Coupling and Collapse

Prof. Guido Chincarini

I introduce the following concepts in a simple way:

1 The coupling between particles and photons, drag force.
2 The gravitational instability and the Jean mass made easy. The instability depends on how the wave moves in the medium, that is from the sound velocity and this is a function of the equation of state.
3 The viscosity and dissipations of small perturbations.

Thermal Equilibrium

- After the formation of Helium and at Temperatures below about 10^9 Degrees the main constituents of the Universe are protons (nuclei of the hydrogen atoms), electrons, Helium nuclei, photons and decoupled neutrinos.
- Ions and electrons can now be treated as non relativistic particles and react with photons vi avarious electromagnetic processes like bremstrahlung, Compton and Thompson scattering, recombination and Coulomb scattering between charged particles.
- To find out whether or not these electromagnetic interactions among the constituents are capable of maintaining thermal equilibrium the rates of the interaction must be faster than the expansion rate. That is we must have $t_{\text{Interaction Rate}} << H^{-1}$.
- Indeed carrying out a farly simple computation it can be shown that these processes keep matter and radiation tightly coupled till the recombination era.

Scattering & Radiation Drag

1. With the Thomson scattering the photon transfers the momentum to the electron bu a negligible amount of Energy.
2. As a consequence the Thomson scattering does not help in thermalization since there is no exchange of energy between the photons and the electrons.
3. Generally scattering with exchange of energy is called Compton scattering.
4. Compton scattering however does not change the number of photons as it could be done, for instance, by free – free transitions. Since it does not change the number of photons it could never lead to a Planck Spectrum if the system had the wronf number of photons for a given total energy.
5. On the other hand the Thomson scattering, for the reasons stated in 1) , will cause a radiation drag on the particle as we will see on slide 6.

Coupling Matter Radiation

- See Padmanabahn Vol 1 Page 271 Vol 2 Page 286-287 and problems.
- If a particle moves in a radiation field from the rest frame of the particle a flux of radiation is investing the particle with velocity v.
- During the particle photon scattering the photon transfer all of its momentum to the electron but a negligible amount of Energy.
- The scattering is accompanied by a force acting on the particle.
- A thermal bath of photons is also equivalent to a random superposition of electromagnetic radiation with $\sigma T^4 = \langle E^2/4\pi \rangle = \langle B^2/4\pi \rangle$
- When an electromagnetic wave hits a particle, it makes the particle to oscillate and radiate. The radiation will exert a damping force, drag, on the particle.
- The phenomenon is important because the coupling of matter and radiation cause the presence of density fluctuation both in the radiation and in the matter.

Continue

- If the particle moves in a radiation bath it will be suffering scattering by the many photons encountered on its path.
- The scattering is anisotropic since the particle is moving in the direction defined by its velocity.
- The particle will be hitting more photons in the front than in the back.
- The transfer of momentum will be in the direction opposite to the velocity of the particle and this is the drag force.
- This means that the radiation drag tend to oppose any motion due to matter unless such motion is coupled to the motion of the radiation.
- If we finally consider an ensemble of particles during collapse of a density fluctuation then the drag force will tend to act in the direction opposite to the collapse and indeed act as a pressure.
- As we will see this effect is dominant in the radiation dominated era when $z > 1000$.
- We take the relevant equations from any textbook describing radiation processes in Astrophysics.

An "other" effect in a simple way:
An electromagnetic wave hits a charged particle

Padmanabahn Vol. I -Page 271 & 164

Average of the force Over one period of the wave

$$Wave \quad + \quad particle$$

Wave makes the particle Oscillate.

$$\langle \bar{f} \rangle = \frac{2}{3}\left(\frac{q^2}{m\,c^2}\right)^2 \langle E^2 \rangle \bar{n} = \frac{8\pi}{3}\left(\frac{q^2}{m\,c^2}\right)^2 \frac{\langle E^2 \rangle}{4\pi}\bar{n} = \sigma_T U_{rad}\bar{n}$$

Flux of Radiation

$$\sigma_T = \frac{8\pi}{3}\left(\frac{q^2}{m\,c^2}\right)^2 = (Electron)\, 6.7\, 10^{-25}\, cm^2$$

$$Particle \quad + \quad Radiation\ field$$

$$\bar{f} = -\frac{4}{3}\sigma_T U_{Rad}\,\gamma^2\left(\frac{\bar{v}}{c}\right) = (non\ relativistic) - \frac{4}{3}\sigma_T U_{Rad}\left(\frac{\bar{v}}{c}\right)$$

See the work done by the drag force (f_{Drag} vel)and the derivation of Compton Scattering

Collapsing Cloud – simple way

- In the collapsing cloud the photons and the electrons move together due to the electrostatic forces generated as soon as they separate.
- To compute the parameters value we will use cosmological densities over the relevant parameters.

