The Luminosity Function

See Math in Tecra 8100

Thanks for Dr. Paolo Saracco for the example of the L.F. Estimate with V_{max}
to Tagliaferri for catching some typos – Ref. to be completed.

By Guido Chincarini

H_o=50 km/s/Mpc

L^*=3.2 10^{10} $L_\odot$

α = -1.25

$Mv_\odot$=-26.78

Mbol=-26.85

Outline

- The Luminosity Function of various classes of astronomical objects is a fundamental distribution function in Astronomy and Cosmology. Knowledge of this function, and its eventual evolution, allows the estimate of how much background light at different wavelengths a given population contributes. The observations allow the estimate of the evolutionary parameters and insights into the formation and evolution of the objects. Finally it might allow the detection of new populations, the evolution of young star forming galaxies and the history of the metal enrichment in the Universe. To tackle these problems:
- I will discuss the Luminosity Function generalities and in particular briefly discuss the L.F. of:
  - Galaxies
  - AGN and QSOs
  - Clusters of Galaxies
  - Seyfert Galaxies

The Background
see also BH file

- Various processes occurring in the Universe cause the emission of photons. These could interact with matter and degrade afterwards. The birth of a galaxy, for instance, is a dissipative process, as far as the baryonic mass is concerned and energy is emitted in the form of photons.
- The Universe therefore has at any epoch also a radiation component which is due to the baryonic mater by which it is populated.
- As we will see however it exists an other component which is the relic of the hotter early phases of the Universe where radiation was dominating. This is the Microwave Background Radiation. PRIMARY BACKGROUND.
- The study of the background radiation is therefore very important since it gives not only information on the past hisotry of the Universe but also information on the objects emitting radiation that were present at the various epochs after formation.
- It must be observed at various wavelengths because different physical phenomena emit radiation at different wavelengths.
- The question comes whether in some cases the background is due to source confusion or it is real (see X ray background as an example).
- The secondary background is fundamental in the study of the formation and evolution of galaxies.

Units

The unit for νI_ν is:

$$nW\ m^{-2}\, sr^{-1} = 10^{-9}\, 10^{7}\, 10^{-4}\, ergs\ s^{-1}\ cm^{-2}\, sr^{-1} =$$

$$=10^{-6}\, ergs\ s^{-1}\ cm^{-2}\, sr^{-1}$$

The Density of Energy is given by:

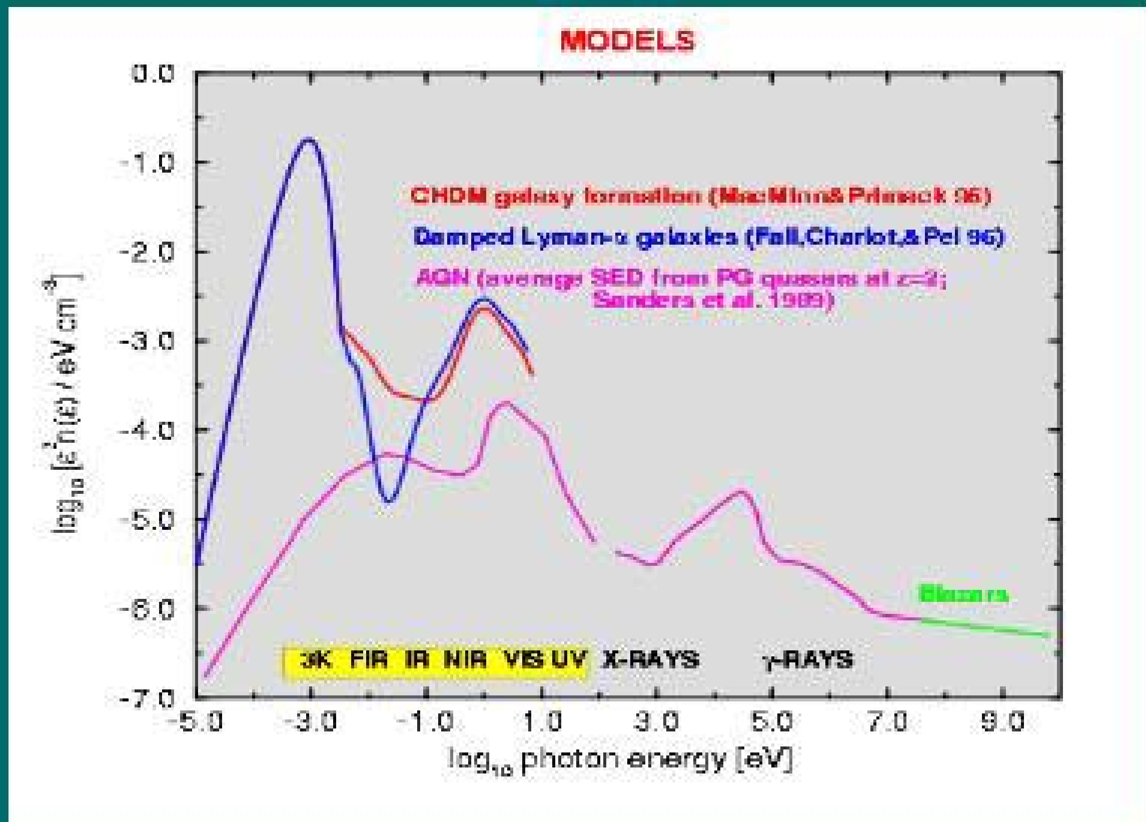
$$4\pi/c\ \nu\ I_\nu\ ergs\ cm^{-3}$$

And I get the density parameter ($\Omega\ h^2$) dividing by $\rho_c\ c^2$. Dividing by c^2 I have a mass and by ρ_c the density parameter we will define later.

I can trasform a plot of $\nu\ I_\nu$ versus wavelength or frequency in a plot of ($\Omega\ h^2$) versus wavelength or frequency.

[see for instance Figure 1.28 in Volume III]

Extragalactic Background Light



Background due to galaxies

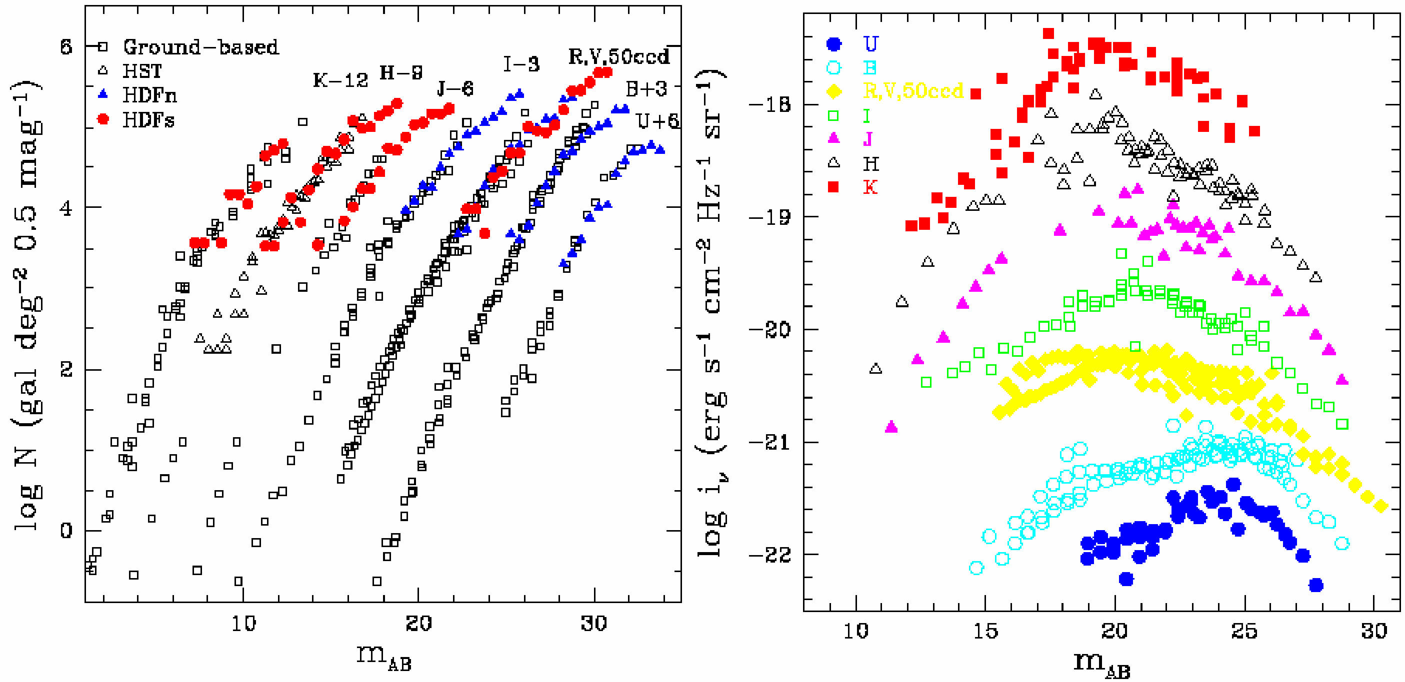


Fig. 1.— *Left:* Differential $UBVIJHK$ galaxy counts as a function of AB magnitudes. The sources of the data points are given in the text. Note the decrease of the logarithmic slope $d \log N/dm$ at faint magnitudes. The flattening is more pronounced at the shortest wavelengths. *Right:* Extragalactic background light per magnitude bin, $i_\nu = 10^{-0.4(m_{AB}+48.6)} N(m)$, as a function of U (*filled circles*), B (*open circles*), V (*filled pentagons*), I (*open squares*), J (*filled triangles*), H (*open triangles*), and K (*filled squares*) magnitudes. For clarity, the $BVIJHK$ measurements have been multiplied by a factor of 2, 6, 15, 50, 150, and 600, respectively.

Sources of photons

- UV to IR originate mainly from star light. The far UV is mainly due to hot massive stars which are the same generating metals. That is the UV is a good tracer of Star Formation and metallicity. Order of magnitude 2-5 nW m^{-2} sr^{-1}. About 10% from AGN and quasars. 50% of the flux may arise from unresolved sources.
- Optical and IR, to about 1000 microns, is complicated. The stellar evolution models show that most of the radiation emission peaks at about 1-1.5 microns. On the other hand here we have also a contribution of light from galaxies that are at high redshift.
- Star formation and star Burst galaxies at high z produce not only high energy photons which are redshifted but, when dust is present, they heats up the dust so that the UV photons are reprocessed and we have a rather intense emission in the far infrared.

Metals

1. The star light (bolometric) is of about 50 nW m^{-2} sr^{-1}.
2. The energy density is [see Lum_Fun_2 in Math] 2.1 10^{-14} erg cm^{-3} => U = 2.1 10^{-14} (1+z) if the emission is occurring at a mean redshift z.
3. Or converting in MeV U=1.3 10^{-8} MeV cm^{-3}
4. Converting Hydrogen to Helium each baryon releases 25 MeV of Energy. Converting to heavy elements Hydrogen finally releases 30 MeV.
5. That is to produce the energy density above I need to transform 1.38 10^{-8}/30 = 4.3 10^{-10} (1+z) baryons cm^{-3}. This is the baryon number density in metals which is involved in the production of the background. N_z = 4.3 10^{-10} (1+z).
6. Assume we have a density in baryons of about 1.1 10^{-7} (this we will derive) then a fraction [(Baryons transformed to form heavy elements)/(Total number of Baryons)] 4.3 10^{-10} (1+z)/ 1.1 10^{-7} = 4 10^{-3} (1+z) in heavy elements produces the light observed.
7. If z=2 Z ~ 0.01

The point

- The point is I can compute the background light which is produced by a population of sources if I know the luminosity function.
- By so doing and by comparing the expected contribution at various wavelengths with the observations I know if the model is correct, if the evolution I use for the sources is reasonable and I may infer the existence of new types of sources.
- In conclusion we must revisit the Luminosity Function.

GALAXIES

- The Luminosity Function describe the Number of galaxies per unit Volume and between Luminosities L and L+dL. I can also define it as the Number of galaxies brighter than the Luminosity L, Integral Luminosity Funcion, and the is the integral of the previous one from L to Infinity.
- Schechter gives the perused analytical formula for the total Luminosity Function, while, as we will see, when we distinguish between Morphological types, we can either fit by a Gauss function or by a Schechter Function. Furthermore I can define the Functions either in Luminosities or in Magnitudes.

$$\Phi(L,x,y,z)dLdV = N(x,y,z)\,\varphi(L)dLdV$$

$$N(x,y,z)\,\varphi(L)dL = \left(\frac{N}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} Exp\left(-\frac{L}{L^*}\right)dL$$

Transforming in Magnitudes

$$\phi(M) \sim \left[10^{0.4\left(M^{*}-M\right)} \right]^{(\alpha+1)} Exp\left(-10\right)^{0.4\left(M^{*}-M\right)}$$

And the Gauss distribution.

$$\phi(M) \sim Exp\left[\frac{\left(M-\mu\right)^{2}}{2\sigma^{2}} \right]$$

An Example

- How many objects do I have within the solid angle $\Delta\Omega$ and redshifts z_1 and z_2? What is the hypothesis I do?

$$N(z_1, z_2) = \Delta\Omega \int_{cz_1}^{cz_2} \left(\frac{cz}{H_o}\right)^2 \frac{cz}{H_o} dz \int_{L_1}^{\infty} \varphi(L) dL$$

  - Where L_1 is the faintest Luminosity I can detect at a given z assuming my sensitivity is limited to a flux f_{lim} corresponding to a limiting magnitude m_l.

$$\int_{L_1}^{\infty} \varphi(L) dL = N\Gamma\left(1+\alpha, {L_1}/{L^*}\right); \quad {L_1}/{L^*} = 4\pi \left(\frac{cz}{H_o}\right)^2 \frac{f_1}{L^*}$$

$$N(0,\infty) = \Delta\Omega \int_0^{\infty} \left(\frac{cz}{H_o}\right)^2 \frac{c}{H_o} dz \int_{L_1}^{\infty} \varphi(L) dL = 10^{0.6\, m_l} = Normalization$$

Functions used

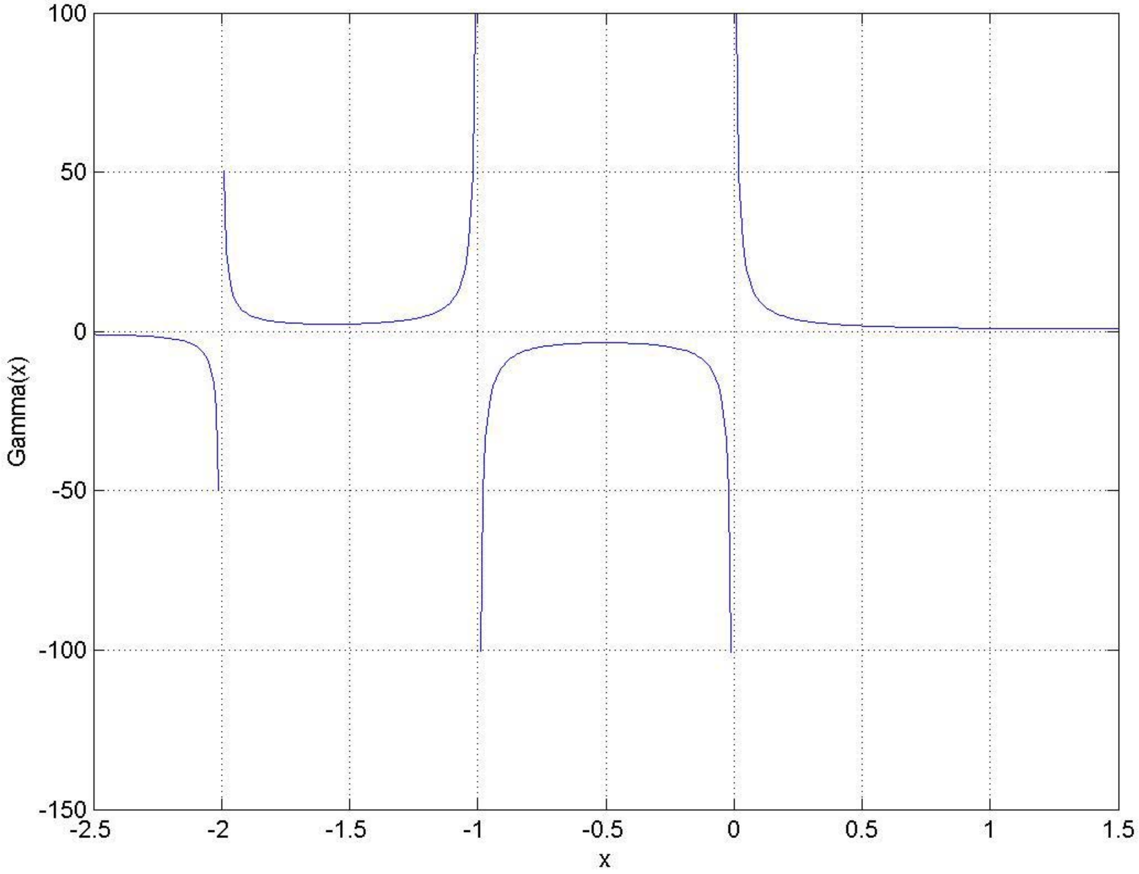
$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} dx; \quad \Gamma(\alpha+1) = \int_0^\infty e^{-x} x^{\alpha} dx$$

