribution function for Fermions is :}$$

$$f(q)=\frac{g_i 4\pi}{h^3}\frac{1}{c^3}\frac{1}{e^{\frac{q}{kT}}+1}\Rightarrow n(q)=\int\frac{g_i 4\pi}{h^3}\frac{1}{e^{\frac{q}{kT}}+1}q^2dq\ ;\ q=E=h\nu \text{ for photons}$$

$$\text{and for the density of Energy}$$

Bernoulli Number

Here I use $g_\nu = 1$

Gamma

Riemann Zeta

$$\rho_\nu=\frac{4\pi}{c^3h^3}\int_0^\infty\frac{1}{e^{\frac{q}{kT}}+1}q^3dq=\begin{cases}\displaystyle\int_0^\infty\frac{x^3}{e^{p\,x}+1}=\left(1-2^{-3}\right)\left(\frac{2\pi}{p}\right)\frac{|B_4|}{8}=5.62\ p^{-4}\\ \displaystyle\int_0^\infty\frac{x^3}{e^{\mu\,x}+1}=\frac{1}{\mu^4}\left(1-2^{-3}\right)\Gamma(4)\zeta(4)=5.68\ \mu^{-4}\\ \displaystyle=\frac{4\pi\ 5.68\left(1.38\ 10^{-16}\right)^4}{\left(3\ 10^{10}\right)^3\left(6.625\ 10^{-27}\right)^3}T^4=3.3\ 10^{-15}\ T^4=\frac{7}{16}\sigma T^4\end{cases}$$

$$=\frac{7\pi^5k^4}{c^3\,30\,h^3}T^4=\frac{7}{16}\sigma T^4\ ;\ \sigma=7.56\ 10^{-15}\ ;\ \text{Density}$$

Conventions

- μ_l is the chemical potential
- $\pm$ + for Fermions and – for Bosons
- g_i Number of spin states
  - Neutrinos and antineutrinos g=1
  - Photons, electrons, muons, nucleons etc g=2
- $\rho_{e-} = \rho_{e+} = 2\ \rho_\nu = 7/8\ \sigma T^4$ (I use in the previous slide $g_e = 2$)
- Neutrinos have no electric charge and are not directly coupled to photons. They do not interact much with baryons either due to the low density of baryons.
- At high temperatures ~ 10^{11}-10^{12} the equilibrium is kept through the reactions

$$\nu_e + \mu^- \rightleftarrows \bar{\nu}_\mu + e^- \;;\; \bar{\nu}_\mu + \mu^+ \rightleftarrows \nu_e + e^+ \;;\cdots$$

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- Later at lower temperature we have electrons and photons in equilibrium ad neutrinos are not coupled anymore
- At 5 10^9 we have the difference before and after as shown in the next slides.

See Weinberg Page 533

Assuming the particles are in thermal equilibrium it is possible to derive an equation stating the constancy of the entropy in a volume a^3 and expressing it as:

$$S\left(a^3,T\right)=\frac{a^3}{T}\left\{\rho_{eq}\left(T\right)+p_{eq}\left(T\right)\right\}$$

At some point during the expansion of the Universe and the decrease of the Temperature we will create, as stated earlier, radiation (Gamma rays), and however the will not be enough Energy to create electron pairs.

We are indeed warming up the photon gas while the neutrino gas remains at a lower Temperature because we are not pumping in ant Energy.

Entropy

$$\textit{At some Temperature we have only } e^{+}\ e^{-}\ \gamma \textit{ in thermal equilibrium}$$

$$s\,a^{3}\left(Volume \propto a^{3}\right) = \frac{a^{3}}{T}\left\{\rho_{e^{+}} + \rho_{e^{-}} + \rho_{\gamma} + p_{e^{+}} + p_{e^{-}} + p_{\gamma}\right\} \quad \&$$

$$p = \frac{1}{3}\rho c^{2} \quad Relativistic\ Regime\ kT > m_{e}c^{2} \quad T > 5\ 10^{9}$$