$$\textit{Mass of the cloud } \mathfrak{M};\ \textit{Radius } r; \cdots \rho = m_p n$$

$$f_{grav} = \frac{G\mathfrak{M}\rho}{r^2} = G\frac{4}{3}\pi r \rho^2;\ \textit{for n electrons}$$

$$\frac{f_{Drag}}{f_{Grav}} = \frac{-\frac{4}{3}\sigma_T U_{Rad}\left(\frac{\overline{v}}{c}\right)n}{G\frac{4}{3}\pi r \rho^2} = \frac{v\, n\, \sigma_T\, a^* T^4}{G\, c\, \pi\, r\, \rho^2} = \frac{v\, \sigma_T\, a^* T^4}{G\, c\, \pi\, r\, \rho m_p} =$$

$$\propto \frac{vT^4}{r\rho}\ \textit{very strong dependence on } T$$

Adding Cosmology

$$T = T_0 (1+z) \qquad T_0 = 2.7\ ^\circ K$$

$$\rho(z) = \rho_0 (1+z)^3 \qquad \rho_0 = 6\,10^{-30}\ ;$$

$$k = 0; t = t_0 (1+z)^{-\frac{3}{2}} \qquad t_0 = 4\,10^{17}\ \mathrm{sec}$$

$$Time\ of\ collapse \propto \frac{2}{3}\frac{r}{v} \sim 4\,10^{17} (1+z)^{-\frac{3}{2}}$$

$$\frac{f_{Drag}}{f_{Grav}} = \frac{vT^4}{r\rho} \propto \frac{(1+z)^{\frac{3}{2}} (1+z)^4}{(1+z)^3} = (1+z)^{\frac{5}{2}}\ ;$$

$$Since\ the\ const\ of\ proportionality \sim 10^{-8}$$

$$\frac{f_{Drag}}{f_{Grav}} = 10^{-8} (1+z)^{\frac{5}{2}}\ ;\ That\ is\ f_{Drag} \succ f_{Grav}\ \ for\ z \succ 1000$$

