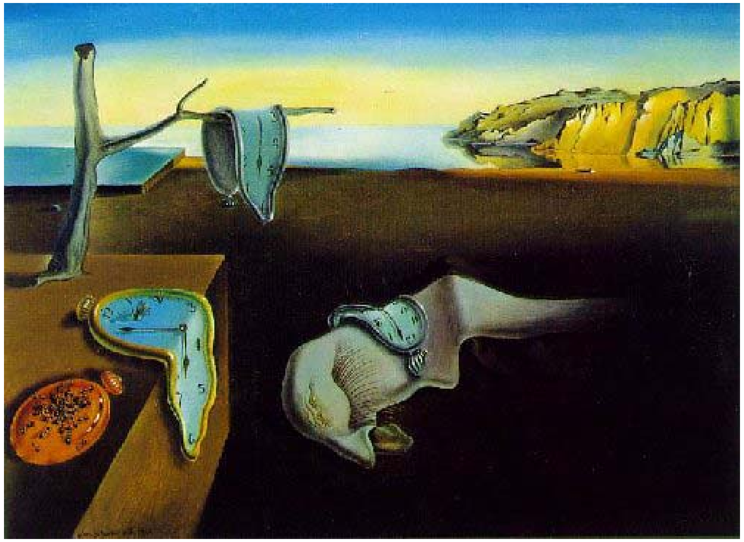


The Horizon

Cosmology 2004 2005

Through the history of the Universe the solution of the Friedmann equations, see the plots or the parametric solution of the equations, give:

$$a(t) \propto t^n \quad with \quad n < 1$$

For instance in a flat Universe:

$$a(t) \propto t^{2/3}$$

And for the Hubble time we have:

$$H^{-1}(t) = \frac{a}{\dot{a}} \propto \frac{t^n}{t^{n-1}} \propto t$$

So that the Hubble radius goes as:

$$c\, H^{-1}(t) \propto t$$

The Hubble Radius grows faster than the distance

Consequences

Assume today I have a scale length λ such that it is contained in the visible Universe, in other words we have:

$$\lambda = a(t_0)\, r < d_H = c\, H^{-1}$$

By going back in time d_H will shrink faster than a(t) so that there will be a time when the scale length is larger than the visible Universe. The Scale length λ is outside the visible Universe.

We compute now the Horizon for different Models.

The distance between two observers in the expanding Universe is proportional to a(t) so that I can write:

$$dr = a(t)\,dx$$

and dx is independent of time

For each observer I have $r(t) = a(t)\,x$.

During the time interval dt the photon coming toward us passes from an observer r+dr to an observer in r and these two observers are separated by a distance that is $c\,dt = a(t)\,dx$ and this means that the comoving distance from us is $x = \int_{t_e}^{t_0} \frac{c\,dt}{a(t)}$ and the distance is:

$$r(t_0, t_e) = a_0 x = a_0 \int_{t_e}^{t_0} \frac{c\,dt}{a(t)} \quad \mathit{for}\ z >> 1\ \mathit{emission\ at}\ t_e = 0$$

$$r(t_0) = a_0 \int_0^{t_0} \frac{c\,dt}{a(t)} \quad \mathit{and\ \ for\ any}\ t \neq t_0 \Rightarrow r(t) = a(t) \int_0^t \frac{c\,dt'}{a(t')} = a(t) \int_0^a \frac{1}{a(t) H} \frac{c\,da}{a(t)}$$

