The Models at work
Merge with Counts et al.

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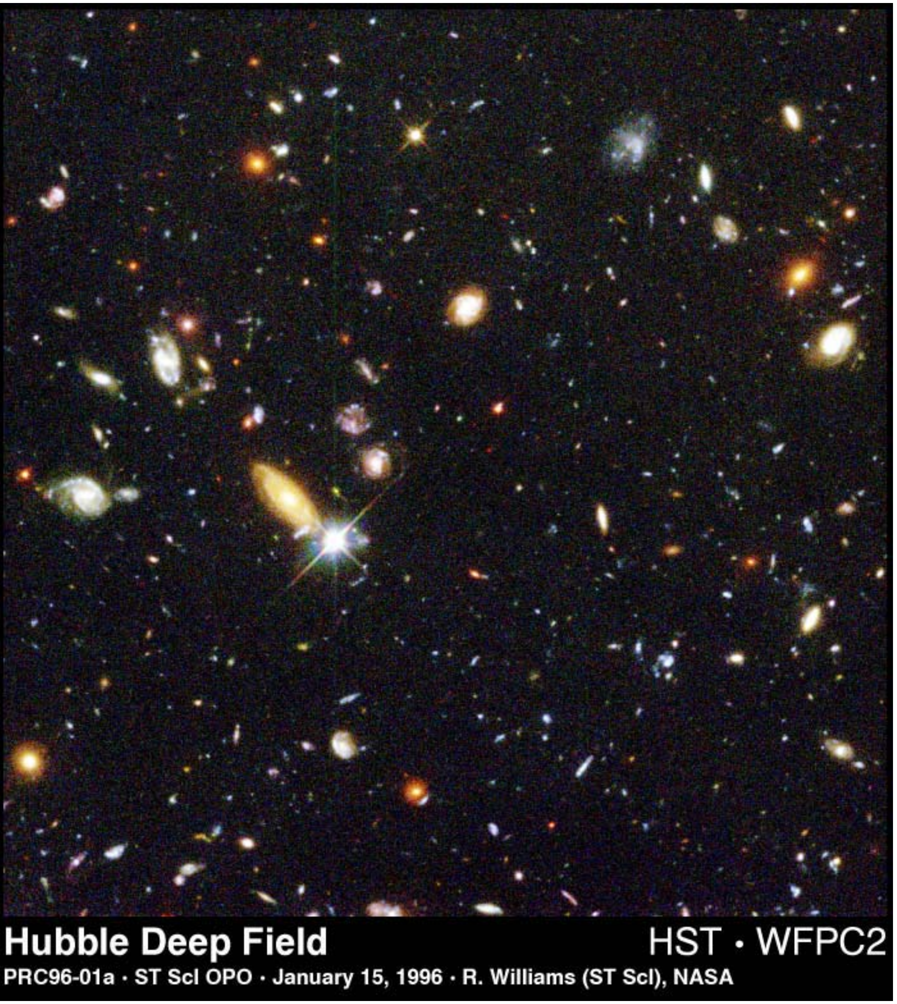
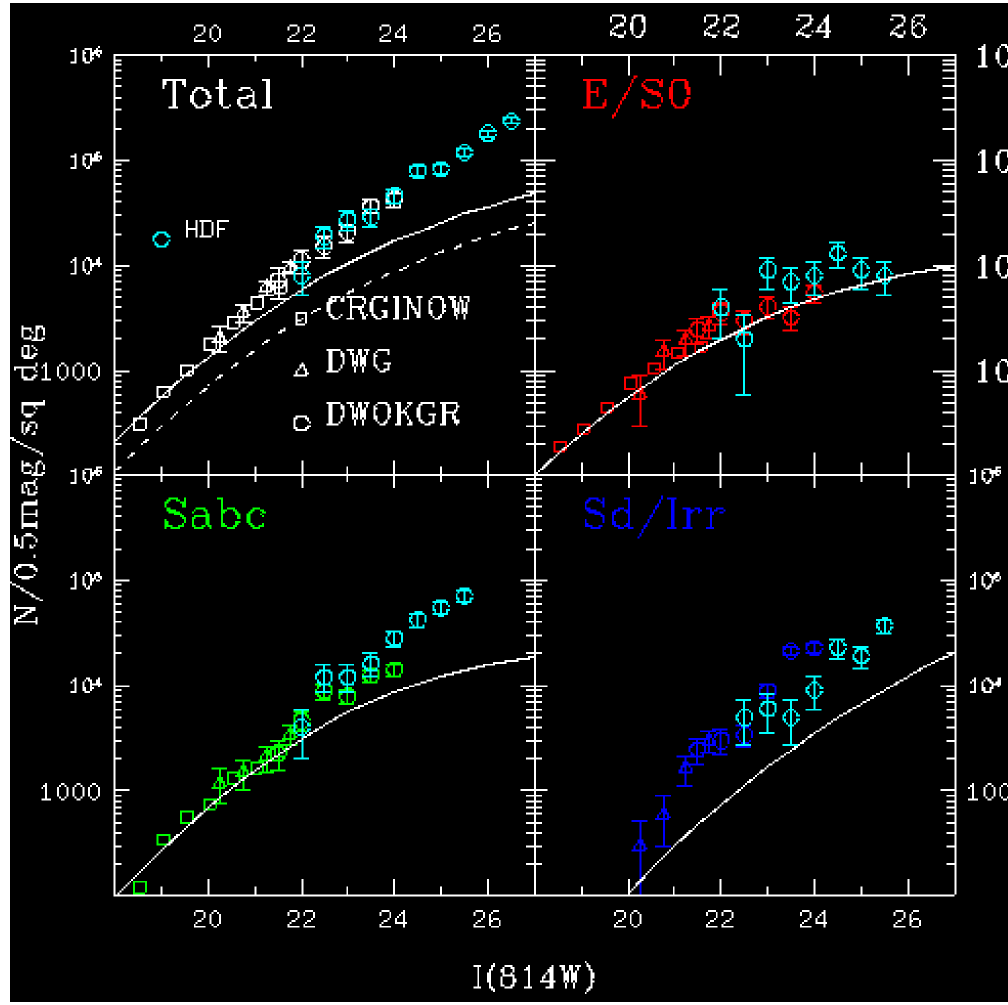
Lectures in Cosmology