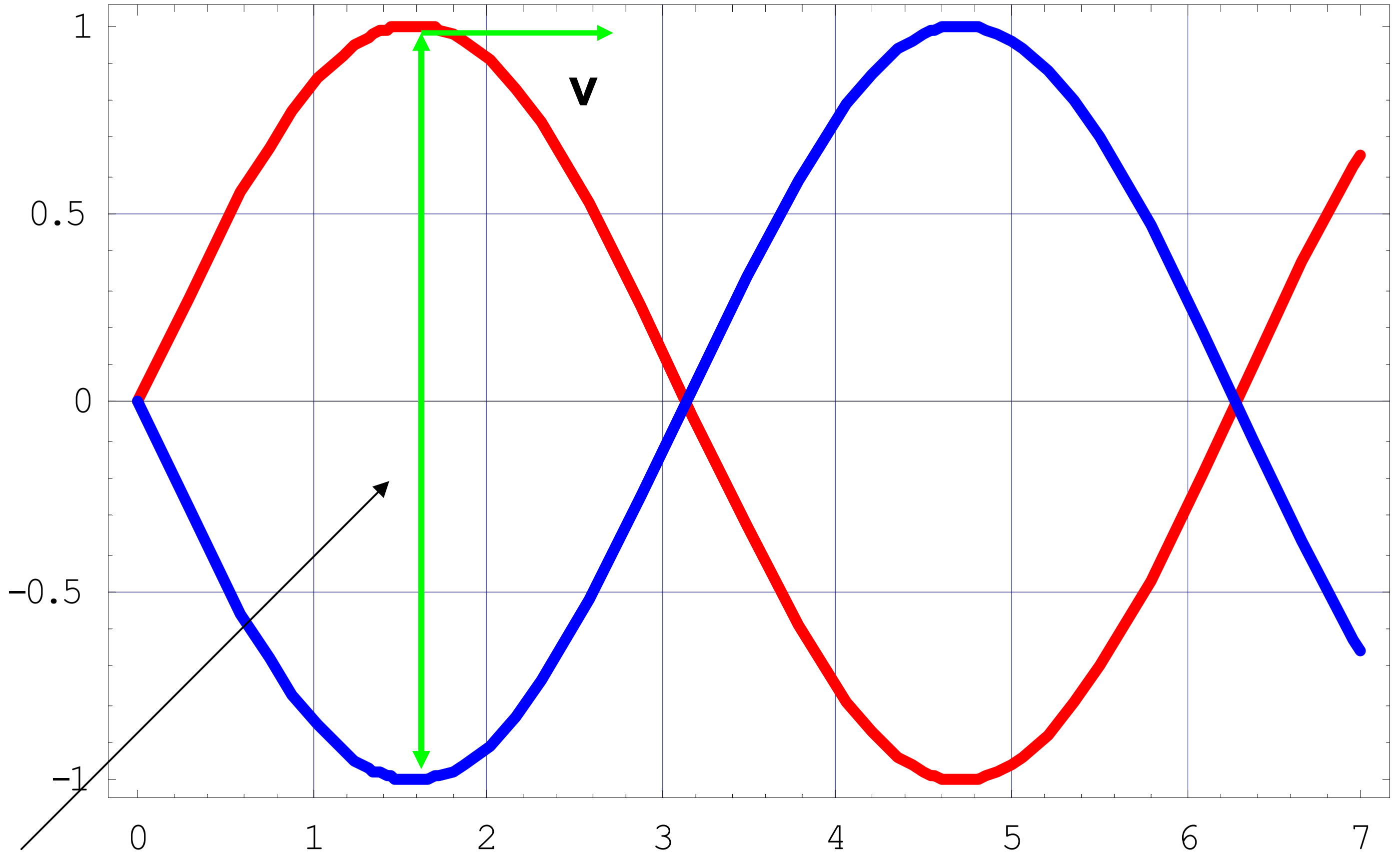
Summary

- The radiation at very high redshifts constrains the motion of the particles.
- It will stop the growth of fluctuations unless matter and radiation move together.
- This situation is valid both for a single electron or for a flow of electrons and protons.

Jean mass simplified

- If I perturb a fluid I will generate a propagation of waves. A pebble in a pound will generate waves that are damped after a while.
- The sound compresses the air while propagates through it. The fluid element through which the wave passes oscillates going through compressions and depressions. After each compression a restoring force tends to bring back the fluid to the original conditions.
- If the wave moves with a velocity v in a time r/v the oscillation repeat. See Figure.
- If the perturbation is characterized by a density ρ and dimension r the free fall time of the perturbation is proportional to $1 / \sqrt{\rho} G$
- Instability occurs when the time of free fall is smaller than the time it takes to restore the compression.

$T = t + \Delta t/2 = t + P/2 = t + \lambda/v = t + r/v$



Oscillation in the fluid

The reasoning

- If a wave passes through a fluid I have oscillations and compressions alternate with depression.
- However if the density of the compression on a given scale length r is high enough that the free fall time is shorter than the restoration time, the fluctuation in density will grow and the fluid continue in a free fall status unless other forces (pressure for instance) stop the fall.
- I will be able therefore to define a characteristic scale length under which for a given density I have free fall and above which the fluid will simply oscillate.
- This very simple reasoning can readily be put on equation apt to define the critical radius or the critical mass .
- This is what we normally define as the Jean mass or the Jean scale length.
- The fluctuation will perturb the Hubble flow.

$$t_{free\ fall} \propto (G\rho)^{-\frac{1}{2}};\ t_{restoring} \propto \frac{r}{v}\ and\ if\ (G\rho)^{-\frac{1}{2}} < \frac{r}{v}$$

$$The\ collapse\ wins;\ Define\ r_{crit} = v\,(G\rho)^{-\frac{1}{2}};\ The\ Jean\ mass$$

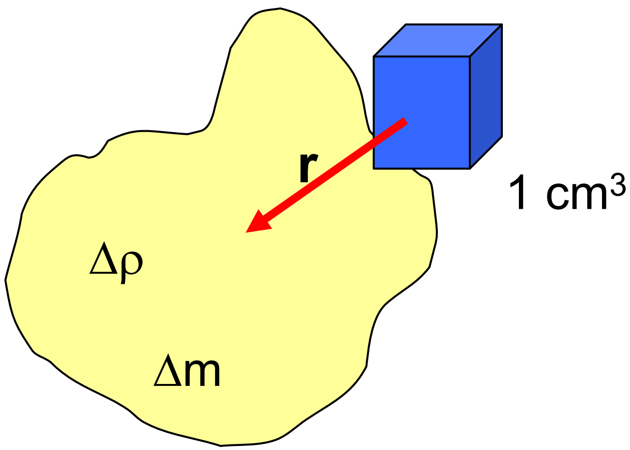
$$\mathfrak{M}_J = \frac{4}{3}\pi r_{crit}{}^3 \rho = \frac{4}{3}\pi\left(v\,(G\rho)^{-\frac{1}{2}}\right)^3 \rho = \frac{4}{3}\pi\,\rho v^3\,(G\rho)^{-\frac{3}{2}}$$

$$note\ [see\ Binney\ \&\ Tremaine]$$

$$t_{free\ fall} = \frac{1}{\sqrt{2}}\sqrt{\frac{3\pi}{16\,G\,\rho}} = \frac{1}{\sqrt{2}}t_{dynamical\ time}$$