$$\Gamma(\alpha, y) = \int_y^\infty e^{-x} x^{\alpha-1} dx$$

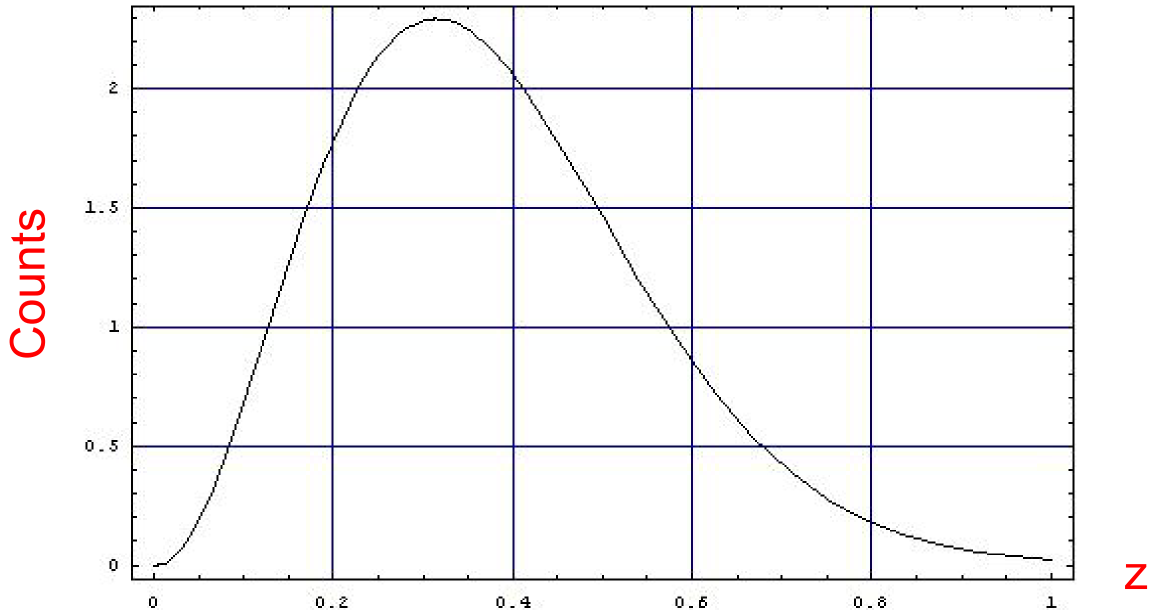
$$N_{Tot} = \int_0^\infty \varphi(L) dL = N \Gamma(\alpha+1)$$

$$L_{Tot} = \int_0^\infty L\varphi(L) dL = N\ L^* \Gamma(\alpha+2)$$

The Gamma Function

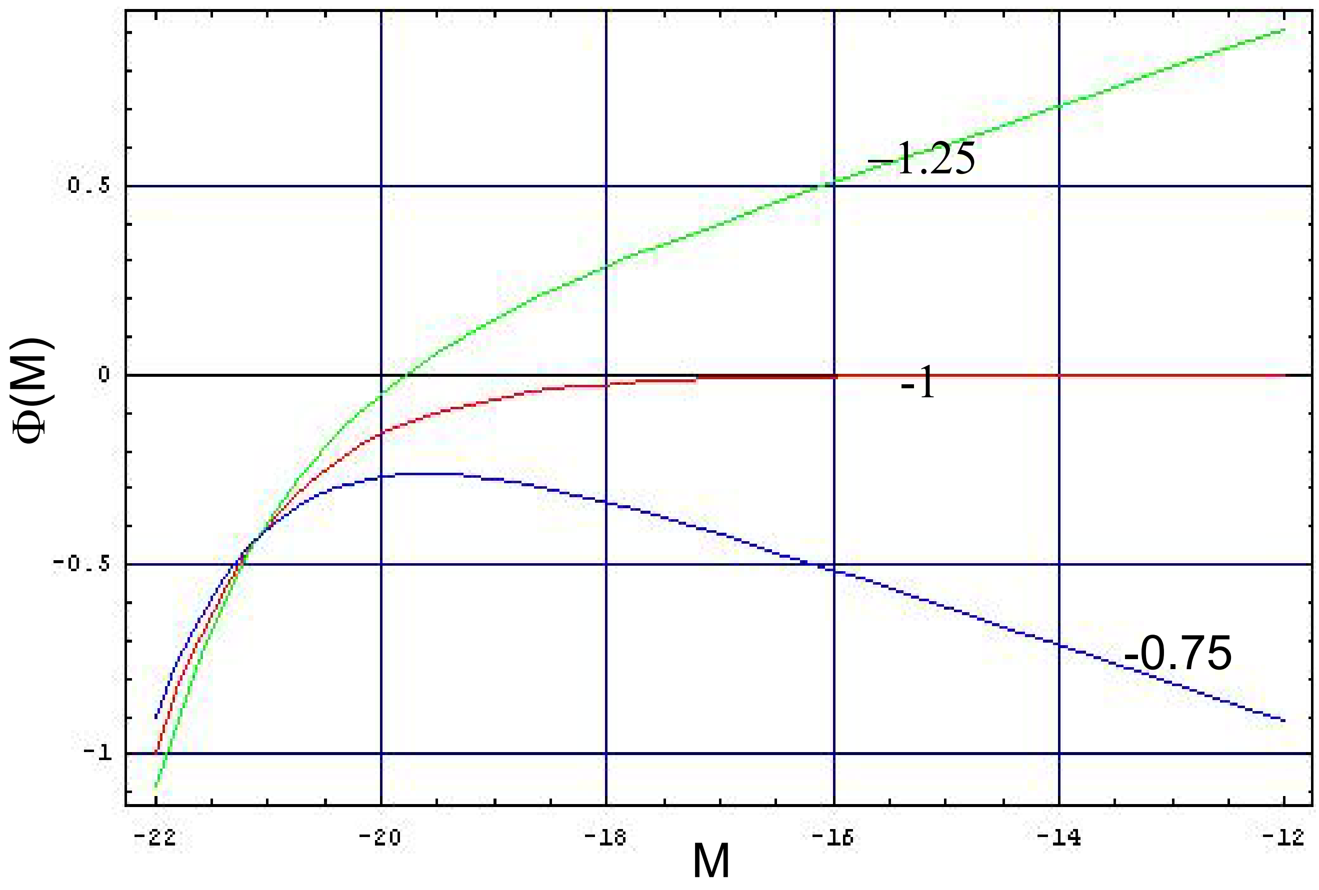


The Plot

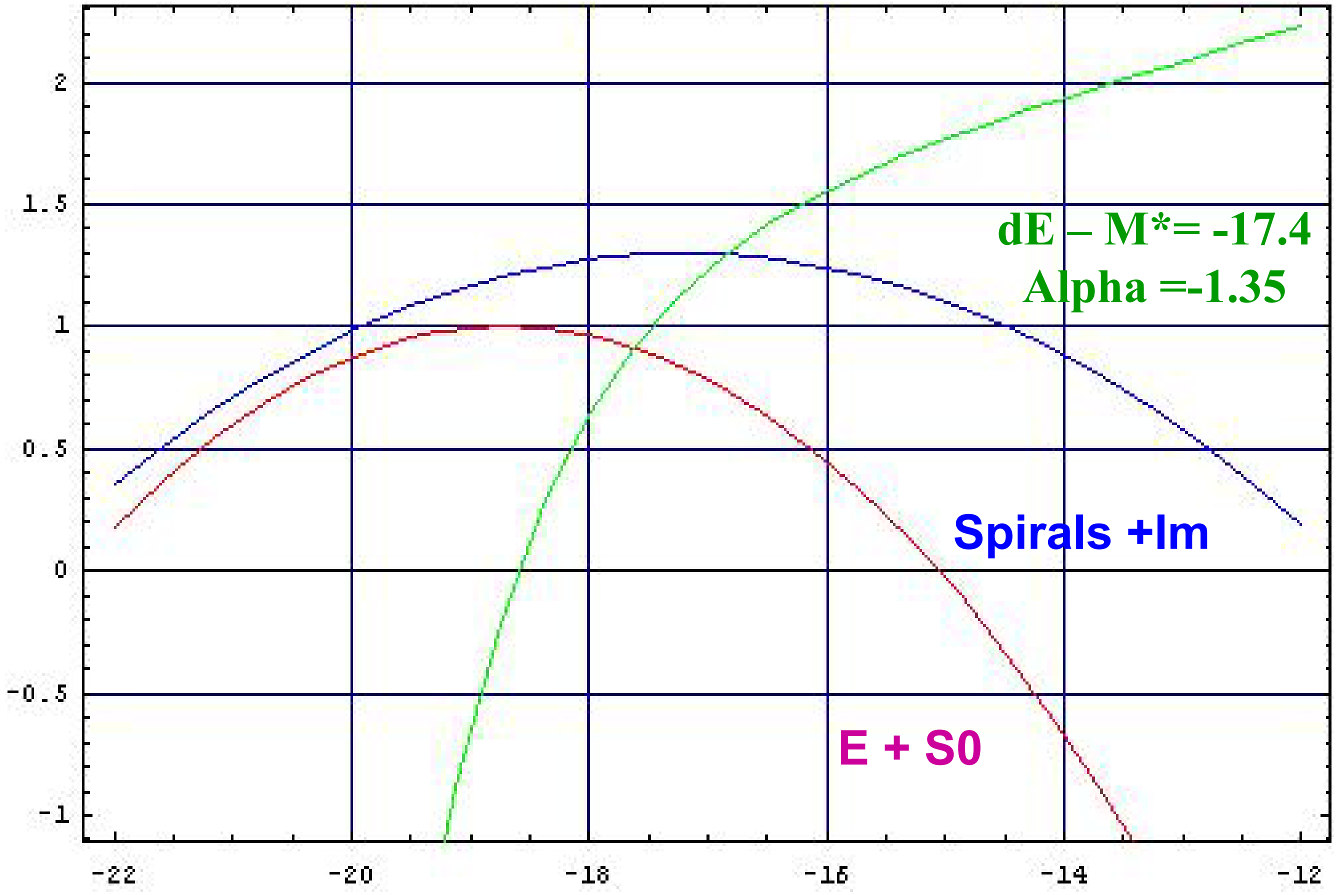


- This is only indicative since we used non cosmological relations and these must be used for z≥0.1. The student practice using the cosmological relation for Luminous distance. In addition look at the counts of galaxies and the redshifts surveys. Compare the redshift distribution of various samples with the distribution expected for the limited magnitude of the sample. See also later slides.

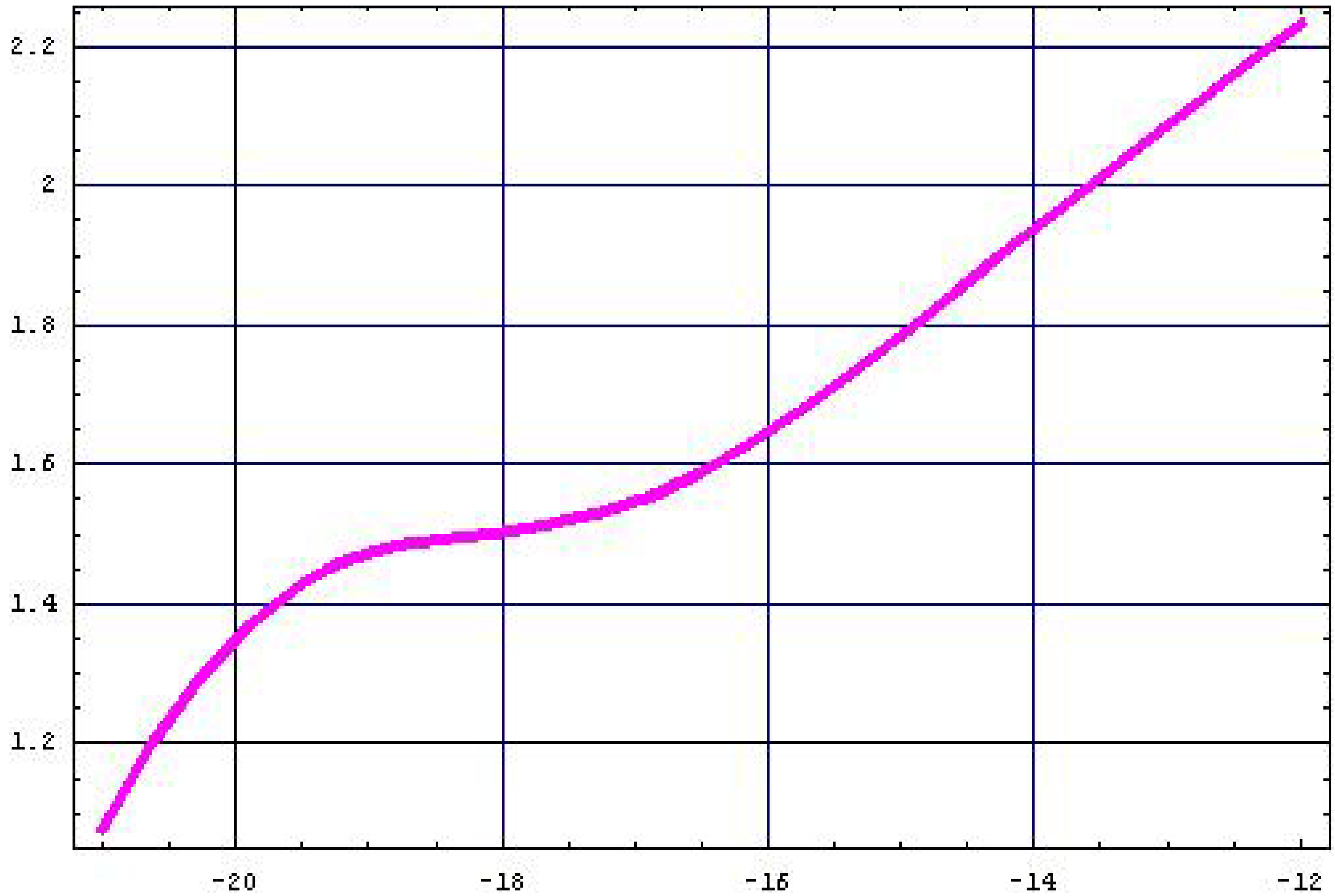
How α works

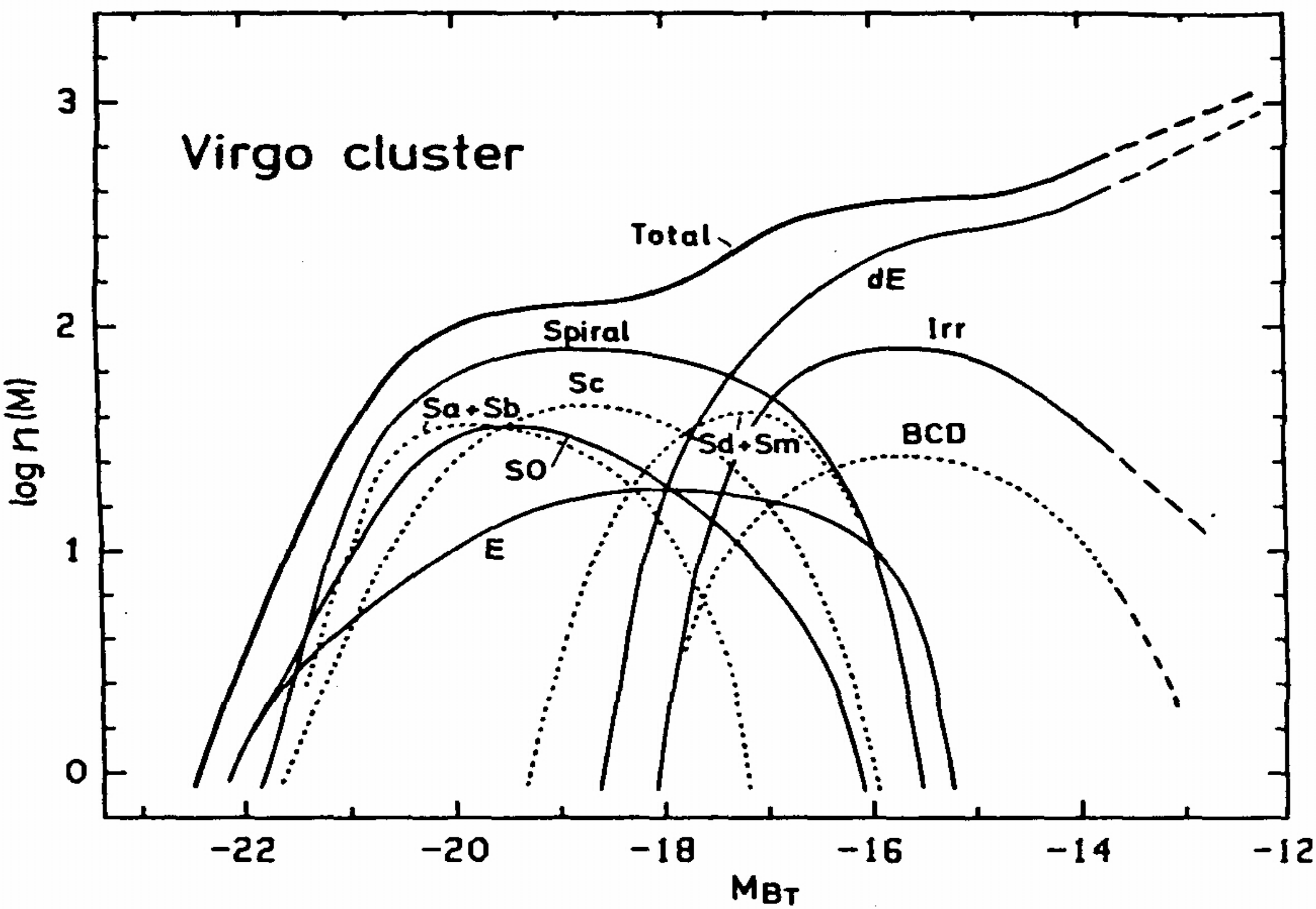


Gauss ~ Virgo



The composite L.F.





Virgo cluster
Total
dE
Spiral
Irr
Sc
Sa+Sb
Sd+Sm
BCD
SO
E
log n (M)
MBT
3
2
1
0
-22
-20
-18
-16
-14
-12

Bias (Malmquist)

- When I have a sample limited in apparent magnitude, as it is usually the case, at a given distance I will be capable to detect only those sources which satisfy the relation:

$$d \le d_{max} = 10^{0.2(m_{lim} - M - 25)}$$

- Each galaxy, depending on its Luminosity, will sample a characteristic Volume – solid angle $\Delta\Omega$, which differs from galaxy to galaxy. Since $\varphi(L)=N(L)/V$ we have $N(L)=V\,\varphi(L)$.
- Due to a selection effect each galaxy contributes in a different way to the volume under consideration because of a simple selection effect.
- This means that when we estimate the function we have to account for the volume in which they can be seen.
- See Schmidt 1975 and Felten 1976.

Derivation

$$\varphi\left(M_j-\frac{\Delta M}{2};M_j+\frac{\Delta M}{2}\right)=\sum_{i=1}^{n_j}\frac{1}{V_{max}\left(M_i\right)};\ M_i\in\left[M_j-\frac{\Delta M}{2};M_j+\frac{\Delta M}{2}\right]$$

$$V_{max}\left(M_i\right)=\frac{\Omega}{3}d_{max}^3\left(M_i\right)=\frac{\Omega}{3}10^{0.6\left(m_{lim}-M_i-25\right)}$$

The above comes directly from the relation of the previous page:

- j — indicate the j bin – width of the bin = ΔM.
- n_j — the number of galaxies that are in the j bin of Mag.
- M_i — the absolute magnitude of the i galaxy within the j bin.