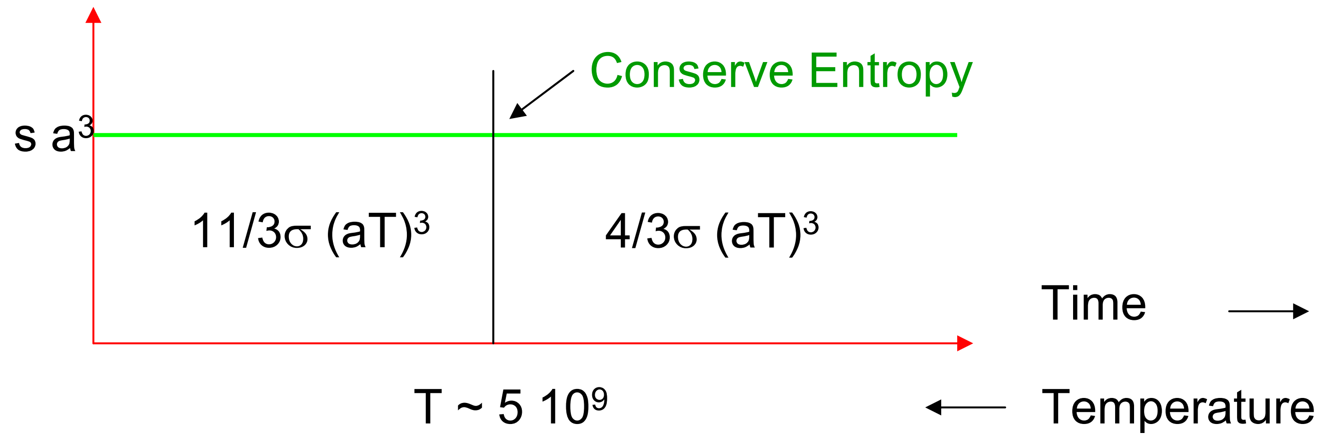
$$electrons\ are\ relativistic\ T \approx \frac{0.5\ 10^{6}\,1.6\ 10^{-12}}{1.38\ 10^{-16}} = 5.9\ 10^{9}\ ;\ \rho_{e^{+}} = \rho_{e^{-}} = \frac{7}{8}\sigma T^{4}$$

$$s\,a^{3} = \frac{a^{3}}{T}\left\{\rho_{Tot} + \frac{1}{3}\rho_{Tot}\right\} = \frac{a^{3}}{T}\frac{4}{3}\rho_{Tot} = \frac{a^{3}}{T}\frac{4}{3}\left\{\rho_{e^{+}} + \rho_{e^{-}} + \rho_{\gamma}\right\} =$$

$$= \frac{a^{3}}{T}\frac{4}{3}\left\{\frac{7}{8}\sigma T^{4} + \frac{7}{8}\sigma T^{4} + \frac{4}{4}\sigma T^{4}\right\} = \frac{11}{3}\sigma\left(aT\right)^{3} \textit{ This quantity will be conserved}$$

$$T < 5\ 10^{9}\ \ e^{+} + e^{-} \Rightarrow \gamma\ warms\ up\ photon\ field - we\ are\ left\ with\ photons$$

$$s\,a^{3} = \frac{a^{3}}{T}\frac{4}{3}\rho_{\gamma} = \frac{4}{3}\sigma\left(aT\right)^{3} \textit{ This quantity will be conserved}$$



$$s\, a^3 = \frac{11}{3}\sigma\left(aT >\right)^3 = \frac{4}{3}\sigma\left(aT <\right)^3 \Rightarrow \frac{\left(aT\right)_{T<10^9}}{\left(aT\right)_{T>10^9}} = \left(\frac{11}{4}\right)^{\frac{1}{3}}$$

while neutrinos & antineutrinos are not warmed up $T_\gamma > T_\nu \propto a^{-1}$

$$\left(\frac{T_\gamma}{T_\nu}\right)_{T<10^9} = \left(\frac{11}{4}\right)^{\frac{1}{3}} = 1.401 \Rightarrow T_{\nu,0} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_{\gamma,0} = 1.9\ {}^\circ K$$