DUST

$$r_H(t) = a(t)\int_0^t \frac{c\,dt'}{a(t')} = a(t)\int_0^a \frac{1}{a(t)H(t)}\frac{c\,da}{a(t)}$$

$$H^2(z) = H_0^{\,2}(1+z)^2\left[\Omega_0(1+z)+(1-\Omega_0)\right] = H_0^{\,2}\left(\frac{a_0}{a}\right)^2\left[\Omega_0\left(\frac{a_0}{a}\right)+(1-\Omega_0)\right]$$

$$r_H(t) = a(t)\int_0^a \frac{1}{a(t)H(t)}\frac{c\,da}{a(t)} = a(t)\int_0^a \frac{1}{a(t)}\frac{c\,da}{a(t)\sqrt{H_0^{\,2}\left(\frac{a_0}{a}\right)^2\left[\Omega_0\left(\frac{a_0}{a}\right)+(1-\Omega_0)\right]}} =$$

$$= \frac{c}{a_0H_0}a(t)\int_0^a \frac{1}{a(t)}\frac{c\,da}{\sqrt{\left[\Omega_0\left(\frac{a_0}{a}\right)+(1-\Omega_0)\right]}} \quad \text{and for } (1-\Omega_0)\approx 0$$

$$r_H(z) = \frac{c}{a_0H_0}a(t)\int_0^a \frac{1}{a(t)}\frac{c\,da}{\sqrt{\left[\Omega_0\left(\frac{a_0}{a}\right)\right]}} = 2\frac{c}{H_0\Omega_0^{1/2}}\left(\frac{a}{a_0}\right)^{3/2} = 2\frac{c}{H_0\Omega_0^{1/2}}\left(\frac{1}{1+z}\right)^{3/2}$$

Dust $\Omega_0 > 0$

$$\int_0^a \frac{1}{a(t)} \frac{c\,da}{\sqrt{\left[\Omega_0\left(\frac{a_0}{a}\right)+(1-\Omega_0)\right]}} = \int_0^a \frac{c\,da}{\sqrt{\left[\Omega_0\, a_0 a+(1-\Omega_0)\,a^2\right]}}$$

$$for\quad \Omega_0 > 1;\quad (1-\Omega_0) = d < 0;\quad \Omega_0\, a_0 = b$$

$$\int_0^a \frac{c\,da}{\sqrt{\left[b\,a+d\,a^2\right]}}(Branshtein\ Page\ 430) =_0^a \left| -\frac{1}{\sqrt{-d}}\, ArcSin\,\frac{2\,d\,a+b}{b} \right. =$$

$$= \frac{1}{\sqrt{-d}}\, ArcCos\,\frac{2\,d\,a+b}{b}$$

$$r_H(t) = \frac{c}{H_0(1+z)} \frac{1}{\sqrt{(\Omega_0-1)}}\, ArcCos\left[1-2\frac{(\Omega_0-1)}{\Omega_0(1+z)}\right]$$

DUST

$$And \quad for \;\; \Omega_0 < 1; \quad (1-\Omega_0) = d > 0; \quad \Omega_0 \, a_0 = b$$

$$Remember\ similar\ derivation\ and\ Cosh(z) = Cos(iz)$$

$$r_H(t) = \frac{c}{H_0(1+z)} \frac{1}{\sqrt{(\Omega_0 - 1)}} ArcCosh\left[1 - 2\frac{(\Omega_0 - 1)}{\Omega_0 (1+z)}\right]$$

The Observer can not have received light signals, at any time of His history, from sources which are situated at proper distances Greater than $r_H(t)$ from him at the time t.

For H(t) I could use the expression with $\Omega_m \;\; \Omega_\Lambda \;\; \Omega_k$

In case of a Radiation dominated Universe I have to use the Proper expression fo H as well.