Example

Data	$M_{1=}$-20.0, $M_{1=}$-20.4, $M_{1=}$-20.2 $m_{lim} = 20$
Bin Size	0.5 mag
Bin Mag	$M_j = -20.25$
φ (-20.5, -20)	$\Sigma_{for\ i=1\ to\ 3}\ 1/V_{max}(M_i) = 1/V_m(M_1) + 1/V_m(M_2) = 1/V_m(M_3)$
$V_m(M_1)$;	$\Omega/3\ 10^{0.6(20-20+25)} \sim \Omega/3\ 10^9;\ 2\ \Omega/3\ 10^9;\ 1.4\ \Omega/3\ 10^9$
φ (-20.5, -20)	$\sim 3/\Omega\ (10^{-9} + 5\ 10^{-10} + 7\ 10^{-10}) \sim 6.6/\Omega\ Mpc^{-3}$

Propagation of Errors

$$\varphi_j = \sum_{i=1}^{n_j} \frac{1}{V_{max}(M_i)} = \int_{M_j-\frac{\Delta M}{2}}^{M_j+\frac{\Delta M}{2}} \frac{\delta_i}{V_{max}(M_i)} dM; \quad \delta_i = \begin{cases} 1 \text{ if } & M_i \in [\Delta M_j] \\ 0 & \text{otherwise} \end{cases}$$

$$f(x,y) \quad \Rightarrow \sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\sigma_x\right) + \left(\frac{\partial f}{\partial y}\sigma_y\right)}$$

$$\sigma_\varphi = \sqrt{\sum_{i=1}^{n_j}\left(\frac{\partial}{\partial \delta_i}\int_{\Delta M_j}\frac{\delta_i}{V_{max}(M_i)}\right)^2 \sigma_{\delta_i}^2} = \sqrt{\sum_{i=1}^{n_j}\left(\frac{1}{V_{max}(M_i)}\right)^2 \sigma_{\delta_i}^2}; \quad \sigma_{\delta_i}^2 = \left(\sqrt{\delta_i}\right)^2 = 1$$

$$\varphi_j = \sum_{i=1}^{n_j} \frac{1}{V_{max}(M_i)} \pm \sqrt{\sum_{i=1}^{n_j}\left(\frac{1}{V_{max}(M_i)}\right)^2}$$

A different way

See Sandage, Tamman & Yahil & possibly my old notes

$$f(m) = \frac{1}{e^{\frac{m-ml}{\Delta ml}} + 1}$$

Completeness Function

If I define with $P(M,V_0)$ the probability that a galaxy at the redshift v_0 Will have an absolute magnitude brighter than M:

$$P(M,v_0) = \frac{\int_{-\infty}^{M} \varphi(M) D(v_0) f(m)\, dM}{\int_{-\infty}^{\infty} \varphi(M) D(v_0) f(m)\, dM}$$

The differential probability is

$$p(M,v_0) = \frac{\partial P(M,v)_0}{\partial M}$$

And I use the Maximum Likelyhood

$$L = \prod_i p(M_i, v_{0,i})$$

Play with it – See also Peebles

In a more general form let's define as follows:

$$\phi(L/L^*) = \left(\frac{N}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} Exp\left(-\frac{L}{L^*}\right) \equiv \varphi^* y^{\alpha} e^{-y} \;;\; \frac{L}{L^*} = y \;;\; \left(\frac{N}{L^*}\right) = \varphi^*$$

$$j = \int_0^\infty L\phi(L/L^*)\, d\frac{L}{L^*} = L^*\varphi^* \int_0^\infty y^{\alpha+1} e^{-y} dy = L^*\varphi^* \Gamma\left(\alpha+2\right)$$

$$\& \cdots \text{for small } z$$

$$r = \frac{cz}{H_0} \;;\; dr = \frac{c}{H_0} dz \;;\; f = \frac{L}{4\pi r^2}$$

Mean z for a flux (f) limited sample
Unit solid angle

$$d^2N = \phi\left(\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right) r^2 dr\,; \qquad \frac{L}{L^*} = \frac{4\pi r^2 f}{L^*} = \frac{4\pi f}{L^*}\frac{c^2 z^2}{H_0^2} = kz^2$$

$$d\left(\frac{L}{L^*}\right) = d\left(kz^2\right) = z^2 dk = \frac{4\pi}{L^*}\frac{c^2 z^2}{H_0^2} df\,;$$

$$d^2N = \phi\left(kz^2\right) \frac{4\pi}{L^*}\frac{c^2 z^2}{H_0^2} df \frac{c^2 z^2}{H_0^2}\frac{c\,dz}{H_0}$$

$$\frac{d^2N}{df\,dz} = \frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5 z^4 \phi\left(kz^2\right)$$

$$< z >= \int_0^\infty z \frac{d^2N}{df\,dz} dz = \sqrt{\left(\frac{H_0 L^*}{4\pi c^2 f}\right)} \frac{\Gamma(\alpha+3)}{\Gamma\left(\alpha + {}^5\!/_2\right)}$$

see next

Derivation

$$z^2 = \frac{y}{k};\ 2\,z\,dz = \frac{1}{k}dy$$

$$\frac{d^2N}{dfdz} = \frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5\left(\frac{y}{k}\right)^2\phi(y)$$

$$\int z\frac{d^2N}{dfdz}dz = \int z\,\frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5\left(\frac{y}{k}\right)^2\phi(y)\,\frac{1}{k\,2z}dy$$

$$= \int y^2\phi(y)\,dy\,\frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5\frac{1}{k^3}\,\frac{1}{2};\quad since\ k = \frac{4\pi fc^2}{H_0^2L^*}$$

$$\int z\frac{d^2N}{dfdz}dz = \int y^2\phi(y)\,dy\,\frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5\frac{H_0^6L^{*3}}{(4\pi f)^3\,c^6}$$

Normalization to the Total Number of objects between 0 and ∞

$$\int \frac{d^2N}{dfdz}dz = \int \frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5 \left(\frac{y}{k}\right)^2 \phi(y)\left(\frac{y}{k}\right)^{-\frac{1}{2}} \frac{1}{k}dy\frac{1}{2} =$$

$$= \int \frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5 \phi(y)\frac{y^{\frac{3}{2}}}{k^{\frac{5}{2}}}dy\frac{1}{2}$$

$$\frac{\int z\frac{d^2N}{dfdz}dz}{\int \frac{d^2N}{dfdz}dz} = \frac{\int y^2\phi(y)dy}{\int y^{\frac{3}{2}}\phi(y)dy}\left(\frac{1}{k}\right)^{\frac{1}{2}}; \quad \text{since} \int y^{\frac{2}{2}}\phi(y)dy = \Gamma(\alpha+2)$$

$$<z> = \frac{\int y^2\phi(y)dy}{\int y^{\frac{3}{2}}\phi(y)dy}\left(\frac{1}{k}\right)^{\frac{1}{2}} = \frac{\Gamma(\alpha+3)}{\Gamma\left(\alpha+\frac{5}{2}\right)}\left(\frac{H_0^2L^*}{4\pi c^2 f}\right)^{\frac{1}{2}}$$

Counts – Galaxies per unit ln flux interval

$$\int \frac{d^2N}{df\,dz}dz = \int \frac{4\pi}{L^*}\left(\frac{c}{H_0}\right)^5 \phi(y)\frac{y^{\frac{3}{2}}}{k^{\frac{5}{2}}}dy\frac{1}{2} \Leftrightarrow \textit{Total Counts}$$

$$= \frac{1}{2}\int y^{\frac{3}{2}}\phi(y)\,dy\,\frac{\left(L^*\right)^{\frac{3}{2}}}{4\pi^{\frac{3}{2}}}\frac{1}{f^{\frac{5}{2}}} = \frac{dN}{df}$$

$$f\frac{dN}{df} = \frac{dN}{d\ln f} = \frac{1}{f^{\frac{3}{2}}}\frac{\left(L^*\right)^{\frac{3}{2}}}{4\pi^{\frac{3}{2}}}\frac{1}{2}\int y^{\frac{3}{2}}\phi(y)\,dy = \frac{\phi^*}{2}\left(\frac{L^*}{4\pi f}\right)^{\frac{3}{2}}\Gamma\left(\alpha+\frac{5}{2}\right)$$

Converting to magnitudes

$$L = L^* 10^{-0.4\left(M - M^*\right)}; \quad at\ 10\ pc\ by\ definition\ \left(m - M\right) = 0; \quad L = f\, 4\pi \left(10\, pc\right)^2$$

$$f = \frac{L^*}{4\pi \left(10\, pc\right)^2} 10^{-0.4\left(m - M^*\right)}$$

$$< z >= \frac{\Gamma\left(\alpha+3\right)}{\Gamma\left(\alpha+\frac{5}{2}\right)} \left(\frac{H_0^2 L^*}{4\pi c^2 f}\right)^{\frac{1}{2}} = \frac{\Gamma\left(\alpha+3\right)}{\Gamma\left(\alpha+\frac{5}{2}\right)} \left[\frac{H_0^2 L^*}{4\pi c^2} \frac{4\pi\left(10\, pc\right)^2}{L^* 10^{-0.4\left(m-M^*\right)}}\right]^{\frac{1}{2}} =$$

$$< z >= \frac{h\, 100}{c} 10^{-0.2\left(m-M^*\right)} \frac{\Gamma\left(\alpha+3\right)}{\Gamma\left(\alpha+\frac{5}{2}\right)} = \frac{h}{3\, 10^8} 10^{-0.2\left(m-M^*\right)} \frac{\Gamma\left(\alpha+3\right)}{\Gamma\left(\alpha+\frac{5}{2}\right)}$$

From the Observations

$$\langle z \rangle = 10^{0.2\,m - 4.53}$$

And $\alpha = 1.07 \pm 0.05$ *see Zucca et al. However*

And comparing $\langle z \rangle_{computed}$ with $\langle z \rangle_{observed}$ we derive

$$M^* = -19.53 \pm 0.25 + 5 \text{ Log } h$$

Using $M_\odot = 5.48$

$$L^* = 1.0\ 10^{10}\ e^{\pm 0.23}\ h^{-2}\ L_\odot$$

And to get φ*

$$\frac{1}{<z>^3}\frac{dN}{dm}=\frac{\Gamma^3\left(\alpha+\frac{5}{2}\right)}{\Gamma^3\left(\alpha+3\right)}\left(\frac{H_0^2L^*}{4\pi c^2 f}\right)^{-\frac{1}{2}}\frac{dN\,\ln 10}{2.5\,d\ln f}=$$

$$=\frac{\Gamma^3\left(\alpha+\frac{5}{2}\right)}{\Gamma^3\left(\alpha+3\right)}\left(\frac{H_0^2L^*}{4\pi c^2 f}\right)^{-\frac{3}{2}}\frac{\ln 10}{2.5}\frac{\phi^*}{2}\left(\frac{L^*}{4\pi f}\right)^{\frac{3}{2}}\Gamma\left(\alpha+\frac{5}{2}\right)$$

$$\frac{1}{<z>^3}\frac{dN}{dm}=0.2\ln 10\quad\phi^*\left(\frac{c}{H_0}\right)^3\frac{\Gamma^4\left(\alpha+\frac{5}{2}\right)}{\Gamma^3\left(\alpha+3\right)}$$

The Observations

$$dN/dM = 10^{-5.7 \pm 0.1 + 0.6\, m} \; str^{-1} mag^{-1}$$

$$\langle z \rangle = 10^{0.2\, m\, -4.53}$$

$$\phi^* = 0.01\, e^{\pm 0.4}\; h^3\, Mpc^{-3}$$

Mean Luminosity per Unit Volume

$$j = \Gamma(\alpha+2)\,\phi^* L^* = (using\ \alpha = 1.05) = 1.0\ 10^8\ e^{\pm 0.26} h\ L_\odot Mpc^{-3}$$

$$\frac{j}{L^*} = n^* = \Gamma(\alpha+2)\,\phi^* = 0.01\, e^{\pm 0.4} h^3\ Mpc^{-3}$$

$$d^* = \left(n^*\right)^{-\frac{1}{3}} = 4.7\ e^{\pm 0.13} h^{-1}\ Mpc; \quad n^* \left(\frac{c}{H_0}\right)^3 \sim 3\ 10^8$$

$$\rho = j\frac{\mathfrak{M}}{L} = 1.0\ 10^8\ e^{\pm 0.26} h\ L_\odot Mpc^{-3}\ 12\ \frac{\mathfrak{M}_\odot}{L_\odot} \Rightarrow see\ next$$

$$= 8\ 10^{32}\ h^2 g\ cm^{-3} = 5\ 10^{-8}\ h^2\, protons\ cm^{-3}$$

$$\Omega = \frac{\rho}{\rho_c} = 0.004\ e^{\pm 0.3}\ ; \quad \rho_c = \frac{3\ H_0^2}{8\pi G}$$

Where to look

From Faber and Gallagher:

$$M/L = 12\, e^{\pm 0.2}\, M_\odot/L_\odot\, h$$

That is within SB = 26.5 Holmberg Radius

O5 Star

$$M/L = 3.3\, 10^{-3}\, M_\odot/L_\odot$$

M5 Star

$$M/L = 200\, M_\odot/L_\odot$$

Abell's Lum. Func.- History –

And a derivation due to Chincarini – See also Rio de Janeiro and LSS

- Preliminaries:
  - Holmberg in: Stars and Stellar systems Vol IX Galaxies and the Universe Page 123
  - Zwicky: Morphological Astronomy - 1957 - Springer Verlag
  - See also Peebles: Physical Cosmology – 1971 – Princeton Series in Physics.

$$N\left(<M^{*}\right)=A\,10^{\alpha M};\quad \alpha=0.75;\quad M<M^{*}$$

$$N\left(>M^{*}\right)=B10^{\beta M};\quad \beta=0.25;\quad M>M^{*}$$

$$A\,10^{\alpha M^{*}}=B10^{\beta M^{*}}$$

$$M^{*}\sim-18.6+5\log h;\quad h=\frac{H}{100}$$

Some Applications

by Chincarini – See Nature and also Rio de Janeiro Lectures

Compute the number of galaxies expected in the velocity bin V_i V_j in a sample which is limited by the apparent magnitude (flux). Assume an uniform distribution.

Ref Nature 272, 515 (1978).

$$N\left(V_i,V_j\right)=\Delta\Omega\int_{\frac{V_i}{H}}^{\frac{V_j}{H}} x^2\phi\left(x\right)dx; \quad x=\frac{V}{H};\ for\ M<M^*$$

$$\phi\left(x\right)=f\left[M=m-5\,log\left(\frac{x}{Mpc}\right)-25\right]=C\,10^{\alpha\left(m-5\,log\,x-25-M^*\right)}$$

$$\beta\ if\ M>M^*;C=A\,10^{\alpha M^*}=B\,10^{\beta M^*}$$

$$\phi(x) = C\,10^{\alpha\left(m-5\log x-25-M^*\right)} = C\,10^{5\alpha\left\{\frac{\left(m-M^*\right)}{5}-5\right\}-5\,\alpha\,\log x} =$$

$$C\,10^{5\alpha\left\{0.2\left(m-M^*\right)-5\right\}}\left(10^{\log x}\right)^{-5\alpha}\ ;\ defining:$$

$$D^* = 10^{\left\{0.2\left(m-M^*\right)-5\right\}};\quad V^* = HD^*;\ and\ having\ \left(10^{\log x}\right)^{-5\alpha} \equiv x^{-5\alpha}$$

$$\phi(x) = \begin{cases} C\left(\dfrac{D^*}{x}\right)^{5\alpha} \\ C\left(\dfrac{D^*}{x}\right)^{5\beta} \end{cases}$$

$$\int_{\frac{V_i}{H}}^{\frac{V_J}{H}} C\left(\frac{D^*}{x}\right)^{5\beta} x^2\,dx = C\left(D^*\right)^{5\beta}\frac{1}{3-5\beta}\Bigg|_{\frac{V_i}{H}}^{\frac{V_j}{H}} x^{3-5\beta} =$$

$$\left(D^*\right)^{5\beta}\frac{C}{3-5\beta}\left\{\left(\frac{V_j}{H}\right)^{3-5\beta} - \left(\frac{V_i}{H}\right)^{3-5\beta}\right\}$$

In conclusion:

$$For\ M > M^* \equiv V_i < V_j < V^*, \quad since \quad H^{3-5\beta} = \frac{\left(V^*\right)^{3-5\beta}}{\left(D^*\right)^{3-5\beta}}$$

$$\int_{\frac{V_i}{H}}^{\frac{V_J}{H}} C\left(\frac{D^*}{x}\right)^{5\beta} x^2 dx = \left(D^*\right)^3 \frac{C}{3-5\beta}\left\{\left(\frac{V_j}{V^*}\right)^{3-5\beta} - \left(\frac{V_i}{V^*}\right)^{3-5\beta}\right\} and$$

$$N\left(V_i, V_j\right) = \frac{\Delta\Omega\, C\left(D^*\right)^3}{3-5\beta}\left\{\left(\frac{V_j}{V^*}\right)^{3-5\beta} - \left(\frac{V_i}{V^*}\right)^{3-5\beta}\right\}$$

& equally

$$For\ M < M^* \equiv V^* < V_i < V_j$$

$$N(V_i,V_j) = \frac{\Delta\Omega\, C\,(D^*)^3}{3-5\alpha}\left\{\left(\frac{V_j}{V^*}\right)^{3-5\alpha} - \left(\frac{V_i}{V^*}\right)^{3-5\alpha}\right\} \quad and$$

$$For\ M < M^* \equiv V_i < V^* < V_j$$

$$N(V_i,V_j) = \frac{\Delta\Omega\, C\,(D^*)^3}{3-5\alpha}\left\{\left(\frac{V_j}{V^*}\right)^{3-5\alpha} - 1\right\} + \frac{\Delta\Omega\, C\,(D^*)^3}{3-5\beta}\left\{1 - \left(\frac{V_i}{V^*}\right)^{3-5\beta}\right\}$$

Normalization &
Peak of the distribution

$$N(0,\infty) = 1.9\,C\left(D^*\right)^3 \Delta\Omega = from\ Counts = Cnst\ 10^{0.6\,m}$$

$$I\ derive\ C$$

$$The\ distribution\ peaks\ at\ V^*\ and\ for\ a\ sample\ limited\ at\ m_l$$

$$V^* = H\ D^* = H\ 10^{0.2\left(m_l - M^*\right)-5} = H\ 10^{0.2\,m_l - 0.2\left(-18.6+5\log h\right)-5}$$

$$V^* = 5.248\ 10^{0.2\,m_l}$$

AGN

- The AGN and QSO are at cosmological distance so that we must work with the Luminosity Distance as defined in Cosmology. This of course is also true for distant galaxies ans celestial objects of any type at $z > 0.1$.
- It becomes also important to consider that the band in which we observe is different by the band in which the radiation has been emitted. That is we must apply the k correction.
- As for distant galaxies looking back in time we might get variations both in the Luminosity and in the Number of galaxies per unit volume. This must be accounted for or, more precisely, the measures of the Luminosity Function will tell us about evolution.
- AGN and starbusrst galaxies, as we will see later on, will give us information about the Background (baryons) and eventually over the accretion rate of Black Holes.