The student can derive this result

Remark:



$$\Delta v = acc * \Delta t$$

$$acc = G\frac{\Delta m}{r^2} = G\frac{\frac{4}{3}\pi r^3 \Delta\rho}{r^2}$$

$$\Delta v \propto \frac{r^3 \Delta\rho\,\Delta t}{r^2} = r\Delta\rho\,\Delta t \sim \frac{r\,\Delta\rho}{H}; \quad \Omega = \frac{\rho}{\rho_c} \propto \frac{\rho}{H^2}$$

$$\frac{\Delta v}{v} = \frac{\Delta v}{Hr} = \frac{\Delta\rho}{H^2} \propto \frac{\Delta\rho}{\rho}\Omega \; proper\ analysis \; \frac{\Delta v}{v} = \frac{\Delta\rho}{\rho}\Omega^{0.6}$$

Improving – See B&T page 289

- I consider a spherical surface in a fluid and I compress the fluid of a certain factor. The volume change from V to $(1-\alpha)$ V.
- This will change the density and the pressure and I consider it as a perturbation to an homogeneous fluid.
- The compression causes a pressure gradient and originates therefore an extra force directed toward the exterior of the surface.
- On the other hand the perturbation in the density causes a gradient of density and a force toward the center of mass.
- From the balance of the two forces we derive as we did before the critical mass: The Jean mass.

$$V \Rightarrow (1-\alpha)V \ \textit{with} \quad \alpha \ll 1 \ \textit{and the density}$$

$$\frac{\mathfrak{M}}{V} = \rho_0 \Rightarrow \frac{\mathfrak{M}}{(1-\alpha)V} \sim \frac{\mathfrak{M}}{V}\left[1 + \frac{1}{(1-\alpha)^2}\right]\alpha + ... \approx \frac{\mathfrak{M}}{V}(1+\alpha) \ \textit{or}$$

$$\rho_{Tot} = \rho_0 + \rho_1 \equiv \rho_0 + \rho_0\,\alpha \ \textit{and the pressure}$$

$$p = p_0 + \left(\frac{\partial p}{\partial \rho}\right)_0 \Delta\rho + ... \Rightarrow p_1 = \left(\frac{\partial p}{\partial \rho}\right)_0 \Delta\rho = \left(\frac{\partial p}{\partial \rho}\right)_0 \rho_1 = \left(\frac{\partial p}{\partial \rho}\right)_0 \rho_0\,\alpha$$

And via the sound velocity the pressure perturbation :

$$\sqrt{\left(\frac{\partial p}{\partial \rho}\right)_0} = v_s \Rightarrow \left(\frac{\partial p}{\partial \rho}\right)_0 = v_s^2 \Rightarrow p_1 = \rho_0\,\alpha v_s^2$$

And the force per unit mass (acc.) :

$$\frac{\Delta p}{\Delta r}\frac{1}{\rho} \Rightarrow \frac{p_1}{r}\frac{1}{\rho_0} \approx \frac{\rho_0\,\alpha v_s^2}{r\rho_0} = \frac{\alpha v_s^2}{r}$$

The perturbation in density causes an inward gravitational force

$$\frac{G\mathfrak{M}}{r^2} = \frac{G\frac{4}{3}\pi r^3 \rho_0}{r^2} \Rightarrow dr = \alpha r \Rightarrow r^2 dr = \alpha r^3 \Rightarrow 4\pi r^2 dr = \alpha V$$

$$Potential \equiv \Phi_{Tot} \Rightarrow \frac{G\mathfrak{M}}{(r - dr)} = \frac{G\mathfrak{M}}{(r - \alpha r)} = \frac{G\mathfrak{M}}{r(1-\alpha)} = \Phi_0 + \Phi_1$$

$$\frac{d}{dr}\Phi_{Tot} = \frac{d}{dr}\frac{G\mathfrak{M}}{r(1-\alpha)} = \frac{G\mathfrak{M}}{r^2}\frac{1}{(1-\alpha)} = \frac{G\mathfrak{M}}{r^2}(1+\alpha)$$

$$Extra\ Force\,(perturbation) = \frac{G\mathfrak{M}}{r^2}\alpha$$

$$F_{p_1} + F_{G_1} > 0 \Rightarrow The\ fluid\ is\ stable$$

$$Directed\ outward \Rightarrow fluid\ exp\,ands \Rightarrow perturbation\ dissipates$$

$$F_{p_1} + F_{G_1} < 0 \Rightarrow The\ fluid\ is\ unstable$$

$$Directed\ inward \Rightarrow fluid\ contracts \Rightarrow perturbation\ increases$$

$$F_{G_1} = \frac{G\mathfrak{M}}{r^2}\alpha \approx G\rho_0 r\alpha > F_{p_1} = \frac{\alpha v_s^2}{r}$$

$$r^2 \geq \frac{v_s^2}{G\rho_0}$$

Summary

- We have found that perturbation with a scale length longer than $v_s/Sqrt[G\rho_0]$ are unstable and grow.
- Since v_s is the sound velocity in the fluid the characteristic time to cross the perturbation is given by r/v_s.
- We have also seen that the free fall time for the perturbation is $\propto 1/Sqrt[G\rho_0]$.
- We can therefore also state that if the dynamical time (or free fall time) is smaller than the time it takes to a wave to cross the perturbation then the perturbation is unstable and collapse.
- The relevance to cosmology is that the velocity of sound in a fluid is a function of the pressure and density and therefore of the equation of state.
- The equation of state therefore changes with the cosmic time so that the critical mass becomes a function of cosmic time.