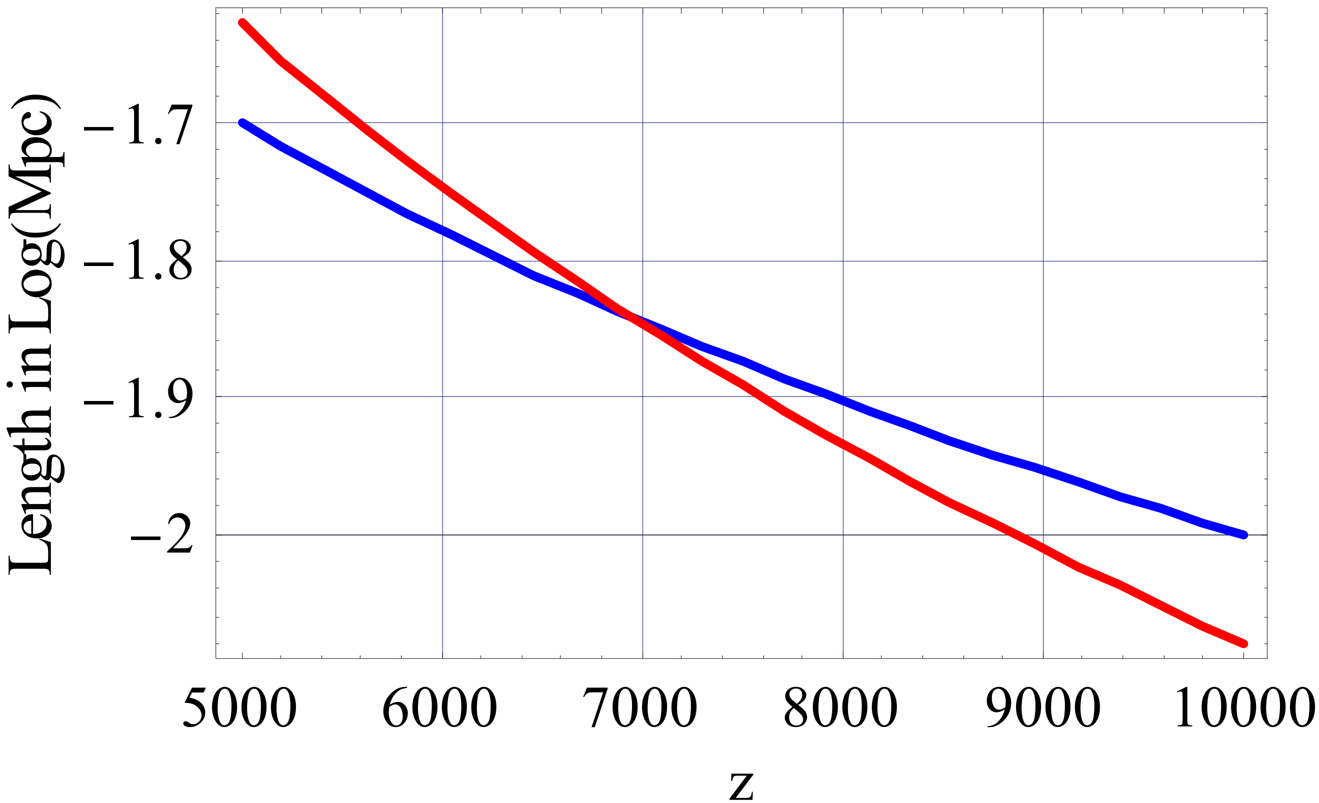
Radiation => p=1/3 ρ c²

$$r_H(t) = a(t)\int_0^a \frac{1}{a(t)H(t)}\frac{c\,da}{a(t)} =$$

$$(general\ case)\frac{c}{a_0 H_0}a(t)\int_0^a \frac{1}{a(t)}\frac{da}{\sqrt{\left[\Omega_0\left(\frac{a_0}{a}\right)^2 + (1-\Omega_0)\right]}} =$$

$$= \frac{c}{(1+z)H_0}\int_0^a \frac{da}{\sqrt{\left[\Omega_0\, a_0^2 + (1-\Omega_0)\,a(t)^2\right]}} \approx (\Omega \approx 1) \approx \frac{c}{(1+z)^2 H_0}$$