Number of Objects

$$d_L = \sqrt{\frac{L}{4\pi F}}$$

$$N(>F) = \int dL\, V[d_L(L,F)]\,\frac{dn}{dL}; \quad V = d_L^3\left(\Delta\Omega/3\right)$$

$$N_{Eucl}\left(>F\right) = \frac{\Delta\Omega}{3\left(4\pi\right)^{3/2}} F^{-3/2} \int dL\, L^{3/2}\,\frac{dn}{dL}$$

And for a non Euclidean Universe

$$N\left(>F\right) = \int dL \int_0^{r_1(d_L)} dr\;\Delta\Omega\,\frac{r^2}{\left(1-kr^2\right)^{1/2}}\,\frac{dn}{dL}\left(r\right)$$

That is

- For an Euclidean Universe and a population that is uniform and spherically symmetric distributed on large scale we have $N(\geq F) \propto F^{-3/2}$ and this is not affected by the shape of the Luminosity Function.
- For large distances, however, we must use the cosmological Luminosity Distance which is strongly model dependent. It becomes practically impossible therefore to disentangle the models from the evolutions of the sources.
- In addition the K correction is generally not very well known for high redshift objects and this cause another uncertainty.
- It seems that the best way is to estimate the world models using different techniques, the MWB for instance, and then use the observations and the proper models to estimate evolution.

The K Correction

$$m_{z=0} = m_{obs} - K(z); \quad K(z) = K_1 + K_2$$

$$K_1 = 2.5\, Log(1+z);\; K_2 = 2.5\, Log \frac{\int_0^\infty F(\lambda)\, S(\lambda)\, d\lambda}{\int_0^\infty F\left(\frac{\lambda}{1+z}\right) S(\lambda)\, d\lambda}$$

$$and\;\; for\; F_\nu = C\, \nu^{-\alpha}$$

$$K(z) = 2.5\,(\alpha - 1)\, Log\,(1+z)$$

O.K. Straight Forward the check – see Notes and Problems for students.

The observations m_B

Units of N Counts per square degree.

$$Log\ N\left(\prec m_B\right) = \begin{cases} 0.88\, m_B - 16.11 & \left(for\ m_B \prec 18.75\right) \\ 0.31\, m_B - 5 & \left(for\ m_B \succ 19.5\ \right) \end{cases}$$

$$or$$

$$Log\ N\left(\prec m_B\right) = 0.58\, m_B - 11.4$$

$$and\ \ for\ high\ z$$

$$\phi(L,z)dL = \phi^{*}\left\{\left[\frac{L}{L^{*}\left(z\right)}\right]^{-\alpha} + \left[\frac{L}{L^{*}\left(z\right)}\right]^{-\beta}\right\}^{-1}\frac{dL}{L^{*}\left(z\right)}$$

In Magnitudes (M_B) for the constants

We must conserve the Number of Objects

$$\phi(M,z)\,dM = \phi(L,z)\,dL = \phi(L,z)\left|\frac{dL}{dM}\right|dM\;;\; M = M^* - 2.5\,Log\left(\frac{L}{L^*(z)}\right)$$

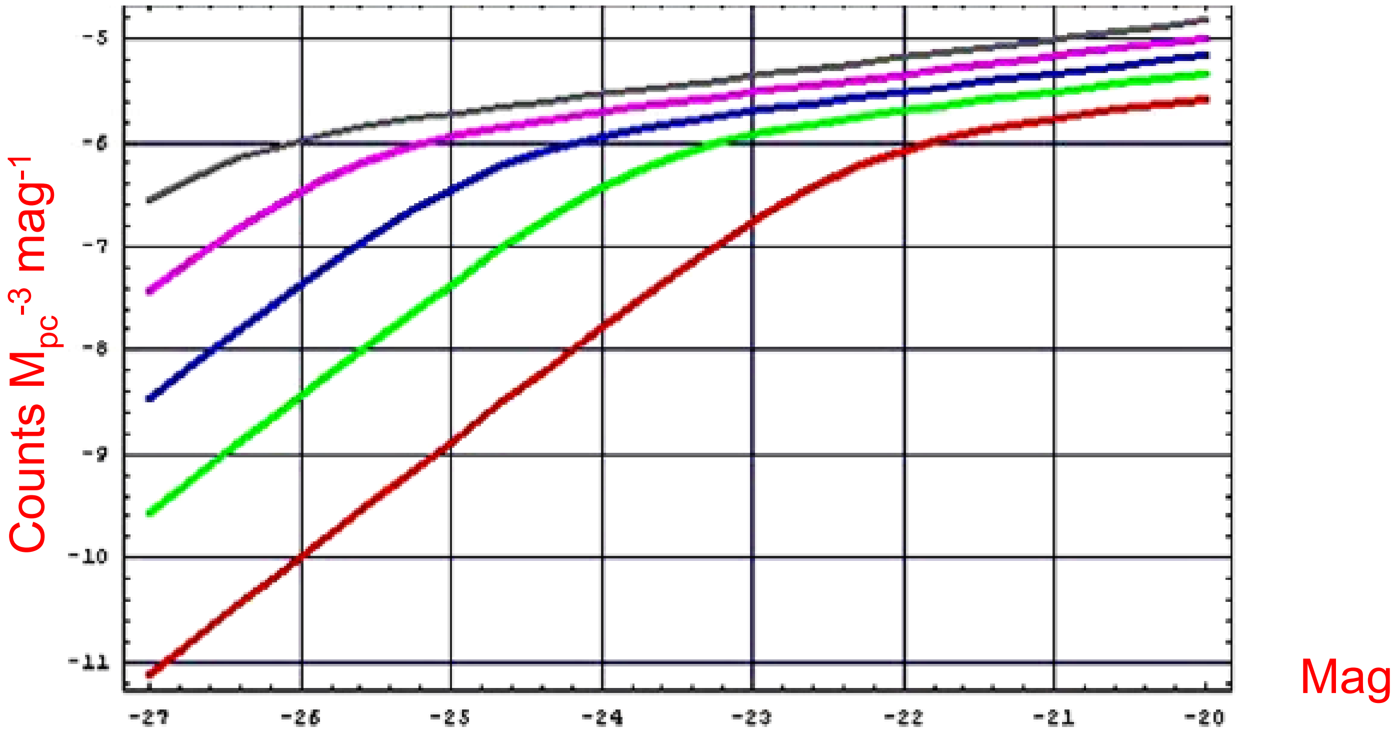
$$\phi(M,z)\,dM = \frac{\phi^*\,dM}{10^{0.4\left[M-M^*(z)\right](\alpha+1)} + 10^{0.4\left[M-M^*(z)\right](\beta+1)}}$$

$$M^*(z) = \begin{cases} A - 2.5\,k\,log(1+z) & (for\ z<z_{max}) \\ M^*(z_{max}) & (for\ z>z_{max}) \end{cases}$$

$$\alpha = -3.9\;;\;\beta = 1.5\;;\;A = -20.9 + 5Logh\;;\;k = 3.45$$

$$z_{max} = 1.9\;;\;\phi^* \sim 6\;10^{-7}\;h^3\;Mpc^{-3}mag^{-1}$$

Counts Plot



- Luminosity Function of Quasars according to the observations and fits carried out by Boyle et al. (1988). From red z=0 to z=0.5, z=1, z=1.65, z=2.45 (Dark Grey).

QSO at high z – z > 2.2

- Warren, Hewett & Osmer (1995) observe 85 quasars with z >2.2 and adding data from the literature analyze a sample of 100 quasars. They also study the complete samples of quasars in the range 2.0 < z <2.2.
- The luminosity function in the range 2.0 < z < 3.5 is welld described by a double power law form with pure evolution similar to what we have given above.
- In this case however they use for $M^*(z)$ the rest frame continuum absolute magnitude at 1216 A.
- $M^*(z)$ is given by the relation $M^*(z) = M_{lim} - 1.086\, k_L \tau$ where τ is the look back time.
- The constants in the equations for a flat Universe (q_o=0.5) with H_o=75 km /sec/Mpc are:
- α =-5.05 ; β =-2.06 ; M_{lim} = -13.21 ; k_L = 10.13 ; log ϕ^* = -0.99.
- By adding objects in the range 3.0 < z < 4.5 the significance of the fit decreases considerably and that may mean that positive luminosity evolution ceases at a ~ 3.3. Non evolving luminosity function for z >3.3.
- The number of objects expected under this assumption in the range 3.5 < z < 4.5 if of 52 objects while only 8 are observed.
- However very small statistics, take all this with caution.

Palomar atransit Grism Survey PTGS

- Schneider Schmidt and Gunn (1994) and Schmidt, Schneider and Gunn (1995) find in their sample [90 quasars in the redshift sample 2.75 < z < 4.75] that the ratio of the effective volume to the accessible volume is $(V_e/V_a)=0.377\pm0.026$.
- Assuming the sample is complete that would mean a decrease of quasars with increasing redshift.
- Due to the small range in Luminosity, the LF is valif only in the range $-27.5 < M_B < -25.5$, they can fit only a power law as given below (Here $H_o = 50$):

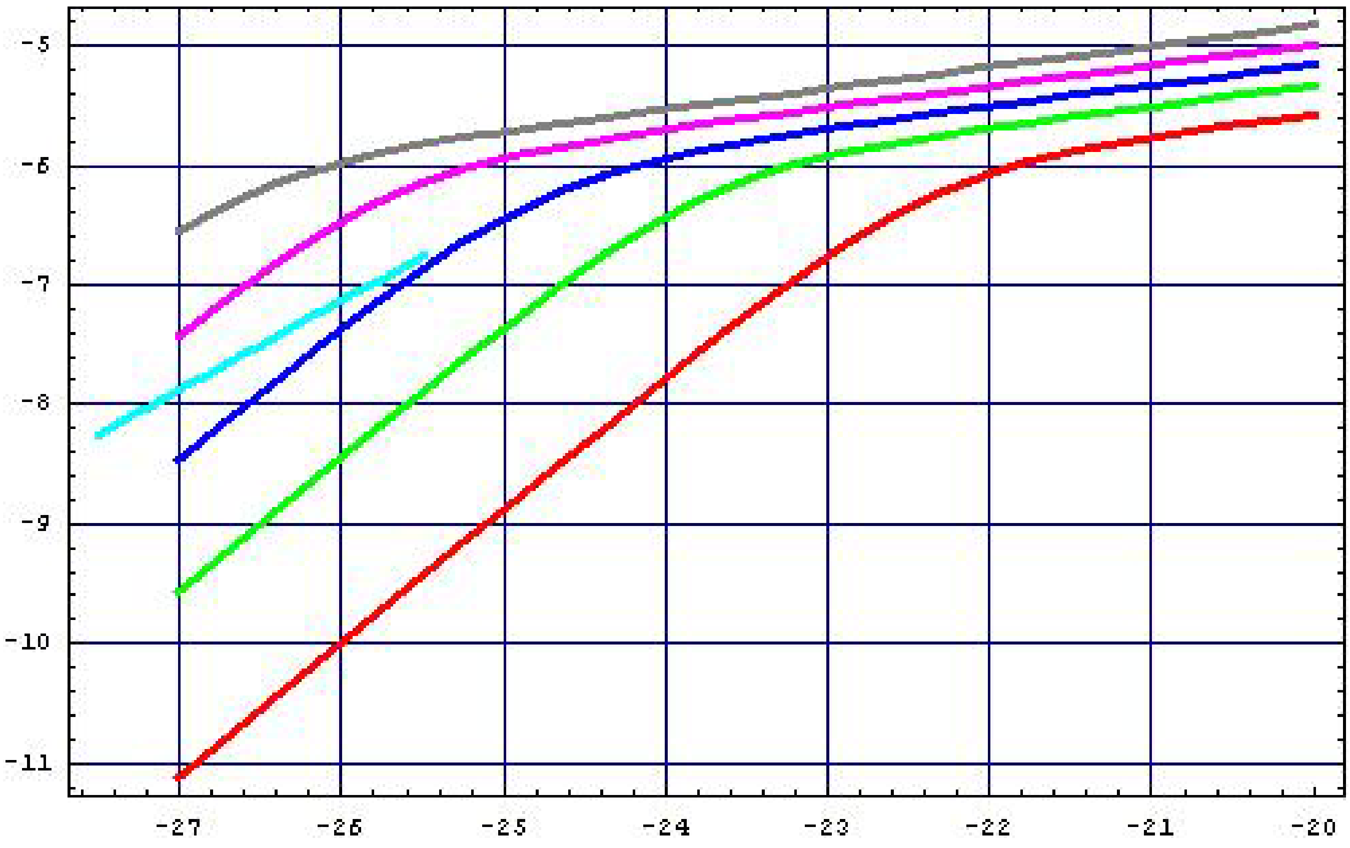
$$Log\ \Phi\left(< M_B, z\right) = 2.165 - 0.43\left(z-3\right) + 0.748\left(M_B + 26\right) - 9$$

$$\Phi\left(< M_B, z\right) = 1.37\ 10^{14+0.748\, M_B - 0.43\, z}$$

$$\Phi\left(L_B, z\right) \propto L_B^{-1.87}\ 10^{-0.43\, z}$$

- Luminosity Functions, for quasars, are not yet understood in a cosmological context. There is today no astrophysical theory of the origin and evolution of quasars in an evolving Universe that lead us to the functional forms we are observing.

Adding High z quasars - ? -

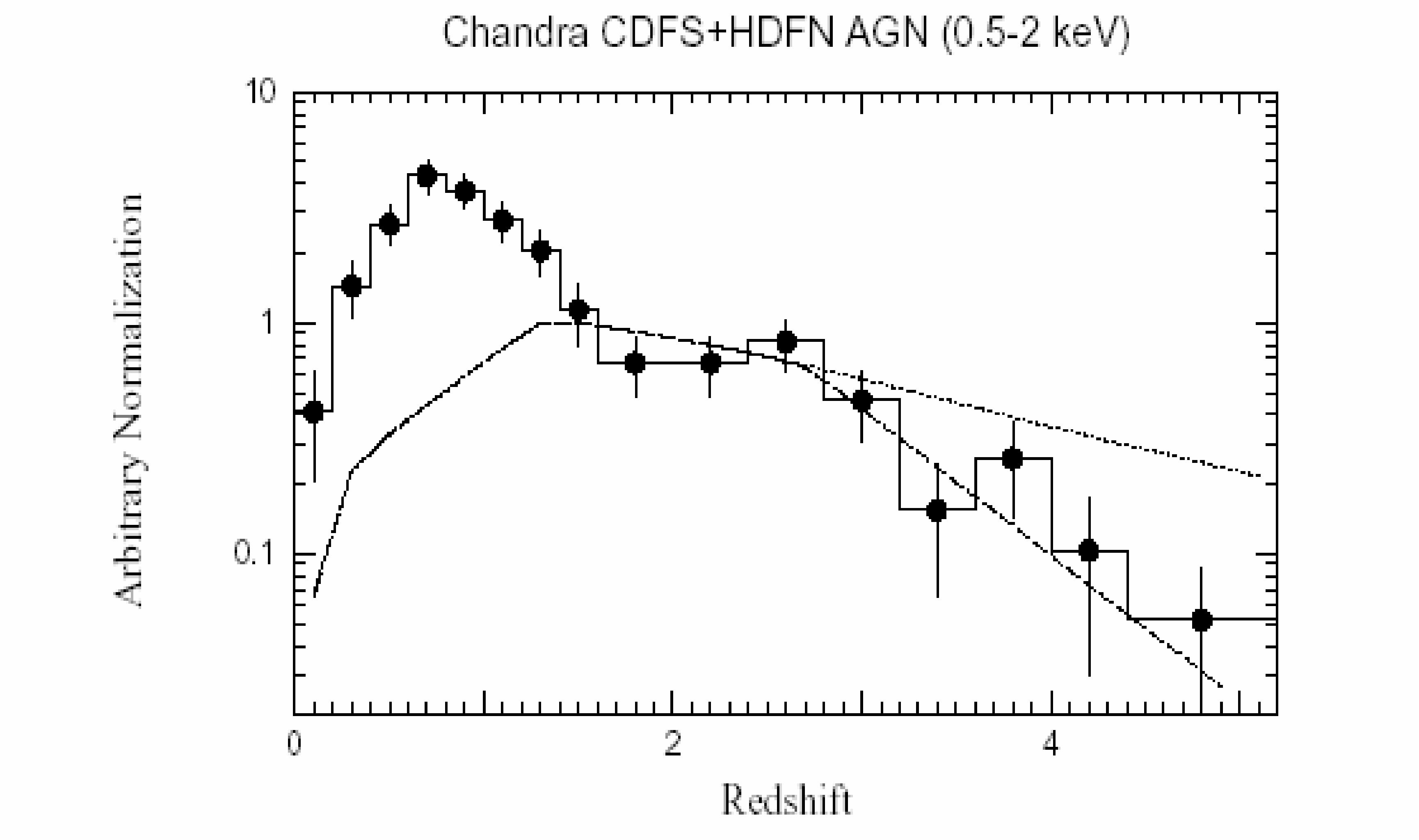


QSO LF for various z

see Peak at 0.7 However – Gunther – Astroph 0301040

- Gilli, Salvati and Hasinger 2001 adopting the AGN Luminosity Function and its evolution as determined from the Rosat Surveys (Miyaji et al. 2000) predict a maximum at redshift around z=1.5 (The students may try to repeat this computation).
- The prediction by Gilli et al. are not matched by a recent sample of 243 AGN selected in the 0.5 – 2 keV band. The observations show a peak at z = 0.7 which is dominated by Seyfert galaxies.
- In the next Figure the dashed line shows the prediction for the model. The comoving space density of high redshift QSO follows the decline above z=2.7 observed in optical samples (Schimidt, Schneider and Gunn, 1995; Fan et al., 2001).
- The dotted line shows a prediction with a constant space density for z> 1.5.
- The two model curves have been normalized at their peak at z=1. The observed curve has been normalizaed as to roughly match the observations in the redshift range 1.5 – 2.5.