Density of Radiation due to neutrinos

$$\rho_{R,\nu} = 3\left(\text{species neutrinos}\right) * 2\left(\text{neut \& antineut}\right) * \frac{7}{16} * \sigma T_\nu^4 = 6\,\frac{7}{16}\left(\frac{4}{11}\right)^{\frac{4}{3}} \sigma T_\gamma^4 =$$

$$= 0.68\ \sigma\ T_\gamma^4$$

$$\int_0^\infty \frac{1}{e^{p\,q}+1} q^2 dq = \frac{3\zeta(3)}{p^3} \; for \; p > 0$$

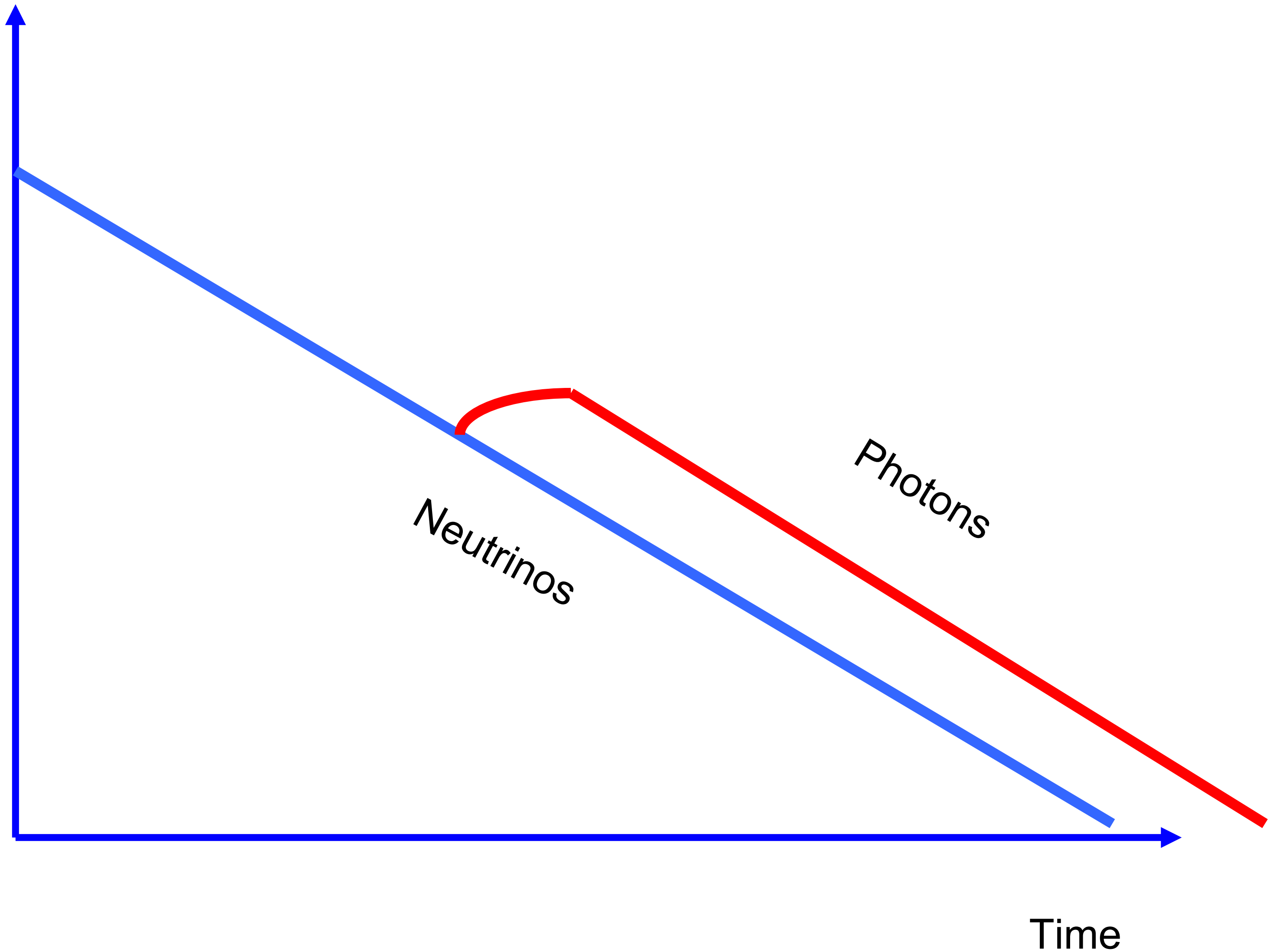
$$n(q) = \int \frac{g_i 4\pi}{h^3} \frac{1}{c^3} \frac{1}{e^{\frac{q}{kT}}+1} q^2 dq = N_\nu \frac{1}{c^3} \frac{4\pi}{h^3} 3\,\zeta(3)(k\,T)^3 = N_\nu \; 108.6 \; cm^{-3} = 326 \; cm^{-3}$$