Note: A few Figures have been elaborated

From Figures by Seb Oliver

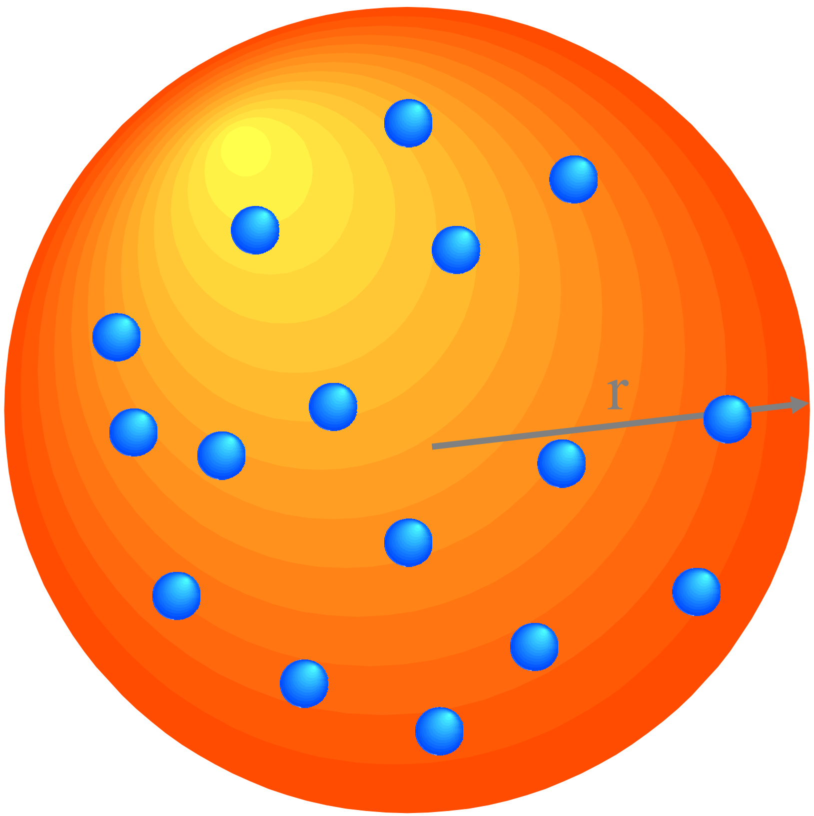
Will we eventually cover the sky with galaxies





And: bluer galaxies with an intrinsically lower luminosity apparently were more numerous in the past

Euclidean Number Counts



Assume a class of objects with L which with a sensitivity f are visible to a distance r