Observed Redshift Distribution of AGN



Quasars & Galaxies

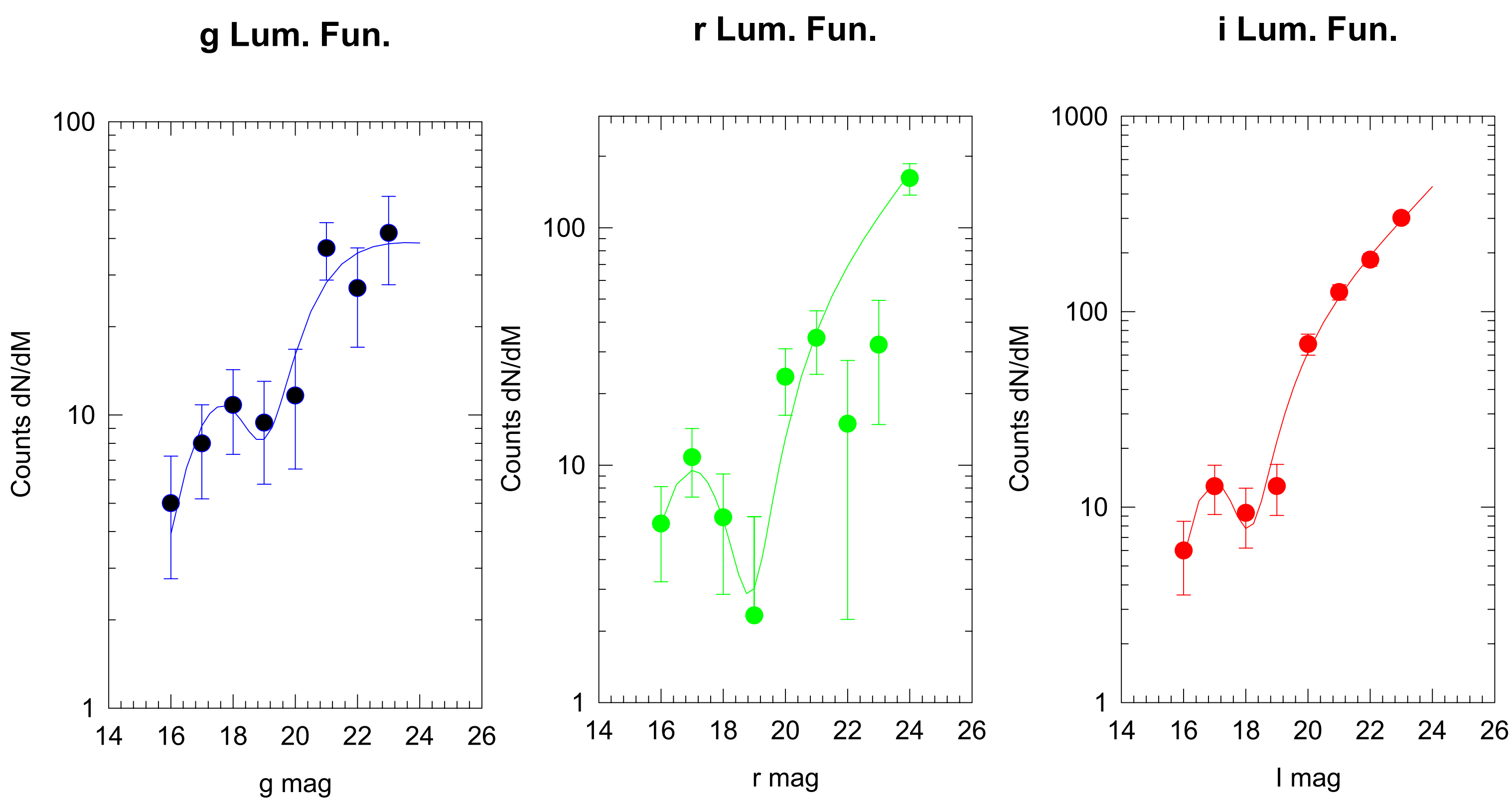
Compute with Kembhavi & Narlikar – See Lum_Fun_2 in Math

- φ^* (galaxies) = 0.02 h^3 Mpc^{-3} as given by Zucca et al.
- φ^* (quasars)= 5.2 10^{-7} h^3 Mpc^{-3} mag^{-1} – See Kembhavi
- Space Density of Galaxies ~ 0.047 h^{-3} Mpc^{-3}
  - α = -1.22; and for L^* > 0.1 [Reasonable for galaxies with BH]
- Space Density of Quasars ~ 5.5 10^{-6} h^{-3} Mpc^{-3}
  - Computed at z=0 and in the range -27 to -20 – See also Allen's Cox.
- Density Rich Clusters ~ 6 10^{-6} h^{-3} Mpc^{-3}

- Note:
  - These numbers are useful to estimate the radiaion due to Black Holes and Galaxy evolution. See Logbook, Padmanahbahn and development in Lum_Fun_2

Galaxies in Clusters

see also Presentation – 1st Module

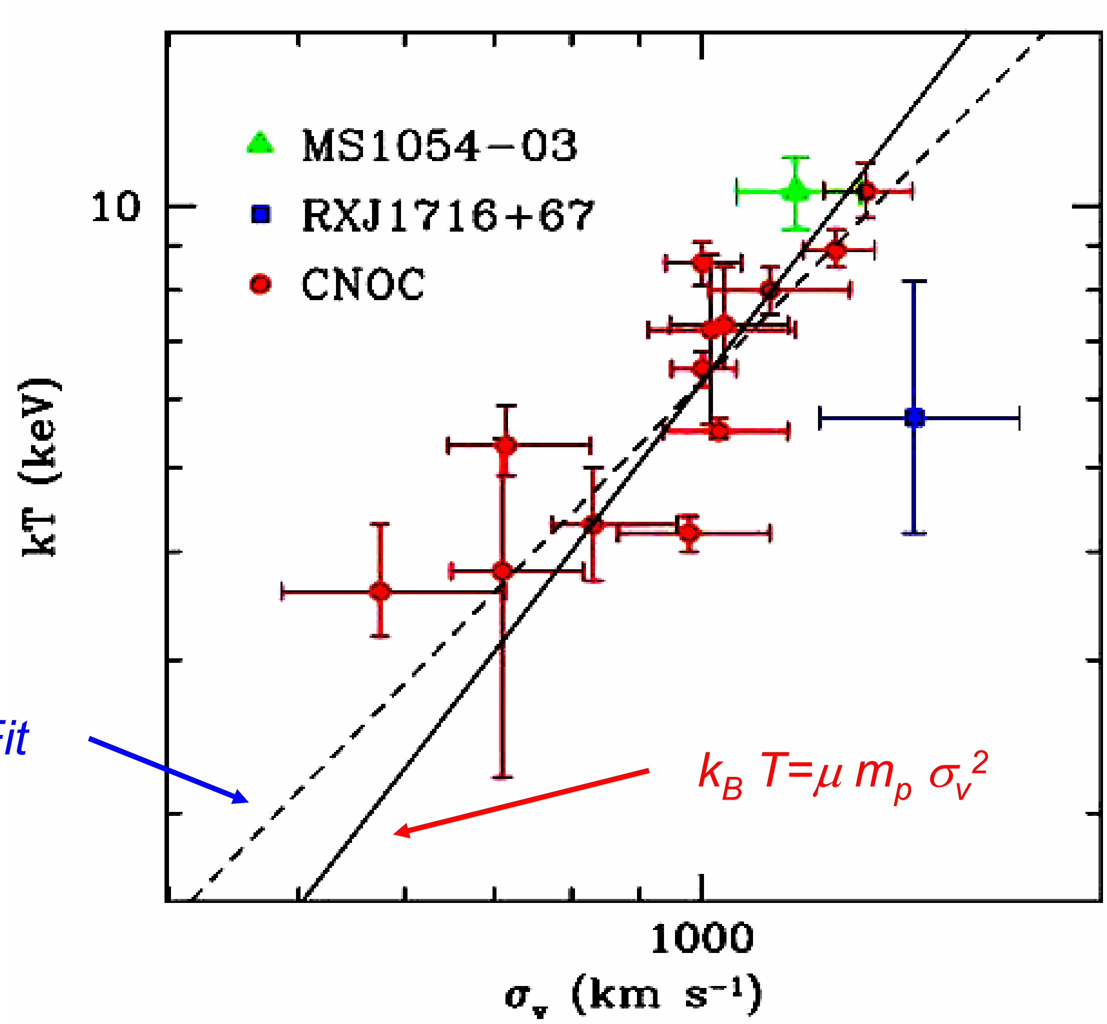


Cluster L.F. in the X Ray

- As we have seen clusters of galaxies appear in the X ray as extended sources due to the gas that is filling the potential well. The estimate of the space density of clusters via the X ray Luminosity distribution function is a very reliable measure due to the high contrast between the cluster luminosity and the X ray background (see X ray background).
- It is convenient to use the same Schechter function as for the galaxies so that the applications are straight forward. It is also to be noticed that the fact that the function fits objects of different scale length must have a deeper meaning.

$$\left.\begin{aligned}
&\varphi(L_X)dL_X = \phi^* \left(\frac{L_X}{L_X{}^*}\right)^{\alpha} Exp\left(-\frac{L_X}{L_X{}^*}\right) d\left(\frac{L_X}{L_X{}^*}\right) \\
&\qquad\qquad \text{equivalent to} \\
&\phi(L_{44}) = K\ Exp\left(-\frac{L_X}{L_X{}^*}\right) L_{44}^{-\alpha}\ ;\ L_{44} = \frac{L_X}{10^{44}}\ ergs\ s^{-1} \\
&\phi^* = K\left(\frac{L}{10^{44}}\right)^{1-\alpha}\ ;\ K\ in\ 10^{-7}\ Mpc^3\left(10^{44}\ erg\ s^{-1}\right)^{\alpha-1}
\end{aligned}\right\}\ \Delta E = (0.5 - 2keV)$$

Z>0.15 – Galaxy velocity dispersion versus ICM Temperature

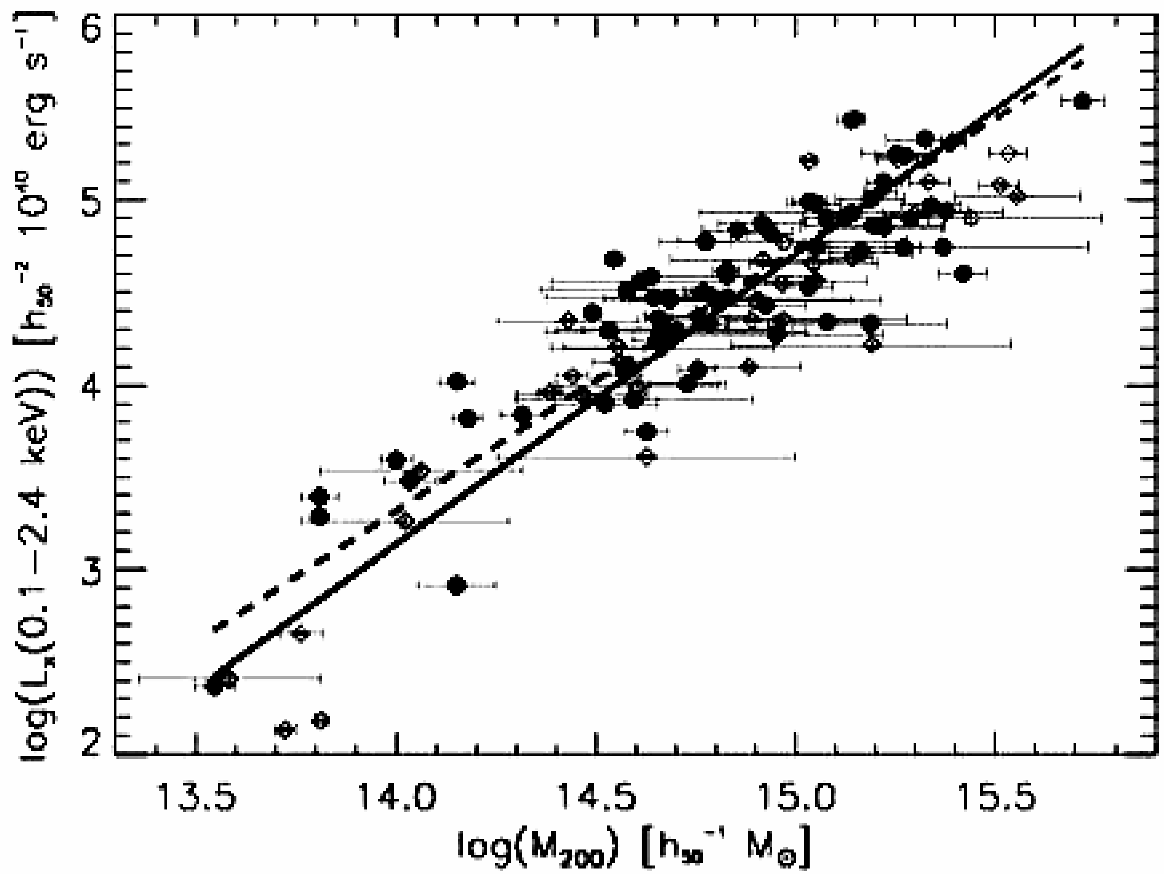


Best Fit

$k_B\,T = \mu\, m_p\, \sigma_v^2$

L_X versus $M(\rho > 200\ \rho_c)$

Fit to two different set of data



Scale Relations

$$k_B T \simeq \mu m_p \sigma_v^2;\ \mu = 0.6\ \ primordial\ composition\ 76\%\ Hydrogen$$

$$\mathfrak{M} \propto R^3 \prec \rho_0 \succ (1+z)^3 \left(\frac{\delta\rho}{\prec \rho_0 \succ}\right)_{R=R\_Vir};\ \left(\frac{\delta\rho}{\prec \rho_0 \succ}\right)_{R=R\_Vir} = const.$$

$$\mathfrak{M} \propto \sigma_v^2 R\ [Virial\ Theorem];\quad R \propto \frac{\mathfrak{M}^{\frac{1}{3}}}{(1+z)};\ from\ above$$

$$T \propto \frac{\mathfrak{M}}{R} \propto \frac{\mathfrak{M}}{\mathfrak{M}^{\frac{1}{3}}}(1+z) \propto \mathfrak{M}^{\frac{2}{3}}(1+z)$$

$$L_X \propto n^2 T^{\frac{1}{2}} R^3 \propto \frac{\mathfrak{M}^2}{R^3} T^{\frac{1}{2}} \propto \frac{\mathfrak{M}}{(1+z)^3}(1+z)^6\, \mathfrak{M}^{\frac{1}{3}}(1+z)^{\frac{1}{2}} \propto \mathfrak{M}^{\frac{4}{3}}(1+z)^{\frac{7}{3}}$$

$$Defining\ gas\ Entropy\ as\ S =$$

Gas Entropy map from Hydrodynamical simulations (Borgani et al. 2001)

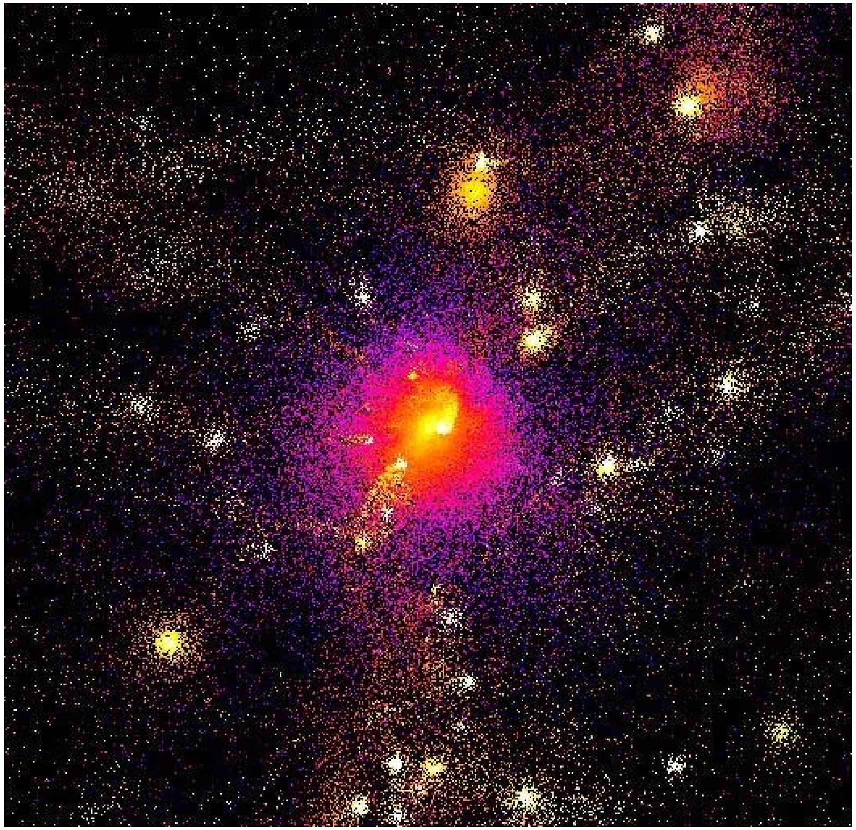
If the process of cluster formation and thermodynamics of the ICM is governed solely by gravitational processes and related phenomena, then there should not be a preferred scale due to cosmology and physics and clusters of different masses should simply be a scaled version of each other.