$$n_{\gamma,0} = 420 \; cm^{-3}$$

$$\rho_{0,c} = \frac{3H_0^{\;2}}{8\pi G} = 9.72 \; 10^{-30} \, g \; cm^{-3} = \frac{9.72 \; 10^{-30} c^2}{1.6 \; 10^{-12}} eVcm^{-3} = 5460 \; eVcm^{-3}$$

$$closure \; with \; \frac{5460}{326} = 16.7 \; eV \equiv \; < m_\nu >$$

$$Observed \approx \quad 0.05\,eV$$


Photons
Neutrinos
Time

Planck Time

- We define this time and all the related variables starting from the indetermination principle. See however Zeldovich and Novikov for discussion and inflation theory.

$$\Delta E \Delta t \simeq \hbar$$

$$\Delta E \Delta t = m_p c^2 t_p \approx \rho_p \left(c\, t_p\right)^3 c^2 t_p \approx \frac{1}{G\, t_p^2}\left(c\, t_p\right)^3 c^2 t_p \approx \frac{c^5 t_p^2}{G} \approx \hbar$$

$$t_p \approx \sqrt{\frac{\hbar\, G}{c^5}} \simeq 10^{-43}\, s$$

$$l_p \approx c\, t_p \approx \sqrt{\frac{\hbar\, G}{c^3}} \approx 1.7\ 10^{-33}\, cm$$

$$\rho_p \approx \frac{1}{G\, t_p^2} \approx \frac{c^5}{G^2 \hbar} \approx 4\ 10^{93}\, g\ cm^{-3}$$

$$m_p \approx \rho_p l_p^3 \approx \sqrt{\frac{\hbar\, c}{G}} \approx 2.5\ 10^{-5}\, g$$

$$n_p \approx l_p^{-3} \approx \frac{\rho_p}{m_p} \approx \left(\frac{c^3}{G\hbar}\right)^{\frac{3}{2}} \approx 10^{98}\, cm^{-3}$$

$$E_p \approx m_p c^2 \approx \sqrt{\left(\frac{\hbar\, c^5}{G}\right)} \approx 1.2\ 10^{19}\, GeV$$

$$T_p \approx \frac{E_p}{k} \approx \sqrt{\left(\frac{\hbar\, c^5}{G}\right)}\ k^{-1} \approx 1.4\ 10^{32}\ {}^\circ K$$

Curiosity – Schwarzschild Radius

- It is of the order of magnitude of the radius that should have a body in order to have Mass Rest Energy = Gravitational Energy.
- And the photons are trapped because the escape velocity is equal to the velocity of light.