$$N \propto V$$

$$f \propto \frac{1}{r^2}$$

$$V \propto r^3$$

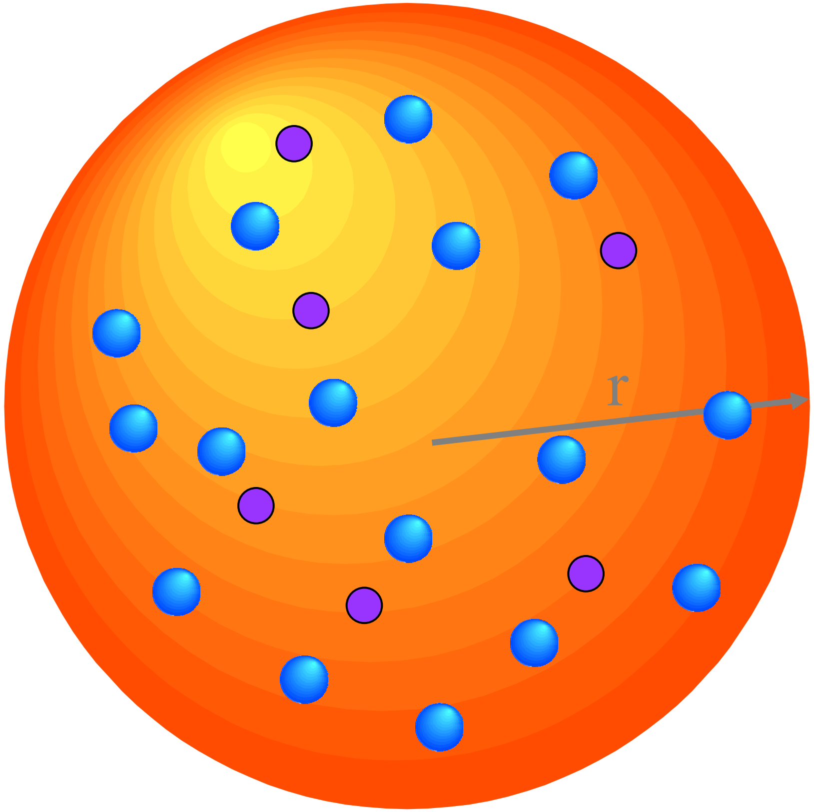
$$r^2 \propto \frac{1}{f}$$

$$\Rightarrow N \propto r^3$$

$$r^3 \propto f^{-3/2}$$

$$N \propto f^{-3/2}$$

Euclidean Number Counts



$$N = N_{0,1} f^{-3/2} + N_{0,2} f^{-3/2} + \cdots$$

$$N = f^{-3/2} \sum N_{0,i}$$

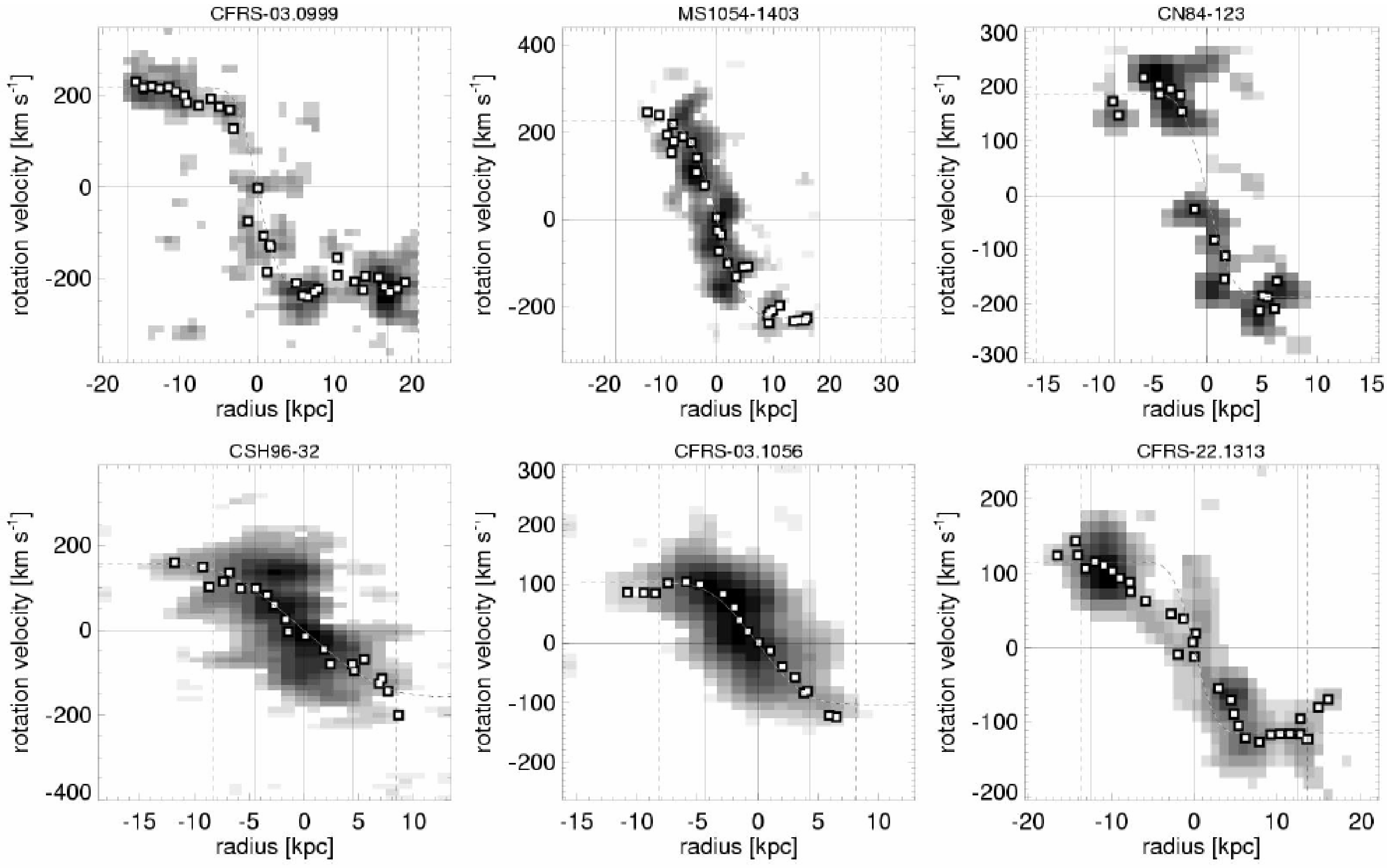
$$N \propto f^{-3/2}$$