Brehmstralung predicts:

$$L_X \propto \mathfrak{M}\rho_{gas}T^{\frac{1}{2}};\ L_X = T_X\left(1+z\right)^{\frac{3}{2}}$$

$$L_X \propto \mathfrak{M}^{\frac{4}{3}}\left(1+z\right)^{\frac{7}{3}};$$

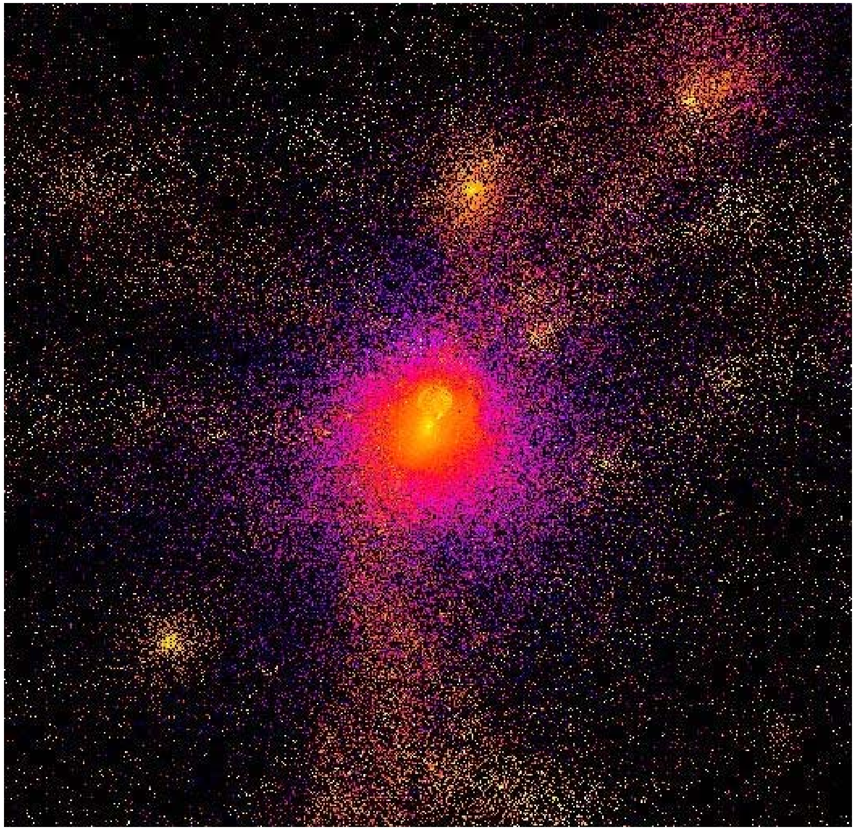
And this is not observed.



Gravitational heating only
Light color = Low Entropy particles
Dark Blue = High Entropy particles

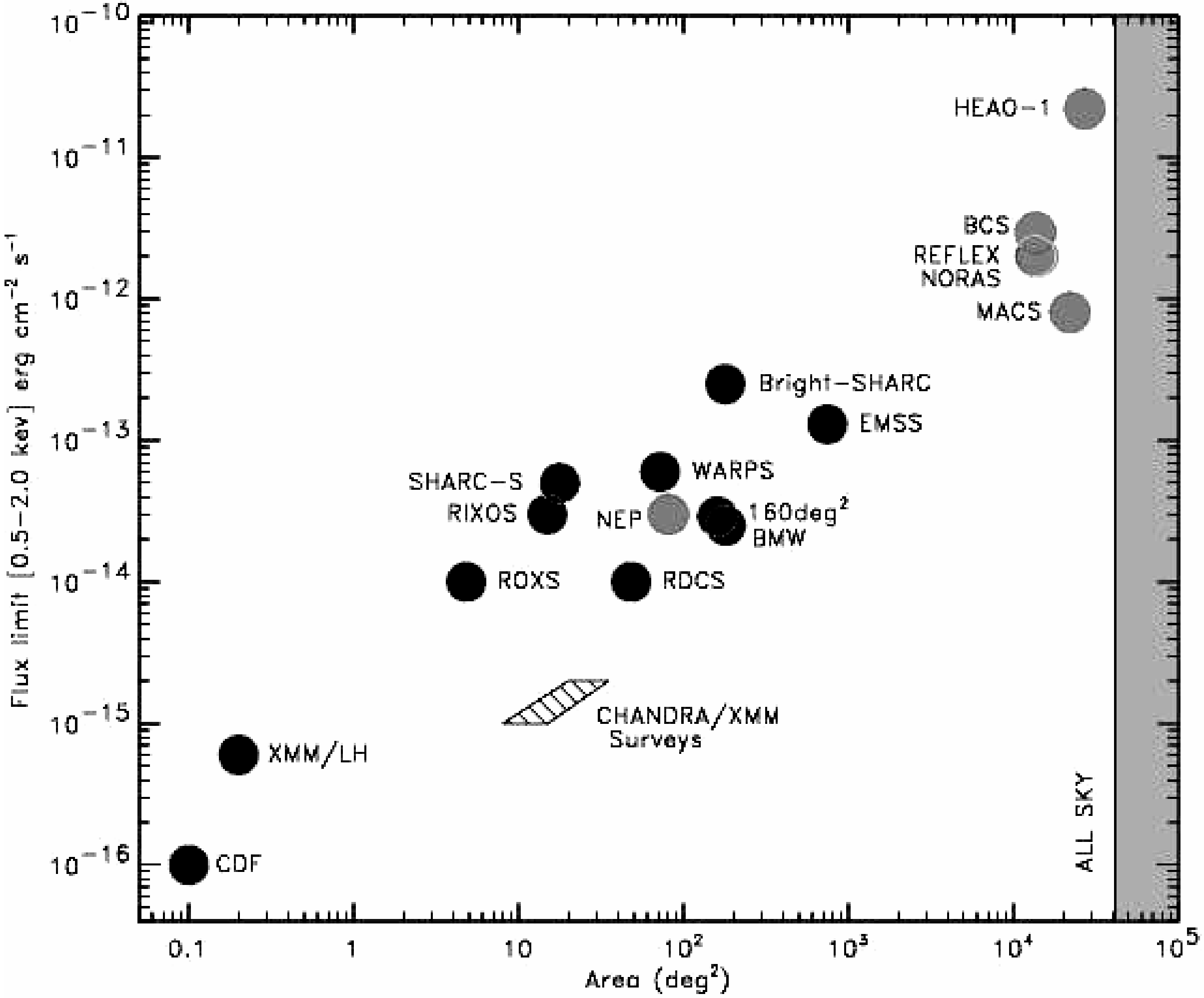
Early heating of the gas from a non gravitational source

- The additional heating results in limiting the central density of the cluster during the gravitational collapse and decrease somewhat the Luminosity of the cluster.
- If the added extra energy per particle is fixed, then the effect is most evident in poor clusters. These have indeed a low velocity dispersion and the extra energy – the extra heating temperature - may become comparable to the virial temperature.
- The result is that the self similar behaviour is preserved in hot rich systems while it is broken in colder systems.
- The simulations seem to indicate that about 1 keV per gas particle of extra energy is required.
- The extra energy injection in addition erases the small clumps associated with accreting groups.



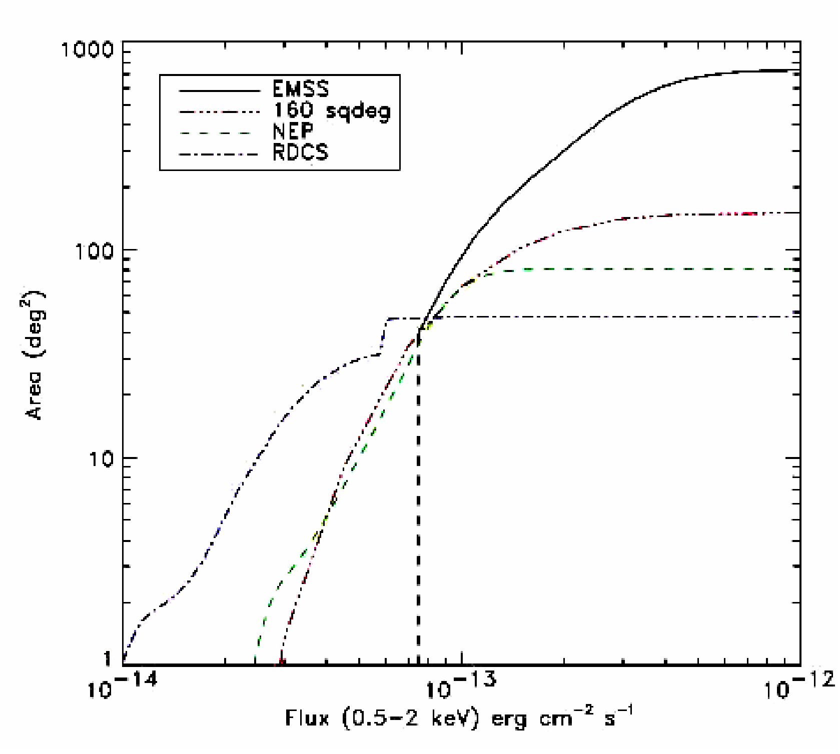
Additional Heating

Sensitivity versus Area of the surveys



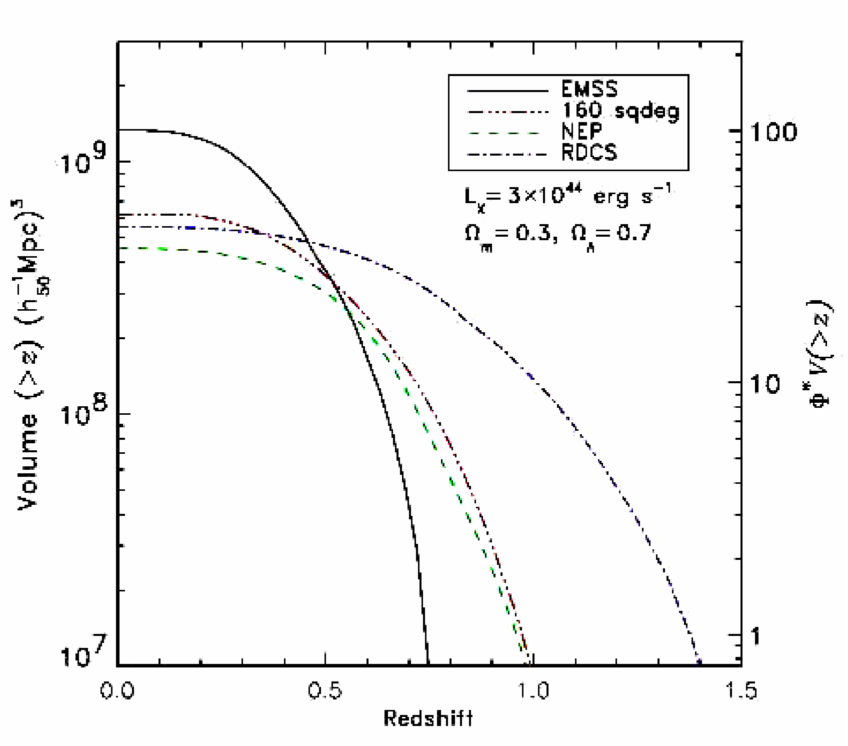
Sky Coverage for Serendipitous surveys

- The serendipitous surveys cover generally an area which is a strong function of the flux. Indeed the spacecraft or the telescope integration time is a function of the particular program which is active.
- Bright sources will be visible almost in all fields while faint sources will be visible only in those fields where deep exposures have been requested.

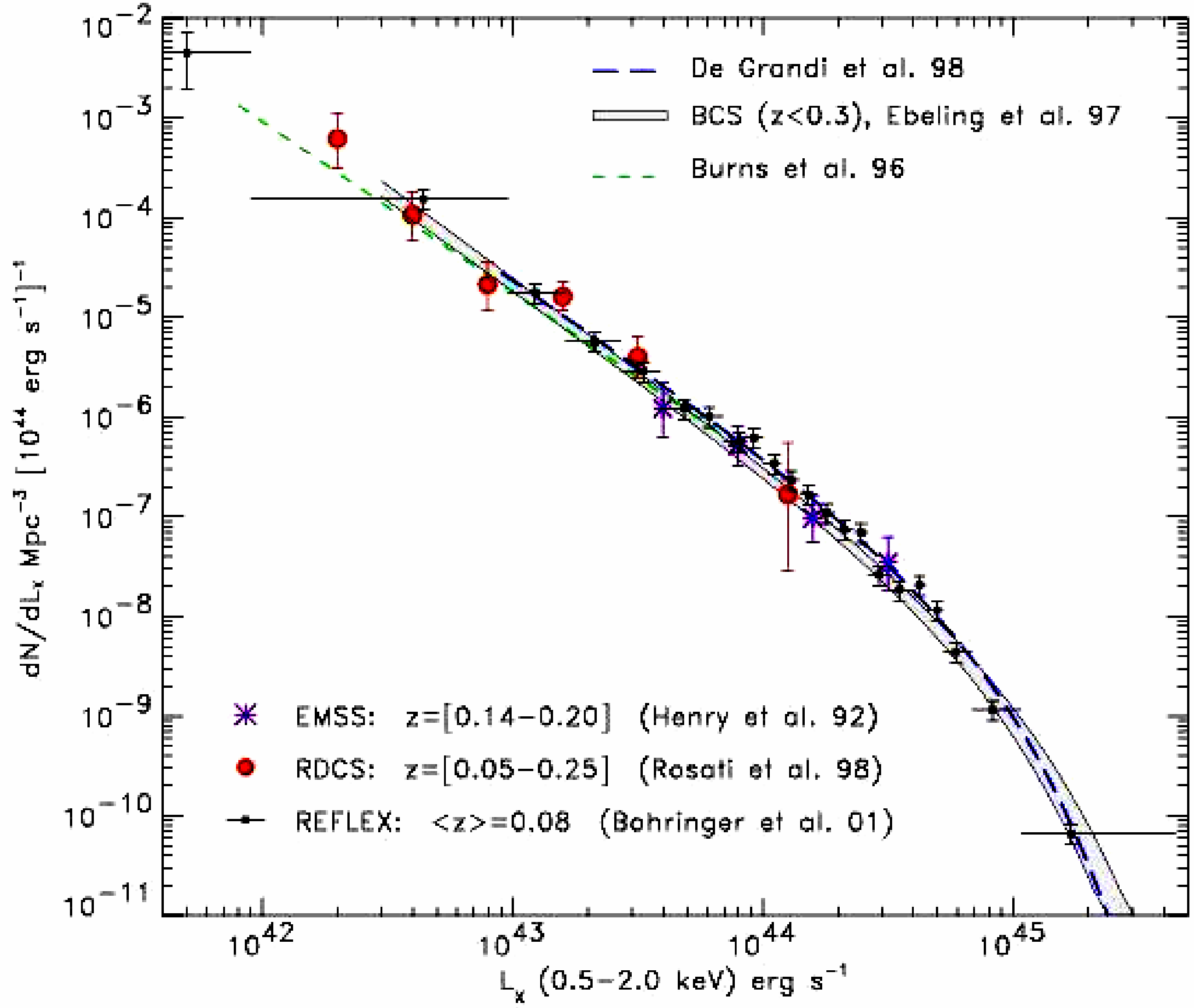


Searched Volume

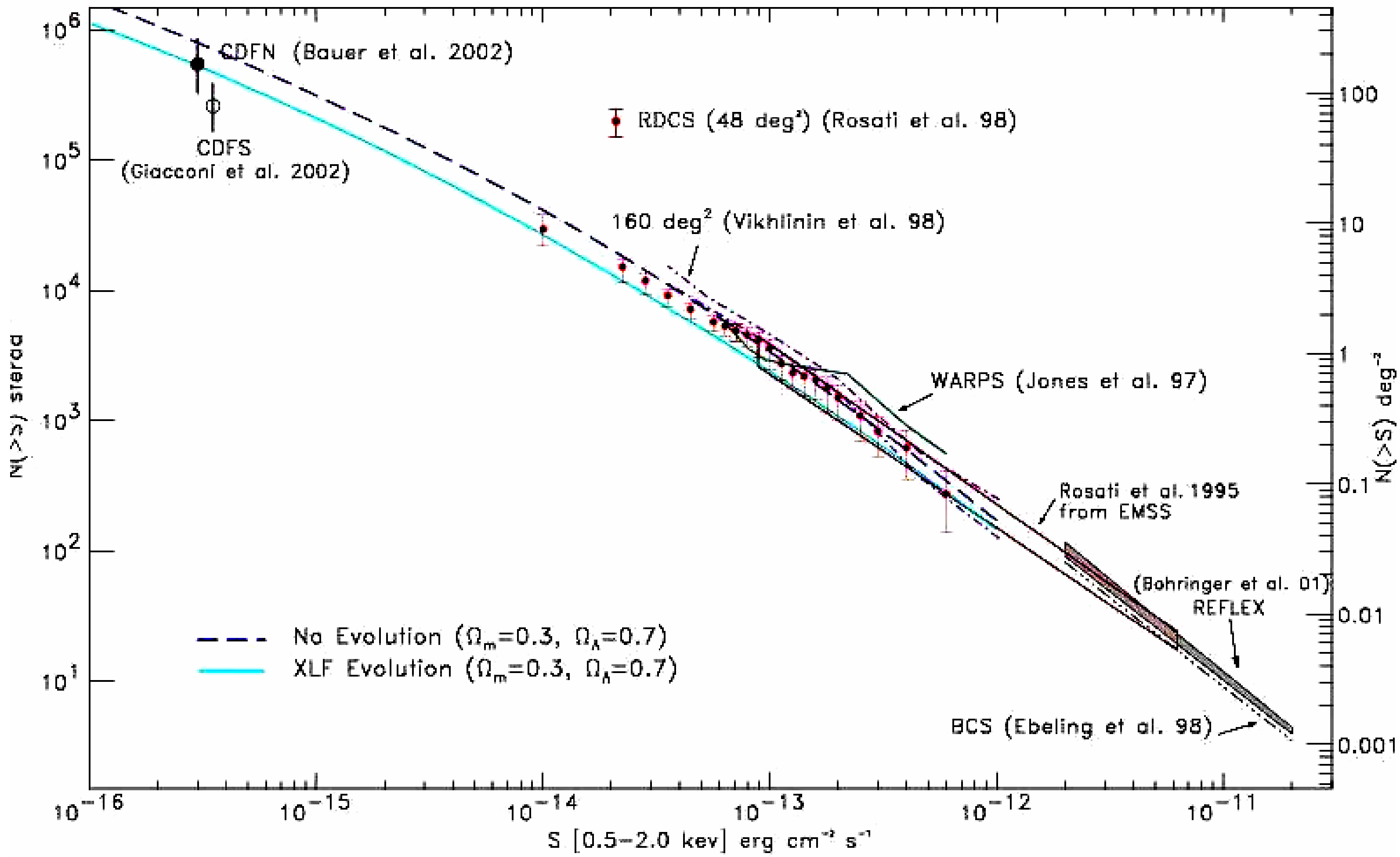
- An analogous concept is the search volume for a cluster of a given luminosity. Here L_X (as given in the Figure) is of the order of L_X^*.
- At a given redshift for a predefined Luminosity I cover the area defined earlier for that flux and I can easily compute the volume.
- Note that the EMSS still covers, at low redshiftd, the largest volume and therefore has the capability of detacteting rather Luminous galaxies (remember how the Luminosity function goes).