$$r_s = \frac{2\,G\,m}{c^2} \qquad m\,c^2 = \frac{G\,m^2}{r_s}$$

$$\frac{c^2}{2} = \frac{G\,m}{r_s} \qquad t_s\left(time\ to\ cross\ r_s\right) = \frac{r_s}{c} = \frac{2\,G\,m}{c^3}$$

$$with\ m = m_p = \sqrt{\frac{\hbar\,c}{G}} \qquad t_s = \frac{2\,G}{c^3}\sqrt{\frac{\hbar\,c}{G}} = 2\sqrt{\frac{\hbar\,G}{c^5}} = 2\,t_p$$

The Compton time

- I define the Compton time as the time during which I can violate the conservation of Energy $\Delta E = mc^2$ $\Delta t = t$. I use the indetermination principle.
- During this time I create a pait of particles $t_c = \hbar / m\, c$.
- In essence it is the same definition as the Planck time for $m = m_p$.

$$l_{Compton} = c\, t_C = \frac{\hbar}{m\, c}\ and\ \ for\ m = m_p = \sqrt{\frac{\hbar\, c}{G}}$$

$$t_C = \frac{\hbar}{m\, c^2} = \frac{\hbar}{c^2}\left(\sqrt{\frac{\hbar\, c}{G}}\right)^{-1} = \sqrt{\frac{\hbar\, G}{c^5}} = t_p$$

Preliminaries

1 Mole: Amount in grams equal to the molecular weight express in amu.

1 amu = 1/12 of the weight of the C^{12} atom = 1.66 10^{-24} g.

N_A = Number of Atoms in 1 Mole.

$$N_A = \frac{Mass\,1\,Mole \equiv M\left(grams\right)}{1.66\,10^{-24} * M\left(mass\,1\,molecule\ or\ atom\ in\ amu\right)} = \frac{1}{amu}$$

$$N_A = 6.02\,10^{23}\ mole^{-1} \quad N \equiv Density; \quad \mu \equiv Molecular\ weight\ in\ amu$$

$$\rho = N\ \mu\ amu \Rightarrow N = \frac{\rho}{\mu\ amu} = \frac{1}{amu}\frac{\rho}{\mu} = \frac{N_A \rho}{\mu}$$

From Thermodynamics

$$P = NkT = \frac{N_A \rho\, k\, T}{\mu} = \frac{R}{\mu}\rho\, T \qquad R = N_A k \; Gas\; cons\tan t \begin{cases} 8.31\; J\; mole^{-1} K^{-1} \\ 5.19\, 10^{19}\; eV\; mole^{-1} K^{-1} \end{cases}$$

$$dQ = dE + PdV$$

$$c_v = \left(\frac{dQ}{dT}\right)_V = \frac{dE}{dT} \qquad c_p = \left(\frac{dQ}{dT}\right)_P = \frac{dE}{dT} + P\frac{dV}{dT}$$

$$E = c_v\, T$$

$$c_P - c_V = \frac{P}{\rho T} \frac{\left(\frac{\partial lnP}{\partial\, ln\, T}\right)_\rho^2 = 1}{\left(\frac{\partial lnP}{\partial\, ln\, T}\right)_T = 1} = \left(Ideal\; Gas\; above\right) \frac{P}{\rho T} = \frac{R}{\mu}$$

$$\left(By\; definition\right) \gamma = \frac{c_P}{c_V}; \quad c_V\left(\gamma - 1\right) = \frac{R}{\mu}; \qquad E = c_V T = \frac{RT}{\mu\left(\gamma - 1\right)}$$

Gas + Black Body

The two systems gas and photons coexist. We neglect any interaction between the two constituents of the mixture except for what is needed to keep thermal equilibrium. They can be considered as two independent systems. For the Black Body (See Cox and Giuli for instance):

$P = 1/3\ a'\ T^4$ and

$E = a'\ T^4\ V = a'\ T^4 / \rho$ per unit mass

So that I finally have, if I consider a mass ρ:

$$E = a' T^4 + \frac{\frac{R}{\mu} T}{\gamma - 1} \rho \qquad Density = \frac{E}{c^2}$$

$$P_{Total} = n\,k\,T + \frac{1}{3} a' T^4$$

$$\rho_{Gas+Rad} = n\,m + \frac{1}{c^2} (\gamma - 1)^{-1}\, n\,k\,T + \frac{1}{c^2} a' T^4$$