$$\frac{dN}{df} \propto -\frac{3}{2} f^{-5/2}$$

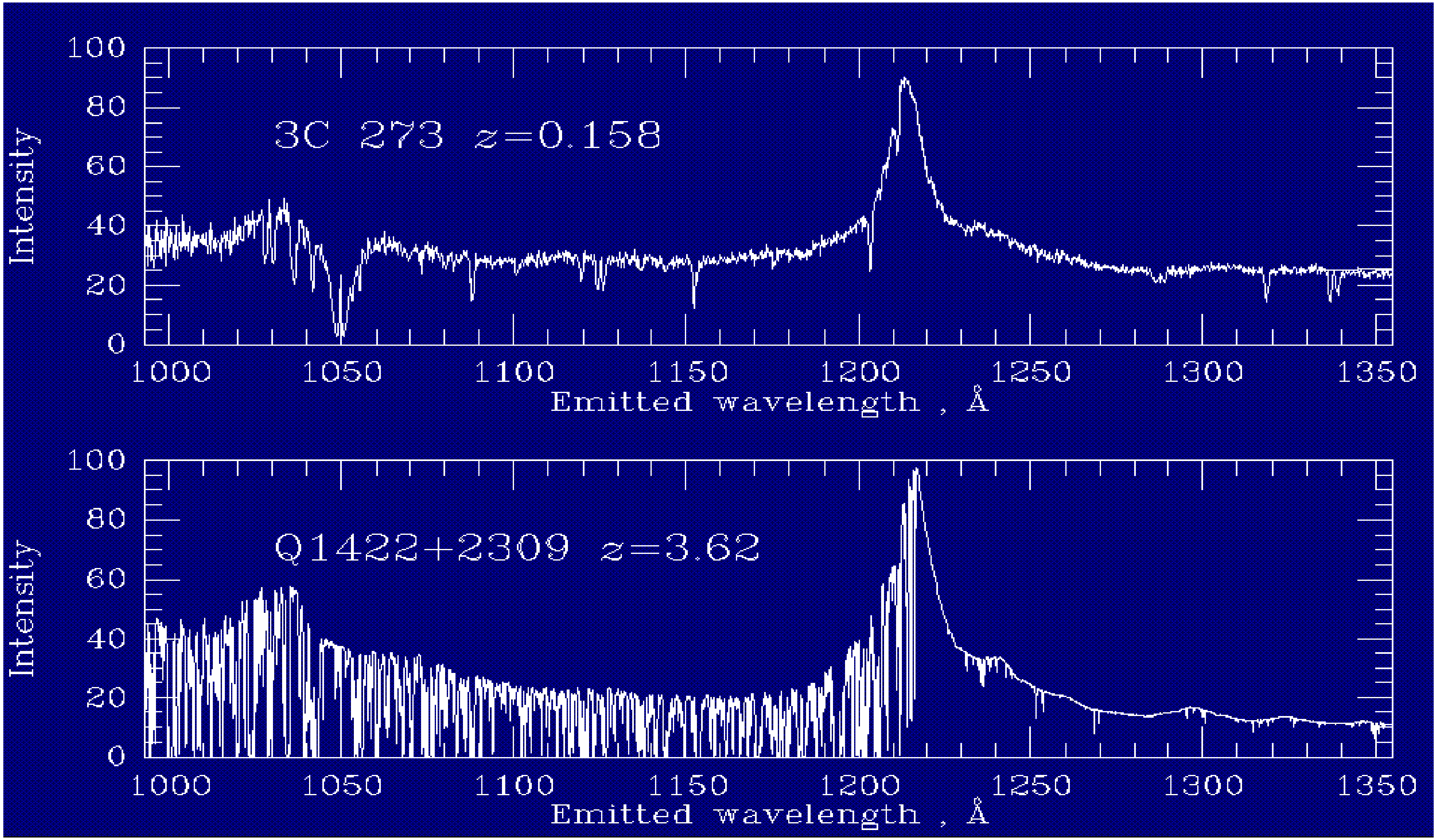
$$d \ln N = -\frac{3}{2} d \ln f$$

$$\frac{d \ln N}{d \ln f} = -\frac{3}{2}$$

Angular Momentum at z = 0.9



The Lyman alpha absorption lines (Lyman alpha Forest) originate from foreground structures. This component evolves strongly with cosmic time, since more absorbers are detected toward higher z.