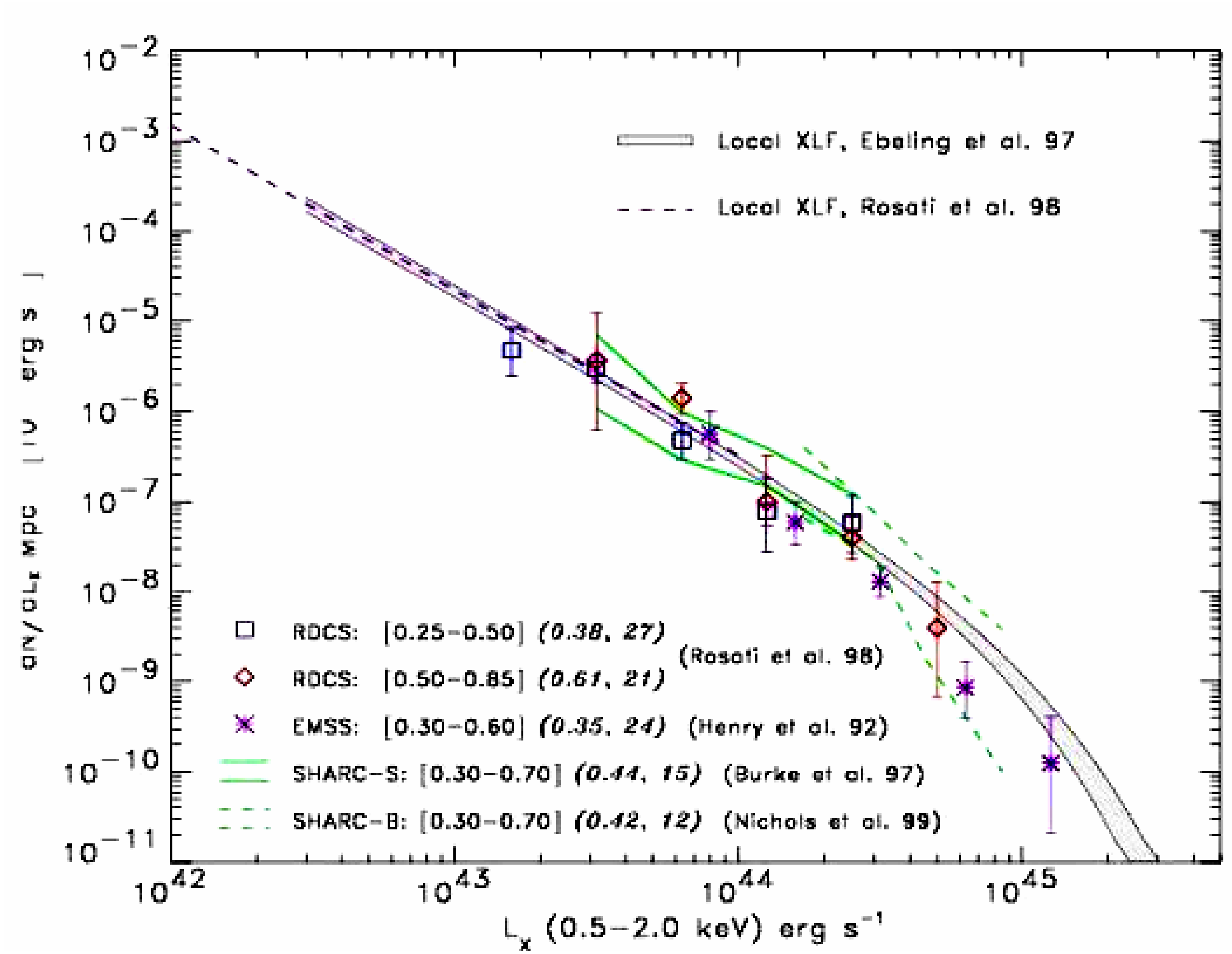
Local X ray Luminosity Function



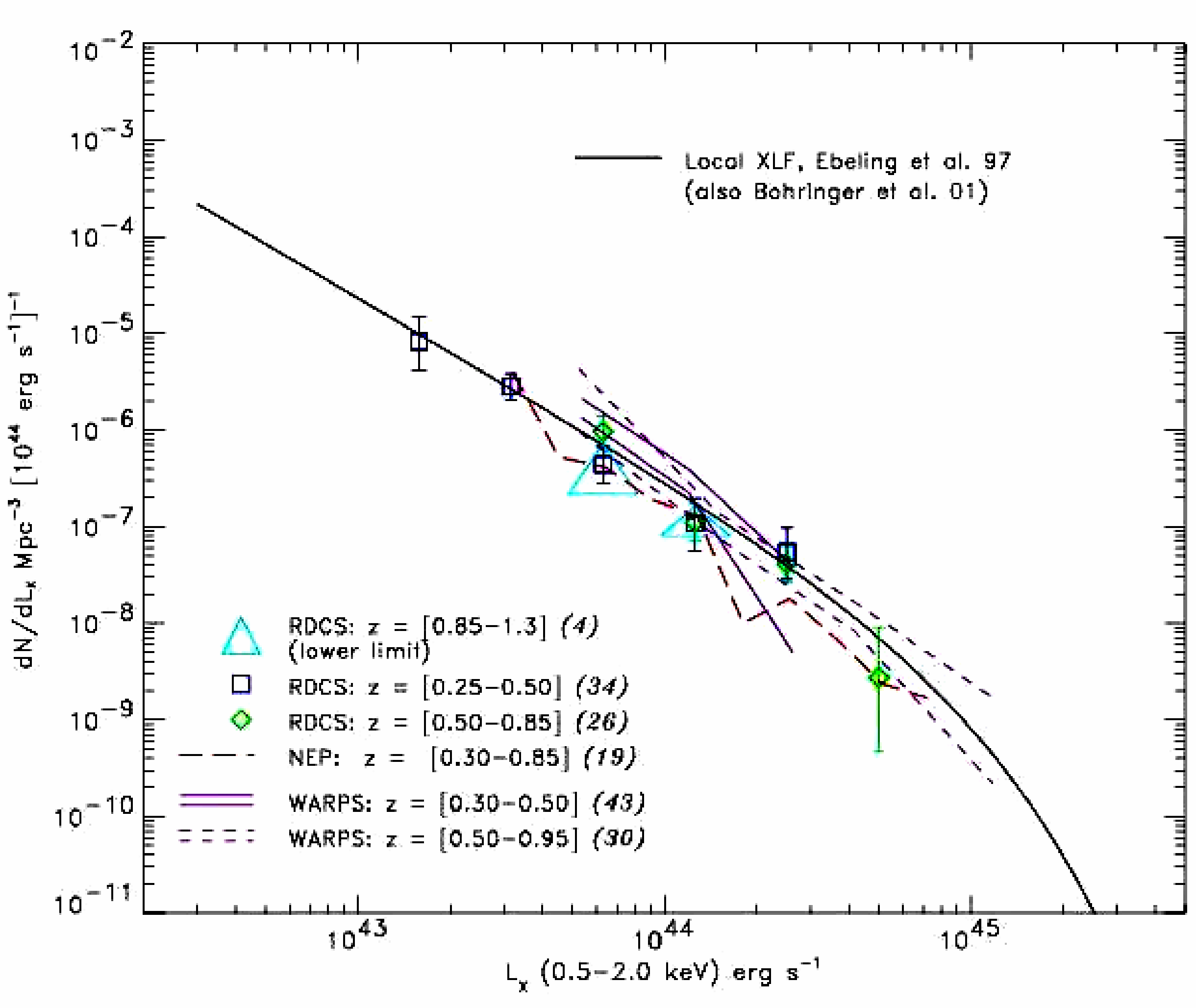
Log N – Log S
Cumulative counts versus flux



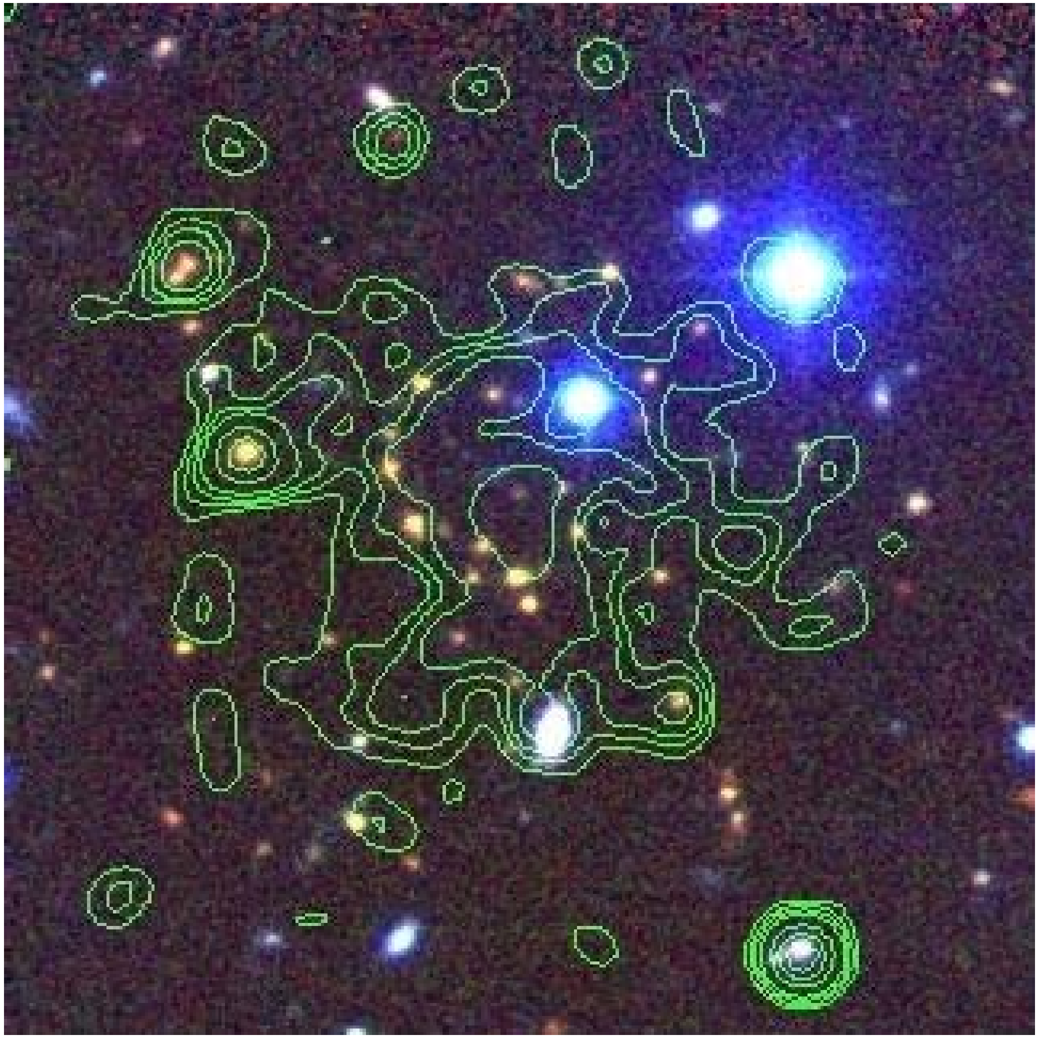
L.F. of distant clusters



RCS from Rosati et al. (2000)

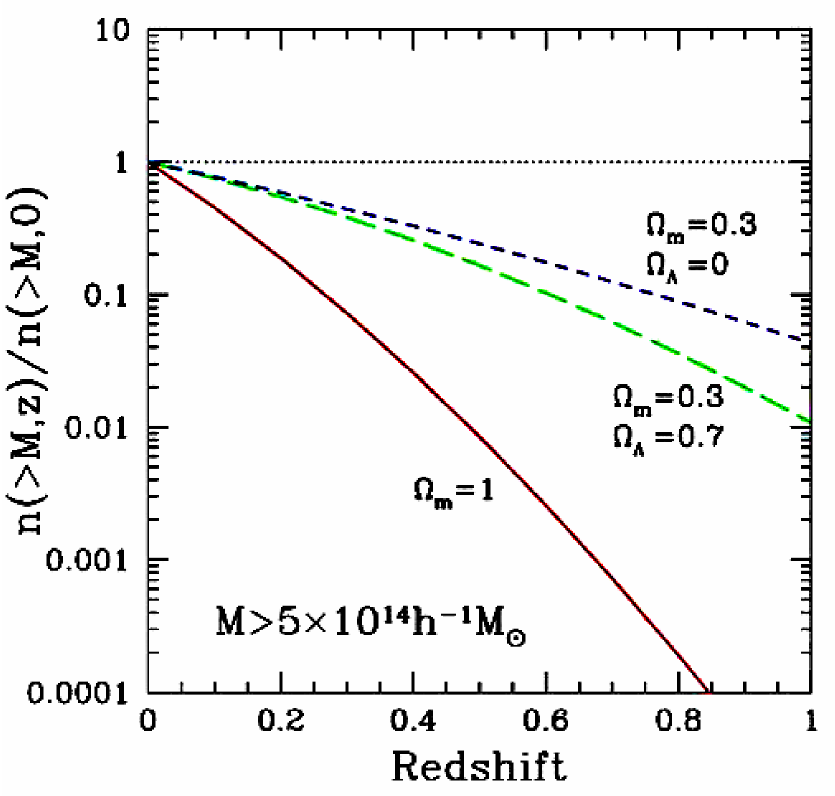


RXJ0910+5422 z=1.11



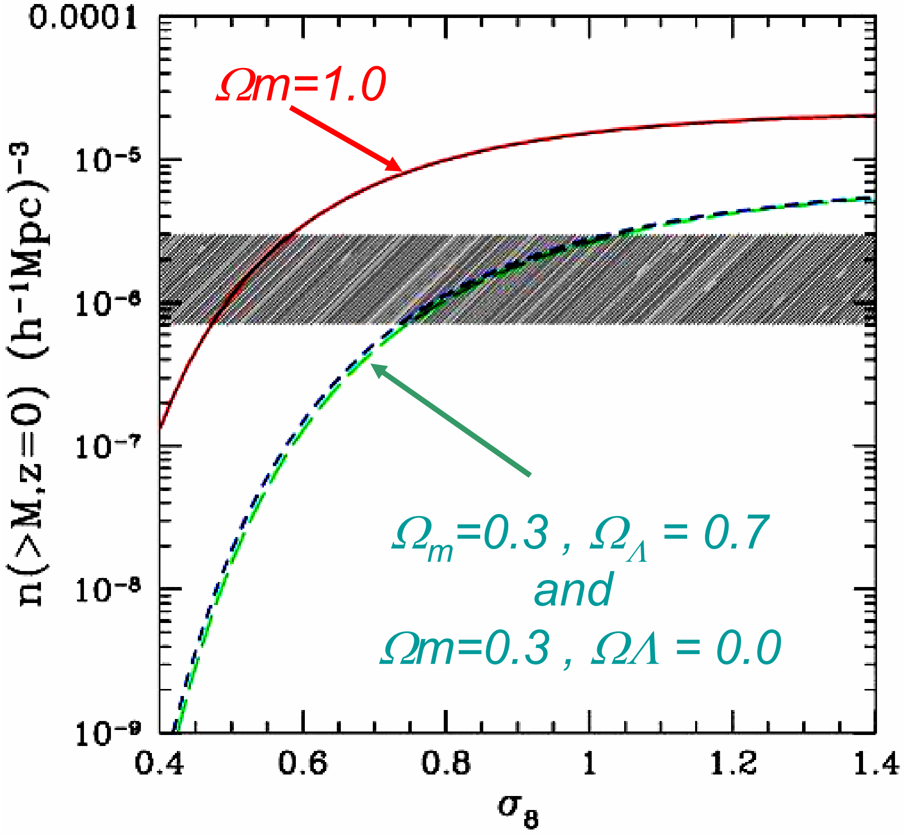
How cluster evolution shows up

- Here is illustrated how the number counts of clusters change as a function of redshift and various cosmological models.
- Rather than use these graphs and the observations to estimate the cosmological parameters (this could be done better using the MWB even if clusters could be an independent method) we should use this to estimate, given a cosmological model, how clusters evolve.
- On the other hand the estimates via clusters are based on different objects, different physics etc so that they still are valuable.
- The theory has been developed first by Press and Schechter 1974. We will work out details later on after the Friedman models.