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

$$\mathfrak{M}_J = \frac{4}{3}\pi\,\rho\, v_s^3 \left(G\rho\right)^{-\frac{3}{2}}$$

See slide 18

$$t \ll t_e$$

$$p = \frac{1}{3}\rho_R c^2; \quad \rho_R = \frac{a^* T^4}{c^2}; \quad \rho_R \approx \rho_{Tot}; \quad v_s^2 = \frac{\partial p}{\partial \rho} = \frac{1}{3}c^2$$

$$\mathfrak{M}_J = \frac{4}{3}\pi\,\rho\, v_s^3 \left(G\rho\right)^{-\frac{3}{2}} = \mathfrak{M}_J = \frac{4}{3}\pi\,\rho \frac{c^3}{3^{\frac{3}{2}}}\left(G\frac{a^* T^4}{c^2}\right)^{-\frac{3}{2}} = \frac{\rho\, c^6}{T_R^6 \left(G a^*\right)^{\frac{3}{2}}}$$

$$\rho(t) = \rho(t_e)\frac{a^3(t_e)}{a^3(t)} = \rho(t_e)\frac{T_R^{\,3}(t)}{T_R^{\,3}(t_e)} = \rho_R(t_e)\frac{T_R^{\,3}(t)}{T_R^{\,3}(t_e)} = \frac{a^* T_R(t_e) T_R^{\,3}(t)}{c^2}$$

$$\mathfrak{M}_J = \frac{\rho\, c^6}{T_R^6 \left(G a^*\right)^{\frac{3}{2}}} = \frac{c^4 T_R(t_e)}{T_R^3 G^{\frac{3}{2}} \left(a^*\right)^{\frac{1}{2}}} \propto T_R^{-3}$$

At the equivalence time t_e

$$\mathfrak{M}_J = \frac{\rho c^6}{T_R^6 \left(Ga^*\right)^{\frac{3}{2}}} = \frac{c^4 T_R\left(t_e\right)}{T_R^3 G^{\frac{3}{2}} \left(a^*\right)^{\frac{1}{2}}} = \frac{c^4}{T_R^2 G^{\frac{3}{2}} \left(a^*\right)^{\frac{1}{2}}} =$$

$$1.74\ 10^{17}\ \mathfrak{M}_\odot$$

A more interesting way to compute the sound velocity

$$v_s^2 = \frac{\delta p}{\delta \rho}$$

$$\rho = n\, m_p + a^* T^4 \; ; \; p = \frac{1}{3} a^* T^4 \; ; \; \sigma_{(photon\ entropy\ per\ particle)} = \frac{4\, a^* T^3}{3\, n\, k} \Rightarrow n = \frac{4\, a^* T^3}{3\, \sigma\, k}$$

$$\delta\rho = \delta n\, m_p + 4\, a^* T^3 \delta T \, ; \; \delta n = \frac{4\, a^*\, 3 T^2 \delta T}{3\, \sigma\, k}$$

$$\delta\rho = \frac{4\, a^*\, T^2 \delta T}{\sigma\, k} m_p + 4\, a^* T^3 \delta T = \left[3\, n\, m_p + 4\, a^* T^4 \right] \frac{\delta T}{T}$$

$$v_s^2 = \frac{\delta p}{\delta \rho} = \frac{\frac{4}{3} a^* T^4 \frac{\delta T}{T}}{\left[3\, n\, m_p + 4\, a^* T^4 \right] \frac{\delta T}{T}} = \frac{\sigma\, k\, n\, T}{\left[3\, n\, m_p + 3\, \sigma\, k\, n\, T \right]} = \frac{1}{3} \frac{\sigma\, k\, n\, T}{\left[m_p + \sigma\, k\, T \right]}$$

$$\mathfrak{M}_J = ...The\ int\ erested\ \ student\ reads\ Weinberg$$

After Recombination

- The Radiation temperature is now about 4000 degrees and matter and radiation are decoupled.
- Therefore the matter is not at the same temperature of the radiation any more but, for simplicity, it is better to refer always to the radiation temperature and transform T_m in T_R. I transform using the equations for an adiabatic expansion.
- I assume a mono atomic ideal gas for which in the polytropic equation of state γ=5/3
- To improve the accuracy of the equation I should use the correct equation for the free fall time in the previous derivations.