The autocorrelation function

- The probability to find an object in the volume δV is:

$$\delta P = n\, \delta V$$

- Where n is the mean number density independent of position.
- The mean number of objects within a finite volume V is:

$$\langle N \rangle = n\, V$$

- The two point correlation function ξ is the joint probability of finding an object in both volume elements δV_1 and δV_2 at a separation r_{12}:

$$\delta P = n^2\, \delta V_1\, \delta V_2\, [1 + \xi(r_{12})]$$

Continue

- In a uniform Poisson distribution of points the probabilities of finding objects in δV_1, δV_2 are independent so that the joint probability is the product of the two probabilities:

$$\delta P = n^2 \, \delta V_1 \, \delta V_2$$

- That is here $\xi(r_{12}) = 0$. If the positions are correlated we expect $\xi(r_{12}) > 0$, if the positions are anti correlated, $-1 < \xi(r_{12}) < 0$.
- The chance of finding an object in δV_1 is $n \, \delta V_1$. Therefore the conditional probability of finding an object in the volume δV_2 at a distance r from the previous object located in δV_1 is:

$$\delta P(2|1) = n\ \delta V_2\ [1+ \xi(r_{12})\ ]$$

- Essentially I divided by n δV_1.

- We can put it in a different way:
- I choose and object at random in my ensemble and I estimate the probability of finding an object at a distance r as:

$$\delta P = n\ \delta V\ [1+ \xi(r)\ ]$$

So that the mean number of objects at a distance r from any randomly chosen object in the sample is (integral of the above:

$$< N >_p = \frac{4}{3}\pi r^3 n + n\int_0^r \xi(r) dV$$

Two Dimensions

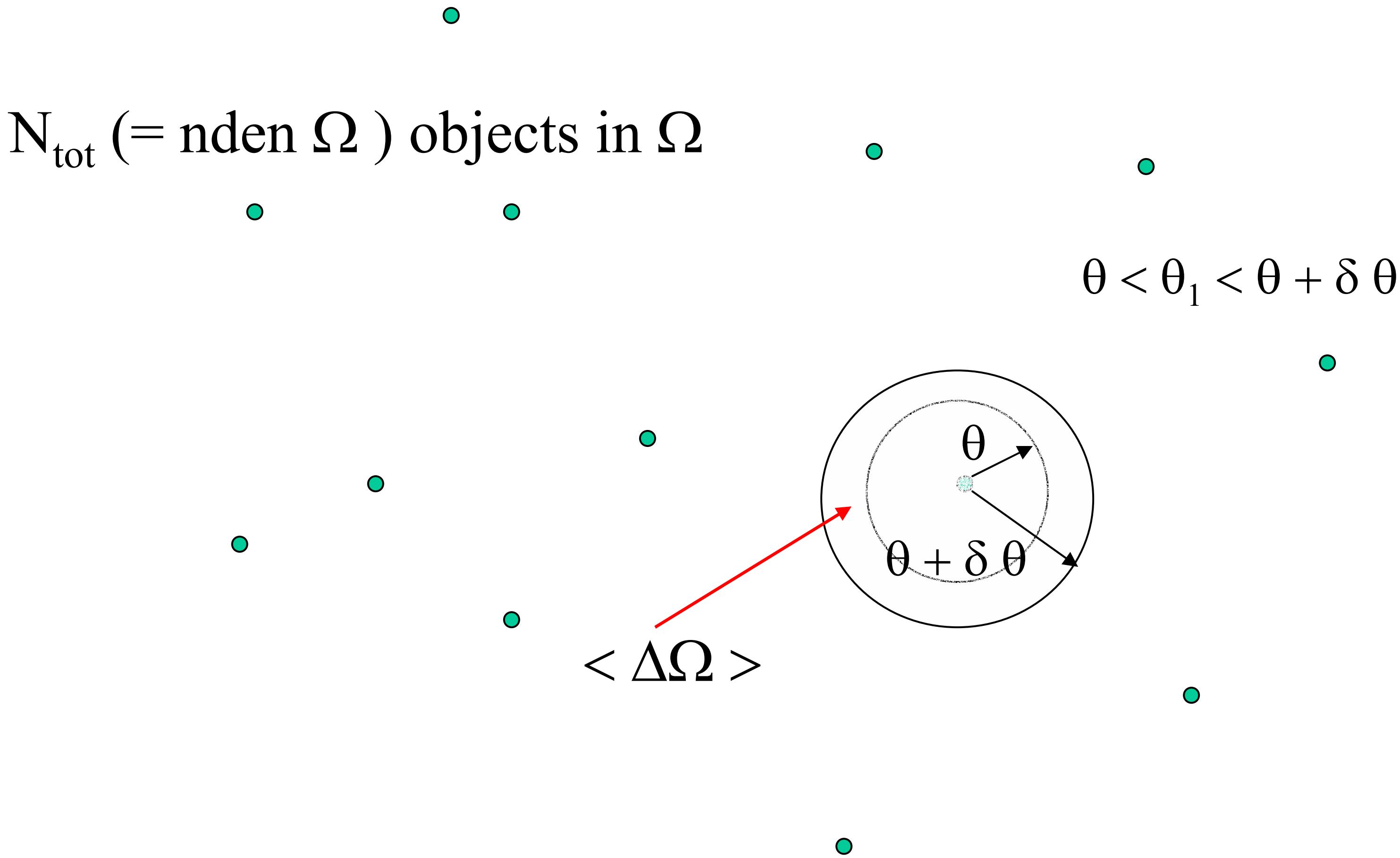
- This is exactly the same as the space two points correlation function except that we use the angle θ rather than the distance r and generally this is indicated as w(θ).