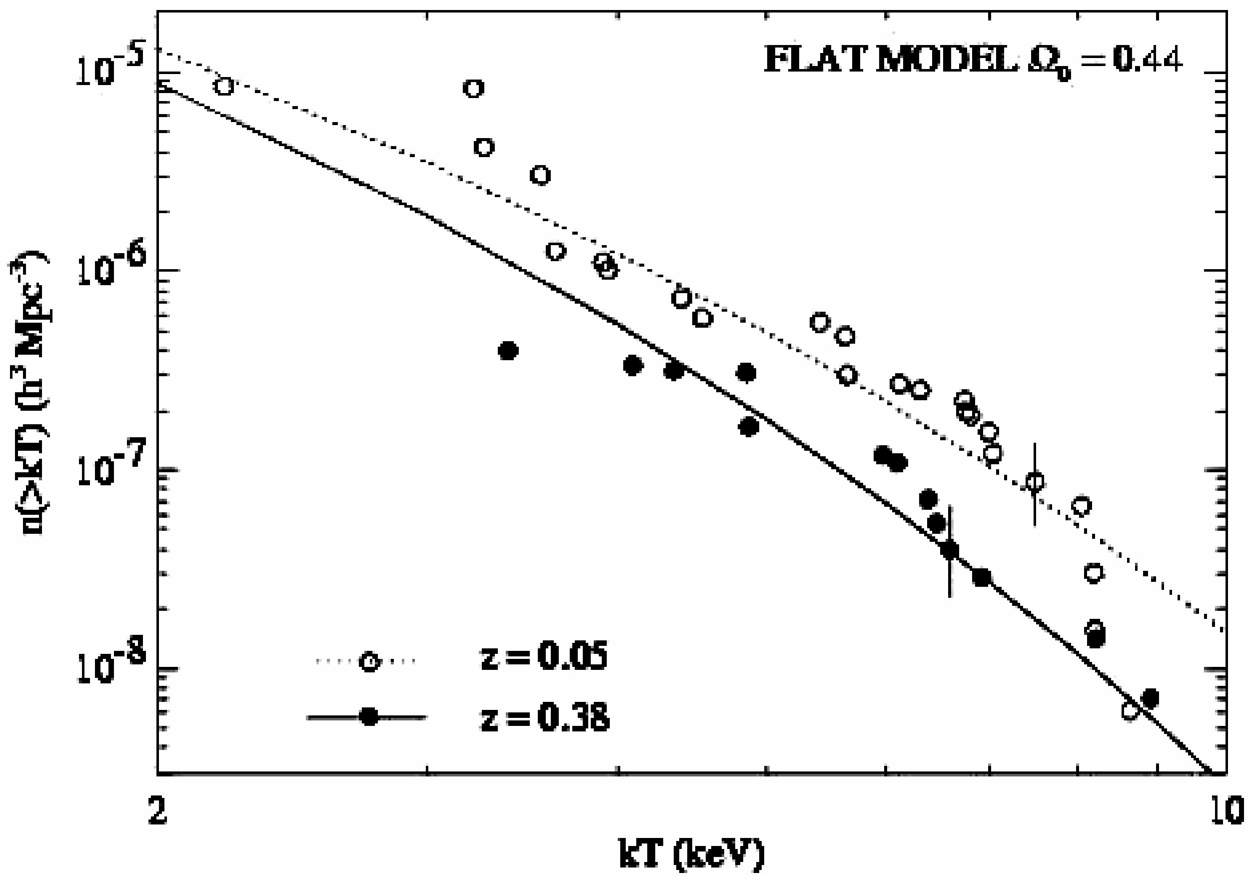


Cumulative Mass function

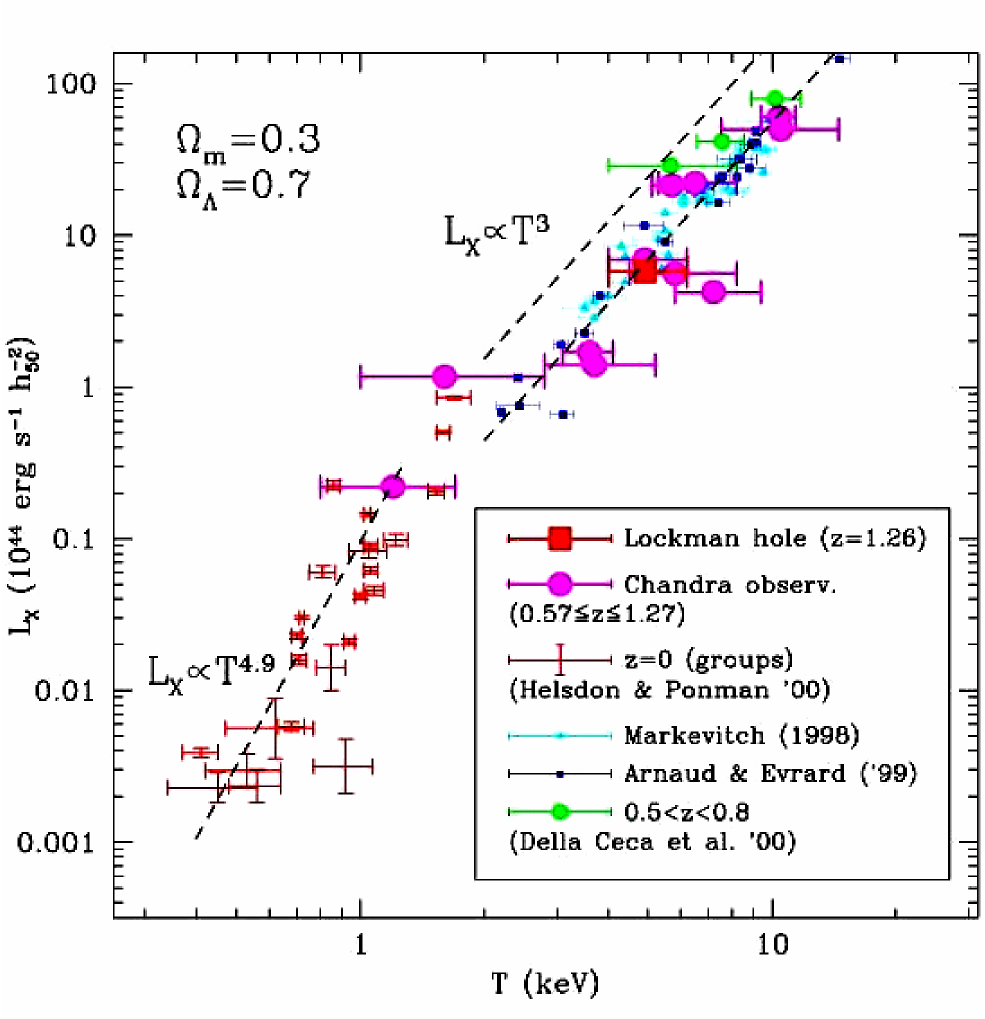
- The cumulative Mass Function , $M > 5\ 10^{14}\ h^{-1}\ M_{\odot}$, for different models at z=0 and as a function of σ_8.
- σ_8 is the rms density fluctuation within a sphere of 8 h^{-1} Mpc radius.
- The shaded area indicate the observational uncertainty.



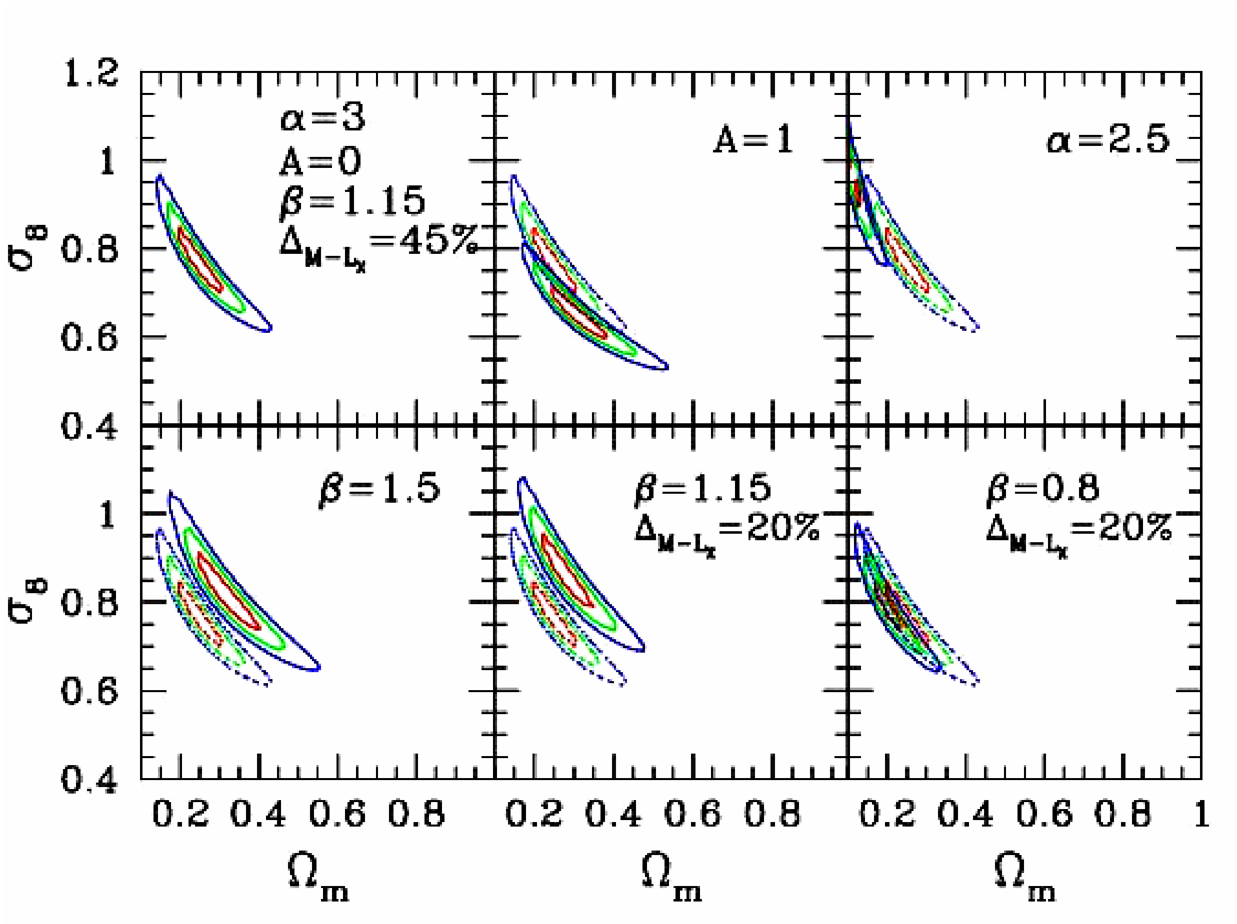
Cumulative X ray Temperature Function



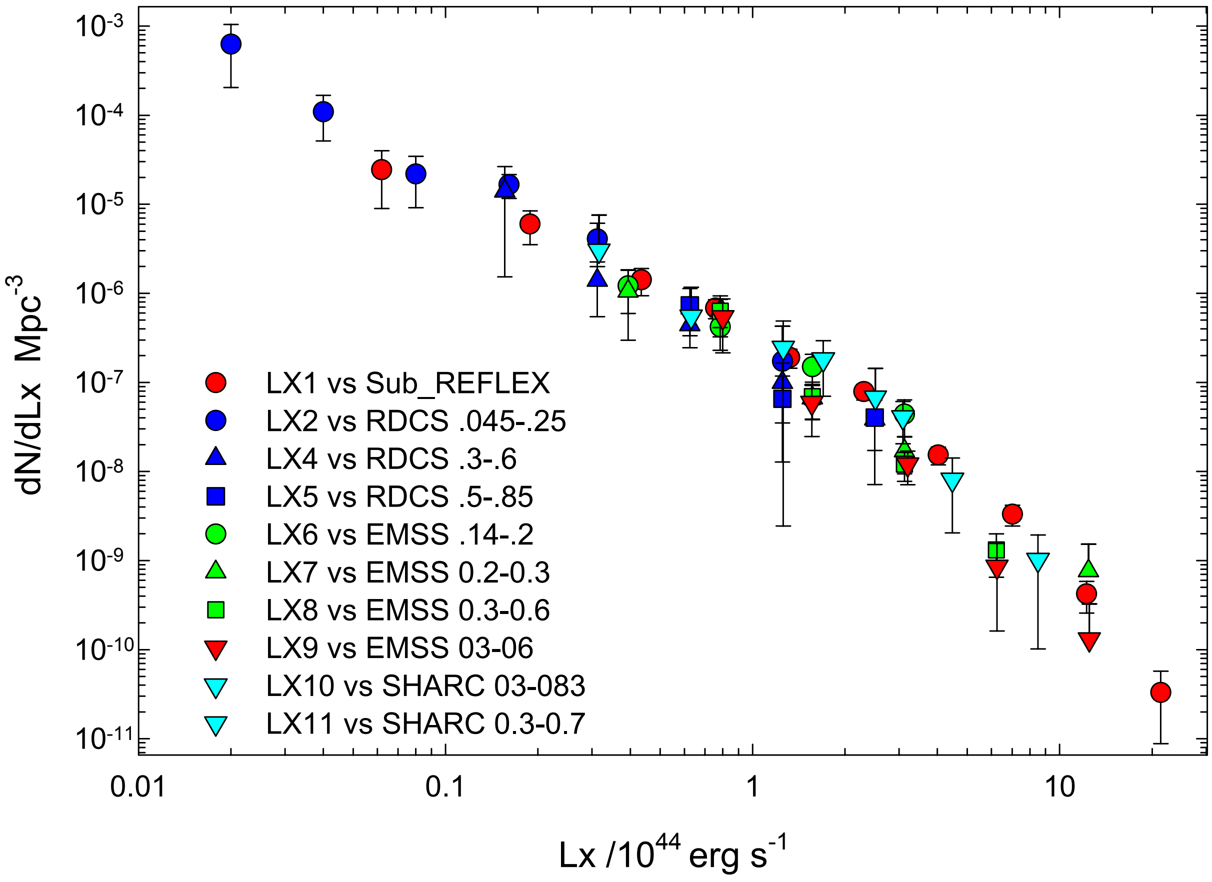
Bolometric Luminosity-Temperature relation.



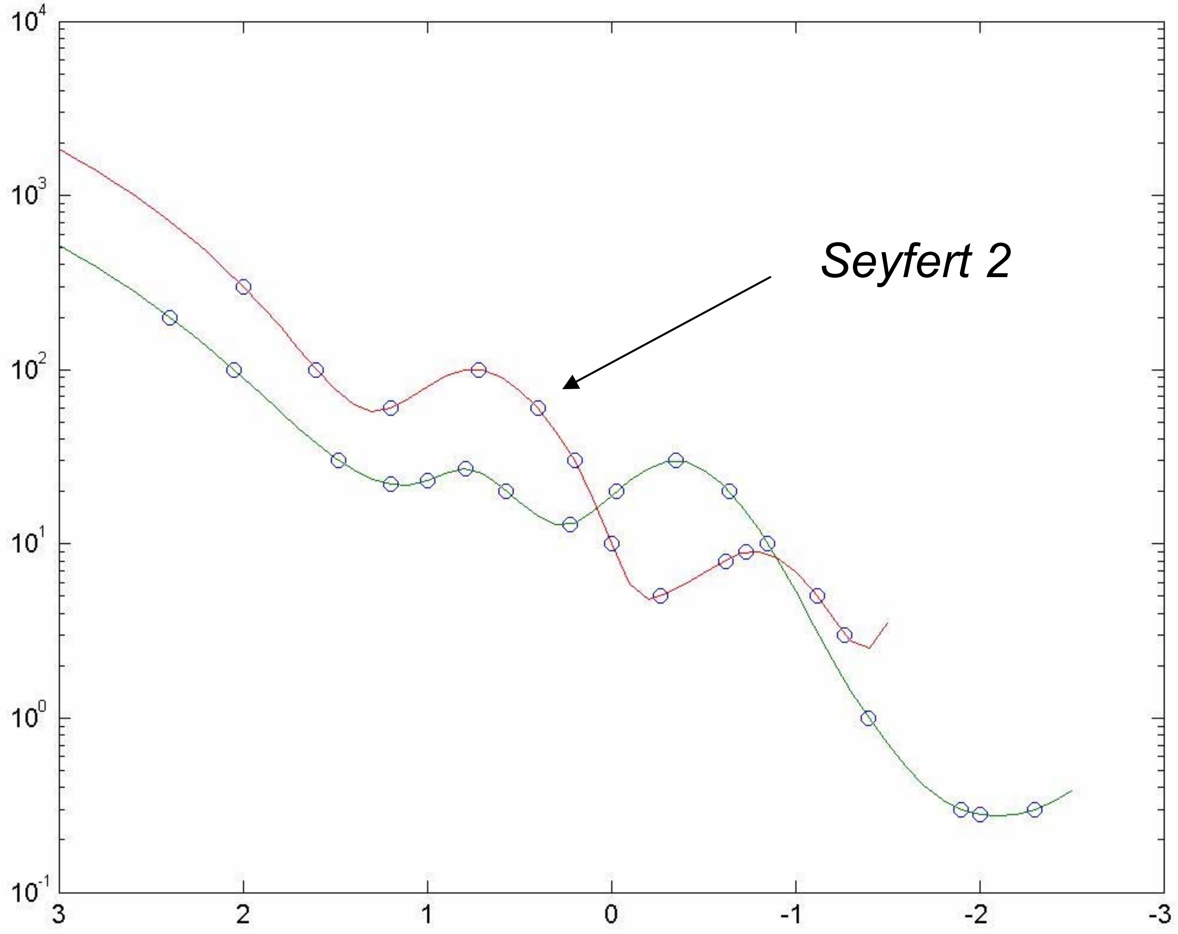
Probability Contours



X-ray Cluster Luminosity Function

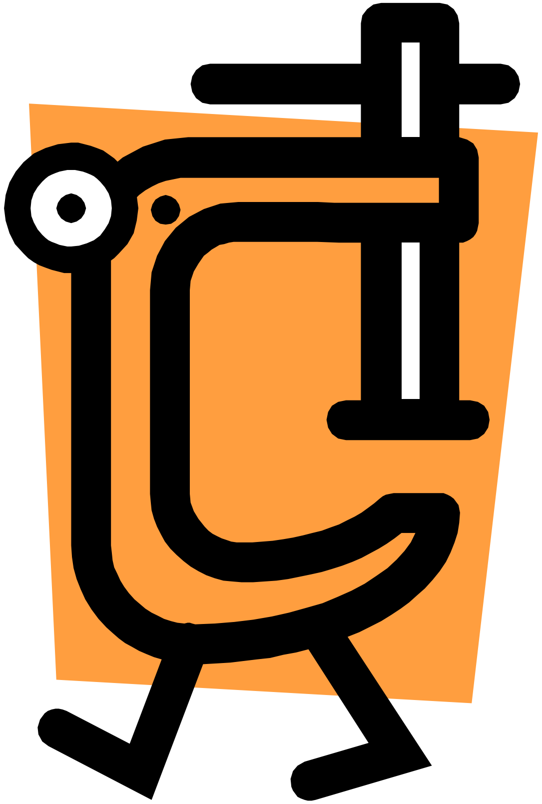


Seyfert Galaxies. The L.F.
see Huchra and Burg (1992) ApJ 393, 90



Students at work

- Generate some random samples using various luminosity functions in different volume of space generated by different cosmological models.
- Use the V/V_{max} test to estimate the value you get for the complete samples you generated.
- Compute the TBD



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- …