T as a function of a

The total pressure of a system is given by: $P = n\,k\,T + \frac{1}{3} a' T^4$

And the density of Energy is $\rho = n\,m + \frac{1}{c^2}(\gamma - 1)^{-1} n\,k\,T + \frac{1}{c^2} a' T^4$

See Cox and Giuli Page 217:

We conserve the number of particles: $n\,a^3 = n_0\,a_0^3$

And Conserve Energy as an adiabatic expansion.

$$dQ = 0 = dE + PdV \quad \Rightarrow \quad dE = -PdV$$

$$d\left(\rho\,a^3\,c^2\right) = -3\,P\,a^2\,da \Rightarrow \frac{d}{da}\left(\rho\,a^3\,c^2\right) = -3\,P\,a^2$$

$$\frac{d}{da}\left[n\,m\,a^3c^2+\frac{1}{c^2}(\gamma-1)^{-1}c^2 n\,k\,T\,a^3+\frac{1}{c^2}a'T^4\,a^3c^2\right]=-3\,n\,k\,Ta^2-a'T^4\,a^2$$

$$\frac{d}{da}\left[n\,m\,a^3c^2\right]=\frac{d}{da}\left(n_0\,m\,a_0{}^3c^2\right)=0$$

$$\frac{d}{da}\left[(\gamma-1)^{-1}n\,k\,T\,a^3\right]=(\gamma-1)^{-1}n_0\,a_0{}^3k\,\frac{dT}{da}$$

$$\frac{d}{da}\left[a'T^4\,a^3\right]=a'T^4\,3\,a^2+a'\,a^3\,4\,T^3\,\frac{dT}{da}\quad \textit{Put together}$$

$$\frac{dT}{da}\left[a'\,a^3\,4\,T+(\gamma-1)^{-1}n_0\,a_0{}^3k\right]=-3\,n\,k\,Ta^2-4\,a'T^4\,a^2$$

$$\frac{dT}{da}\left[a'\,a^3\,4\,T+(\gamma-1)^{-1}n_0\,a_0{}^3k\right]=\frac{T}{a}\left[-3\,n\,k\,a^3-4\,a'T^3\,a^2\right]$$

Divide by $3\,n\,k\,a^3$

$$\frac{dT}{da}\left[\frac{4}{3}\frac{a'\,T^3}{n\,k}+\frac{1}{3}(\gamma-1)^{-1}\right]=-\frac{T}{a}\left[1+\frac{4\,a'T^3}{3\,n\,k}\right]$$

$$\frac{4}{3}\frac{a'\,T^3}{n\,k}=\sigma=74\frac{\left[T_{degrees}\right]^3}{n\left[cm^{-3}\right]};\quad \sigma\,k\equiv \textit{photon entropy per gas particle}$$

$$\frac{dT}{da}\left[\frac{4}{3}\frac{a'\,T^3}{n\,k}+\frac{1}{3}(\gamma-1)^{-1}\right]=-\frac{T}{a}\left[1+\frac{4\,a'T^3}{3\,n\,k}\right]$$

$$\frac{a}{T}\frac{dT}{da}\left[\sigma+\frac{1}{3}(\gamma-1)^{-1}\right]=-[1+\sigma]$$

$$\frac{a}{T}\frac{dT}{da}=-\left[\frac{1+\sigma}{\sigma+\frac{1}{3}(\gamma-1)^{-1}}\right]\begin{cases}\sigma<<1\\ \frac{a}{T}\frac{dT}{da}=-3(\gamma-1)\,;\,\frac{dT}{T}=-3(\gamma-1)\frac{da}{a}\\ T\propto a^{-3(\gamma-1)}\,adiabatic\ exp\,ansion\ ideal\ gas\\ \sigma>>1\\ \frac{a}{T}\frac{dT}{da}=-1\quad T\propto a^{-1}\end{cases}$$