$$n_p = \tfrac{1}{2}\, nden^2 \Omega < \delta\Omega > [1 + w(\theta_1)]$$

$$1 + w(\theta_1) = 2n_p \, \Omega \big/ \left(N_{tot}{}^2 < \delta\Omega > \right) \quad ; \quad or\, use\; N(N-1)$$

- Here N_{tot} is the total number of objects, nden the mean density of objects (nden=N_{tot}/ Ω ,Ω the solid angle of the sample and n_p the number of pairs. ½ because each pair represents 2 neighbors.

Visually


N_{tot} (= nden Ω) objects in Ω
$\theta < \theta_1 < \theta + \delta\,\theta$
θ
θ + δ θ
< ΔΩ >

An easy way to compute it:

- The most direct way is the Monte Carlo method and its most recent developments;
- We can place N_r random points in the survey area. Between θ and $\theta + \delta\theta$ we will then count n_{pr} number of pairs. In the real catalogue of N objects, on the other hand, we will count n_p pairs. By definition for the random distribution of points we will have $w(\theta)=0$ so that:

$$\frac{1+w(\theta_1)}{1+0} = \frac{2n_p\Omega}{N^2 < \delta\Omega >}\frac{N_r^2 < \delta\Omega >}{2n_{pr}\Omega}$$

$$1+w(\theta_1) = \frac{n_p N_r^2}{n_{pr} N^2}$$

Results – However see Esp & Reflex

- A good approximation of the catalogues of Galaxies are:

$$w(\theta) = A\theta^{-\delta} \;;\; \delta = 0.77 \pm 0.04$$

&

$$\xi = (r/r_0)^{-\gamma} \;;\; \gamma = \delta + 1 = 1.77 \mp 0.04 \;;$$

$$r_0 = 4.23 \pm 0.26\ h^{-1}\ \mathrm{Mpc}$$

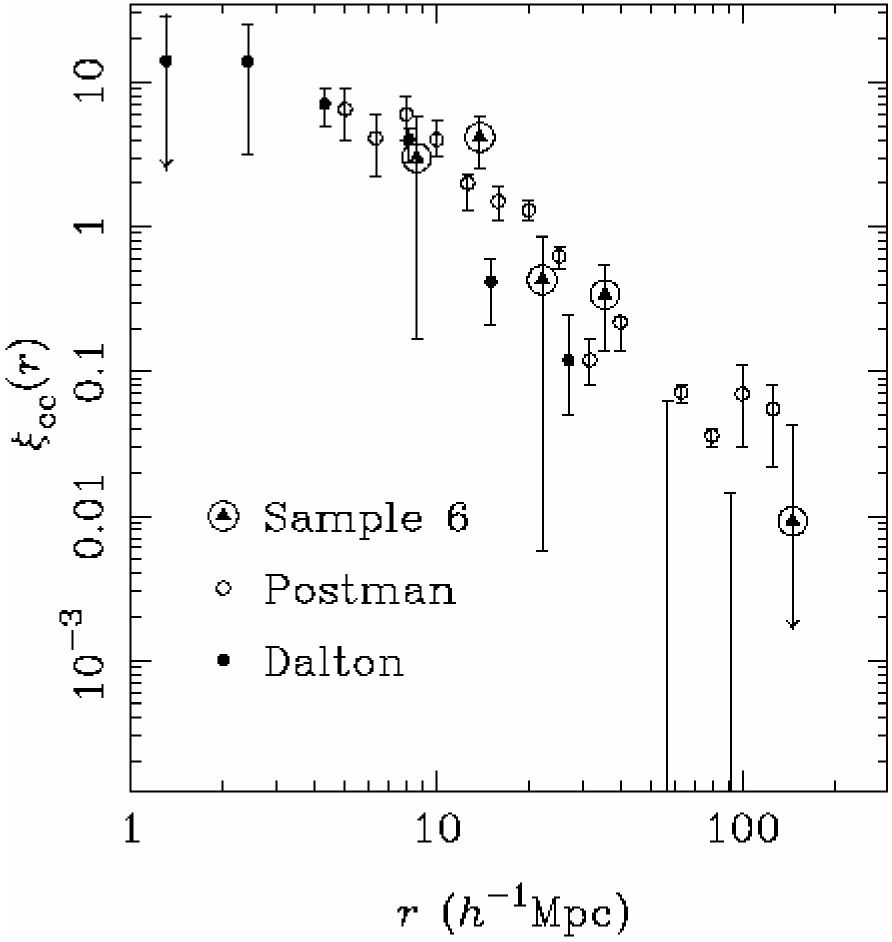
- And for the Clusters of Galaxies:

About the same slope

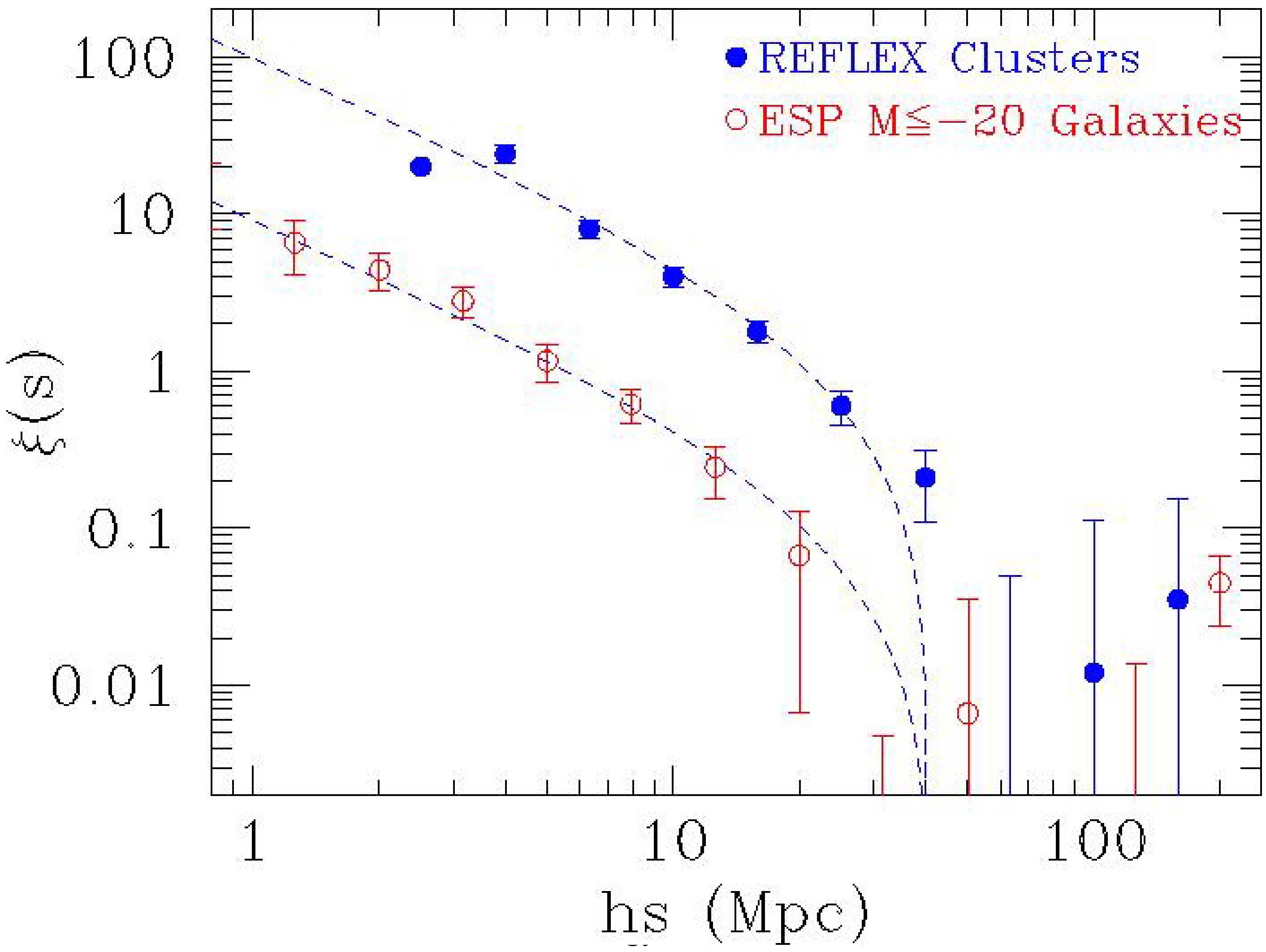
Much higher amplitude, $r_0 \sim 26$ Mpc