$$P = K\rho^{\gamma}\,;\ \frac{\partial P}{\partial \rho} = v_s^2 = \gamma K \rho^{\gamma-1} = \frac{\gamma K \rho^{\gamma}}{\rho} = \gamma \frac{P}{\rho}$$

$$\frac{P}{\rho} = \frac{kT_m}{m_p}\,;\ v_s^2 = \frac{\partial P}{\partial \rho} = \gamma \frac{kT_m}{m_p} = \frac{5}{3}\frac{kT_m}{m_p}$$

$$\mathfrak{M}_J = \frac{4}{3}\pi\,\rho\, v_s^3 \left(G\rho\right)^{-\frac{3}{2}} = \frac{4}{3}\pi\left[\frac{5}{3}\frac{kT_m}{m_p}\right]^{\frac{3}{2}} G^{-\frac{3}{2}}\rho^{-\frac{1}{2}} =$$

$$= \frac{4\pi}{3^{\frac{5}{2}}}\left[\frac{5kT_m}{m_p}\right]^{\frac{3}{2}} n^{-\frac{1}{2}} m_p^{-2}$$

For an adiabatic expansion

$$P = \frac{\rho}{m_p} k T_m = K \rho^{\gamma} \Rightarrow T_m \propto \rho^{\gamma - 1} \; or \; T_m V^{\gamma - 1} = const; \gamma = \frac{5}{3}$$

$$at \; t = t_e \Rightarrow T_m = T_e \Rightarrow \frac{T_m(t)}{\rho(t)^{\frac{2}{3}}} = const = \frac{T_m(t_e)}{\rho(t_e)^{\frac{2}{3}}} = \frac{T_R(t_e)}{\rho(t_e)^{\frac{2}{3}}}$$

$$T_m(t) = T_R(t_e) \frac{\rho(t)^{\frac{2}{3}}}{\rho(t_e)^{\frac{2}{3}}} = \left\{ \rho \propto a^{-3}; T_R \propto a^{-1} \right\} = T_R(t_e) \left(\frac{T_R(t)}{T_R(t_e)} \right)^2$$

$$\mathfrak{M}_J = \frac{4}{3}\pi\rho v_s^3\left(\frac{3/32*\pi}{G\rho}\right)^{\frac{3}{2}};\left[3/32*\pi \text{ added more accurate free fall}\right]$$

$$\mathfrak{M}_J = \frac{4}{3}\left(\frac{3}{32}\frac{5}{3}\right)^{\frac{3}{2}}\pi^{\frac{5}{2}}\left[\frac{kT_m}{m_p}\right]^{\frac{3}{2}}G^{-\frac{3}{2}}\rho^{-\frac{1}{2}} =$$

$$= \frac{4}{3}\left(\frac{5}{32}\right)^{\frac{3}{2}}\pi^{\frac{5}{2}}\left[\frac{k}{m_pG}\right]^{\frac{3}{2}}T_R^{\frac{3}{2}}(t_e)\left(\frac{T_R(t)}{T_R(t_e)}\right)^{\frac{3}{2}*2}\rho^{-\frac{1}{2}}(t_e)\left(\frac{T_R(t)}{T_R(t_e)}\right)^{-\frac{3}{2}} =$$

$$= \frac{4}{3}\left(\frac{5}{32}\right)^{\frac{3}{2}}\pi^{\frac{5}{2}}\left[\frac{k}{m_pG}\right]^{\frac{3}{2}}\rho^{-\frac{1}{2}}(t_e)T_R^{\frac{3}{2}}(t) \propto T_R^{\frac{3}{2}}$$

$$\rho^{-\frac{1}{2}}(t_e) = \rho_m(t_0)^{-\frac{1}{2}}(1+z_e)^{-\frac{3}{2}} = \begin{cases}\rho_m(t_0) = 2.91\ 10^{-30} \\ 1+z_e = 6540\end{cases} = \frac{1}{9.0\ 10^{-10}}$$

$$\frac{4}{3}\left(\frac{5}{32}\right)^{\frac{3}{2}}\pi^{\frac{5}{2}}\left[\frac{k}{m_pG}\right]^{\frac{3}{2}}\rho^{-\frac{1}{2}}(t_e)T_R^{\frac{3}{2}}(t_e) = 1.29\ 10^4\ \mathfrak{M}_\odot$$