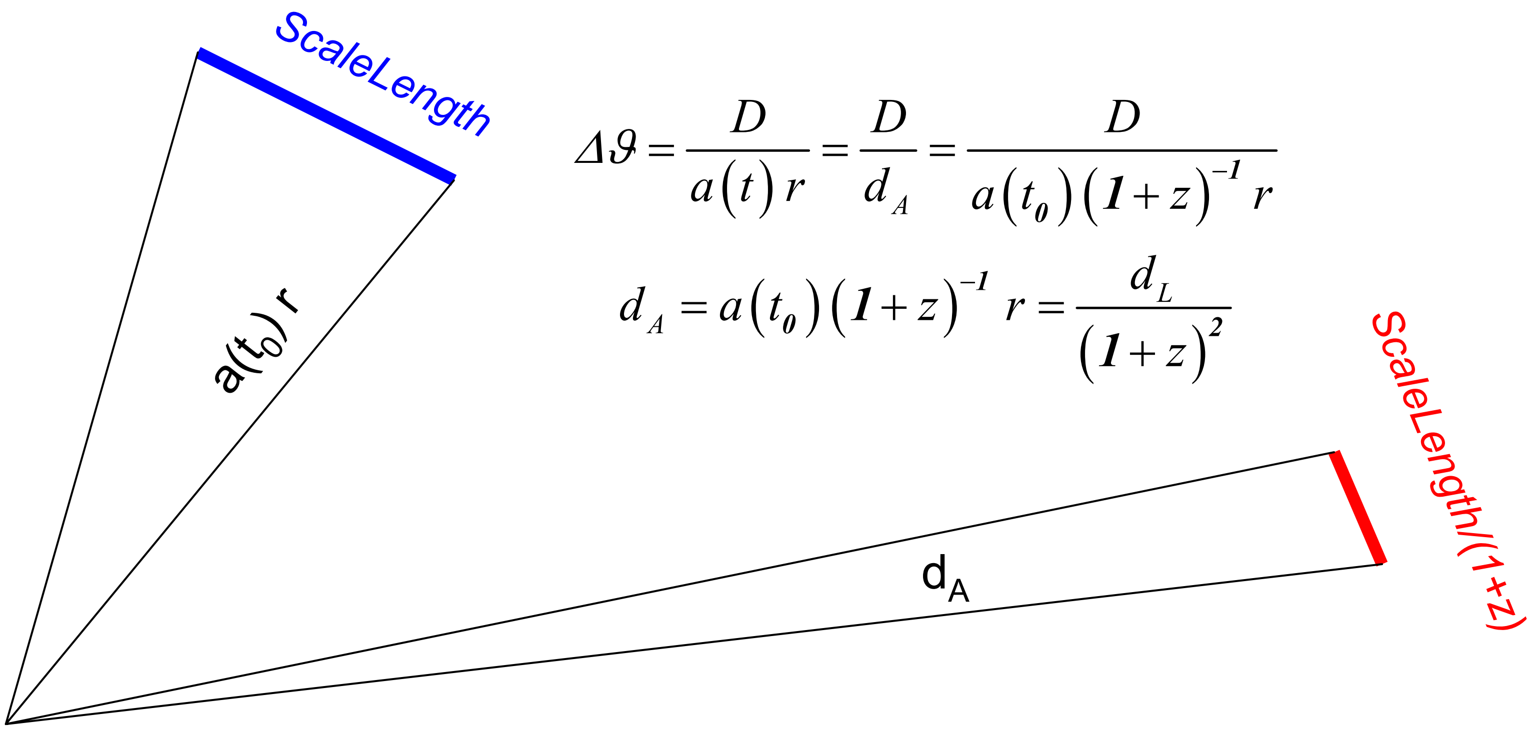
The Reference file is in Class Cosmology Horizon (Math). Rh at Z ~ 6450 is 0.016 Mpc or about 100 Mpc today. In Red how the Horizon goes as a function of redhift and in Blue how the scale length, 100 Mpc today in the present case, varies as a function of redshift in dusty Universe.