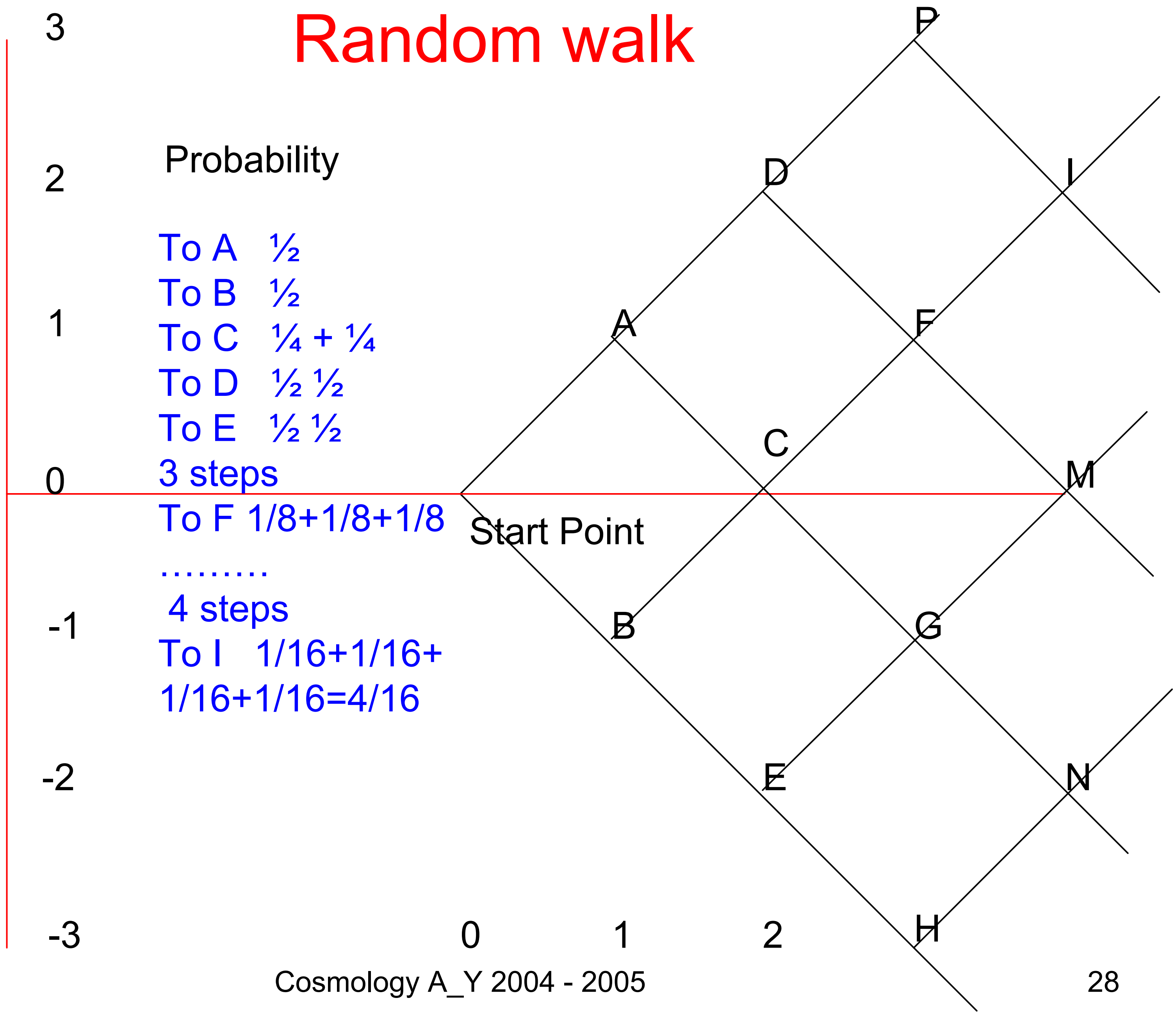
Viscosity in the radiation era

- Assume I have a fluctuation in density or density perturbation in the radiation dominated era.
- The photons moving in the perturbation will suffer various encounters with the particles and as we mentioned the particles tend to follow the motion of the photons.
- On the other hand if the mean free path of the photon is large compared to the fluctuation the photon escapes easily from the over-density.
- As a consequence the fluctuation tend to dissipate and diffuse away. That is the process tend to damp small fluctuations.
- The photons make a random walk through the fluctuations and escape from over-dense to under-dense regions. They drag the tightly charged particles.
- No baryonic perturbation carrying mass below a critical value, the Silk mass, survive this damping process.
- We present a simple minded derivation, for a complete treatment and other effects see Padmanabahn in Structure Formation in the Universe.

Random walk

Probability

To A ½
To B ½
To C ¼ + ¼
To D ½ ½
To E ½ ½
3 steps
To F 1/8+1/8+1/8
………
4 steps
To I 1/16+1/16+1/16+1/16=4/16



Probability to reach a distance m after n steps

$$P(m,n)=\frac{\#\,of\ \ possibilities\ to\ get\ to\ m}{2^n};\ P(-k,n)=P(+k,n)$$

$$After\ n\ steps$$

$$\Delta(N\,steps)=\sqrt{\frac{\sum_{-n}^{n}k^2P(k,n)}{\sum_{-n}^{n}P(k,n)}}=\sqrt{N}$$

UNIVERSITA' DEGLI STUDI DI MILANO BICOCCA

3 steps / 4 steps

k	where	P(k,n)	k^2 P(k,n)		k	where	P(k,n)	k^2 P(k,n)
					4	L	1/16	16/16
3	P	1/8	9/8		2	I	4/16	16/16
1	F	3/8	3/8		0	M	6/16	0
-1	G	3/8	3/8		-2	N	4/16	16/16
-3	H	1/8	9/8		-4	O	1/16	16/16
	Σ	8/8=1	24/8=3 $\Delta=\sqrt{3}$				16/16=1	4 $\Delta=\sqrt{4}$

- In order for the fluctuation to survive it must be that the time to dissipate must be larger than the time it takes to the photon to cross the perturbation.
- It can be demonstrated that if X is the size of the perturbation then the time it takes to dissipate the perturbation is about 5 time the size X of the perturbation. This number is an approximation and obviously could be computed exactly.
- Since the Jean Mass before the era of equivalence is of the order of the barionic mass within the horizon (check) the time taken to cross the fluctuation can be approximated with the cosmic time.
- The photon does a random walk with mean free path = λ.

$$\#of\ steps\ or\ scattering = \left(\frac{X}{\lambda}\right)^2 ;\ time\ per\ scattering = \frac{\lambda}{c} ; \lambda = \frac{1}{\sigma n}$$

$$t_{cross} = \left(\frac{X}{\lambda}\right)^2 * \frac{\lambda}{c} ; t_{dissipate} = 5 * t_{cross}$$

$$R_{diffuse} = \sqrt{c\,5\,t_{cross}\lambda} = \sqrt{c\,5\,t_{cross}\frac{1}{\sigma n}}$$

$$\mathfrak{M}_D = \frac{4}{3}\pi R_d^3 \rho = \frac{4}{3}\pi\left(c\,5\,t_{cross}\frac{1}{\sigma n}\right)^{\frac{3}{2}} n\,m_p = \frac{4}{3}\pi\left(\frac{5\,c\,t_{cross}}{\sigma}\right)^{\frac{3}{2}} n^{-\frac{1}{2}}\,m_p$$

$$\mathfrak{M}_D = \frac{4}{3}\pi\left(\frac{5\,c\,t_{cross}}{\sigma}\right)^{\frac{3}{2}} n^{-\frac{1}{2}}\, m_p\,; \quad \sigma = 0.665\,10^{-24}\ \&\ Before\ equivalence$$

$$t_{cosmic} \equiv t_c \sim t_{z_e} \propto a^2 \propto T^{-2}\,; t_{z_e}^{\frac{3}{2}} \sim \left(T^{-2}\right)^{\frac{3}{2}}\,; t_{z_e}^{\frac{3}{2}} \approx \left(H_0^{-1}\left(\frac{T_0}{T_{z_e}}\right)^2\right)^{\frac{3}{2}}$$

$$n = \frac{\rho_B}{m_p} = \frac{\rho_c \Omega_B}{m_p} = \frac{2.91\,10^{-30}\left(\Omega_B h_0^2\right)}{1.66\ 10^{-24}}\left(1+z\right)^3 = 1.75\ 10^{-6}\left(\Omega_B h_0^2\right)\frac{T_R^3}{T_0^3}$$

$$\mathfrak{M}_D = \frac{4}{3}\pi\left(\frac{5\,c}{0.665\,10^{-24}}\right)^{\frac{3}{2}}\left(H_0^{-1}\left(\frac{T_0}{T_R}\right)^2\right)^{\frac{3}{2}}\left(1.75\ 10^{-6}\left(\Omega_B h_0^2\right)\frac{T_R^3}{T_0^3}\right)^{-\frac{1}{2}} 1.66\,10^{-24}$$

$$= 5.3\ 10^{58}\left(\Omega_B h_0^2\right)^{-\frac{1}{2}}\left(\frac{T_R}{T_0}\right)^{-\frac{9}{2}}$$

$$\mathfrak{M}_D\left(t_e\right) = 8.34\ 10^{11}\left(\Omega_B h_0^2\right)^{-\frac{1}{2}}$$

Conclusions

- Naturally if the perturbation is smaller than the mean free path the photons diffuse instantaneously and no perturbation can survive for smaller scale lengths (or masses).
- Assuming a scale length for which the scale length corresponds to the travel carried out in a random walk by a photon in the cosmic time, we find that at the epoch of equivalence all masse below 10^12 solar masse are damped. The fluctuations can not survive.
- It is interesting to note that the Silk mass at the equivalence time is of the order of the mass of a galaxy. These structure have been allowed to grow.

Graphic Summary