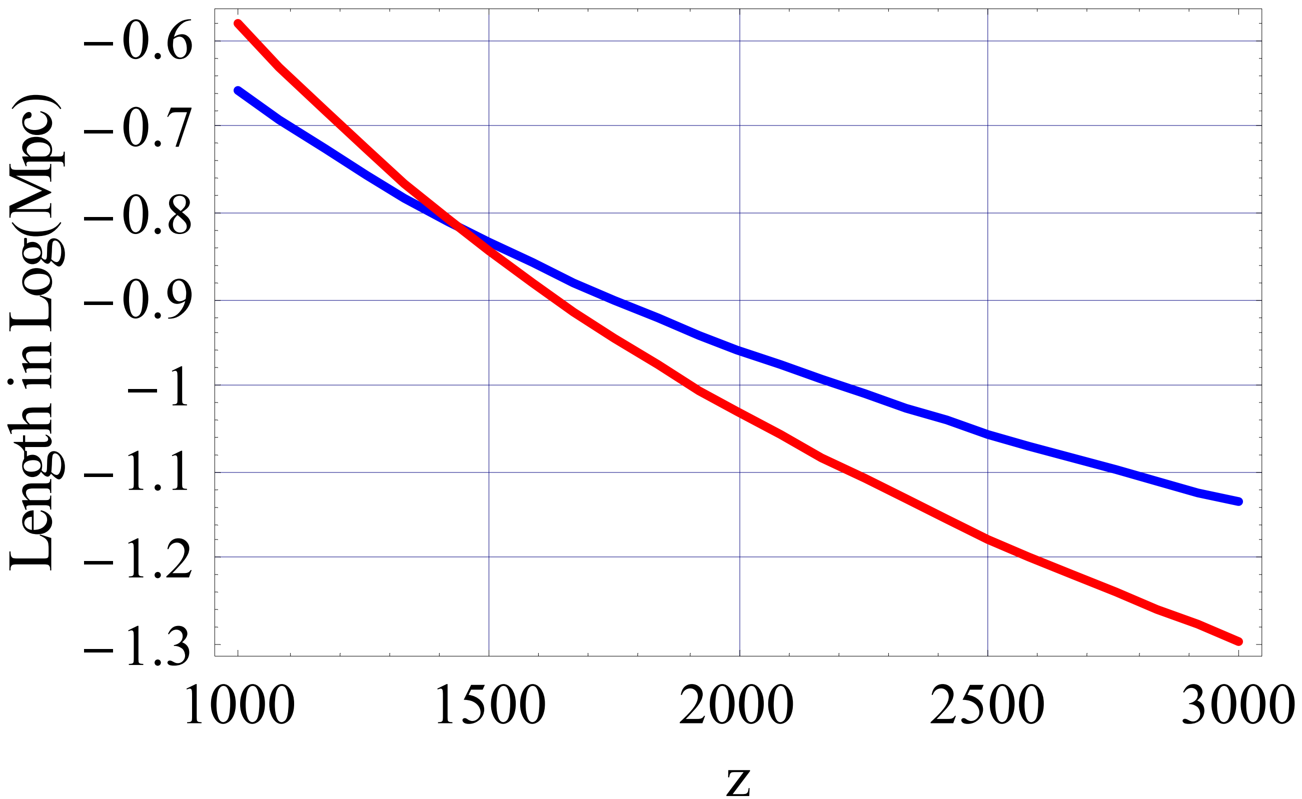
Angular Distance

$$\Delta\vartheta = \frac{D}{a(t)\,r} = \frac{D}{d_A} = \frac{D}{a(t_0)(1+z)^{-1}\,r}$$

$$d_A = a(t_0)(1+z)^{-1}\,r = \frac{d_L}{(1+z)^2}$$



Naturally a large Scale Length would enter the Horizon at later times and I show here the crossing done by a 220 Mpc Scale Length at about the recombination time.
The angle subtended by such Scale Length at the recombination epoch is of 0.94 Degrees, Theta=Length/AngDist.



Mathematica Files

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Or more recent Horizon

The student should not confuse a relaxed auto gravitating where, apart from evolution, the physica parameters are fixed with a scale length changing as (1+z).

Before making a few examples starts the Thermal History and talk about the time of equivalence. Completed the Thermal History and also in relation to the fluctuation of the Microwave Background it may be useful to discuss the Time of Enter of various scale length.