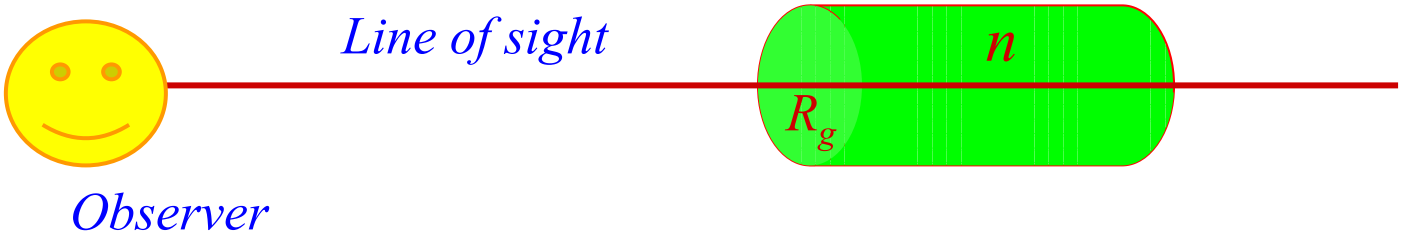
The Galaxy Autocorrelation Function

The Cluster Cluster Correlation Function



ξ(r) for Galaxies and Clusters





$$dN_l = \pi R_g^2\, n\, dl \quad \Rightarrow \quad dP = \pi R_g^2\, n\, dl = \pi R_g^2\, n \frac{dl}{dz} dz$$

$$\frac{a}{a_0} = \frac{1}{1+z} \Rightarrow \frac{da}{a_0} = \frac{dz}{(1+z)^2} \Rightarrow \frac{da}{a(1+z)} = \frac{dz}{(1+z)^2} \Rightarrow da = \frac{a\, dz}{(1+z)}$$

$$dl = c\, dt = c \frac{da}{\dot{a}} = c \left(\frac{a}{\dot{a}} \right) \frac{dz}{(1+z)} = R_H \frac{dz}{(1+z)}$$

$$\frac{dP}{dz} = \pi R_g^2\, n\, R_H \frac{1}{(1+z)} \quad ; \; n(z) = n_0 (1+z)^3$$

I assume to conserve the Number of galaxies

$$\frac{dP}{dz} = \left(\pi R_g^2\, H_0^{-1} n_0 \right) \left[H_0\, R_H\, (1+z)^2 \right]$$

$$\tau(z) = \int_0^z dP = \int_0^z \pi\, R_g^2\; n\; R_H \frac{dz}{(1+z)}$$

$$R_H(z) = \frac{c}{a_0 H_0} a(t) \int_0^a \frac{1}{a(t)} \frac{c\, da}{\sqrt{\left[\Omega_0 \left(\frac{a_0}{a}\right)\right]}} = 2\frac{c}{H_0 \Omega_0^{1/2}} \left(\frac{a}{a_0}\right)^{3/2} = 2\frac{c}{H_0 \Omega_0^{1/2}} \left(\frac{1}{1+z}\right)^{3/2}$$

$$\tau(z) = \int_0^z \pi\, R_g^2\; n_0\; (1+z)^3 \frac{dz}{(1+z)} 2\frac{c}{H_0 \Omega_0^{1/2}} \left(\frac{1}{1+z}\right)^{3/2} = \{for\ \Omega_0 \approx 1\}$$

$$\tau(z) = \frac{2}{3} \pi\, R_g^2\; n_0\, 2\frac{c}{H_0} (1+z)^{\frac{3}{2}}$$

Assuming $n_0 \sim 0.02\ h^3\ Mpc^{-3}$ (see the Luminosity Function) and R_g about $10\ h^{-1}$ (see our galaxy for instance) we find that even at $z = 2$ the fraction of the sky covered by galaxies is rather limited. Also note that the computation is straight forward since small h cancels out.

The Optical depth = 1 for $z \sim 20$ and however long before that our hypothesis, conservations of the number of galaxies and no evolution will break down.

Or I can simply define $R_H = c / H$ In this case I have, See F_Models

$$R_H = \frac{c}{H} =$$

$$= \frac{c}{H_0 \sqrt{\left[\Omega_{0,R} (1+z)^4 + \Omega_{0,B} (1+z)^3 + \Omega_{0,DM} (1+z)^3 + \Omega_K (1+z)^2 + \Omega_\Lambda \right]}}$$

$$\Omega_{0,R} + \Omega_{0,B} + \Omega_{0,DM} + \Omega_K + \Omega_\Lambda = 1$$

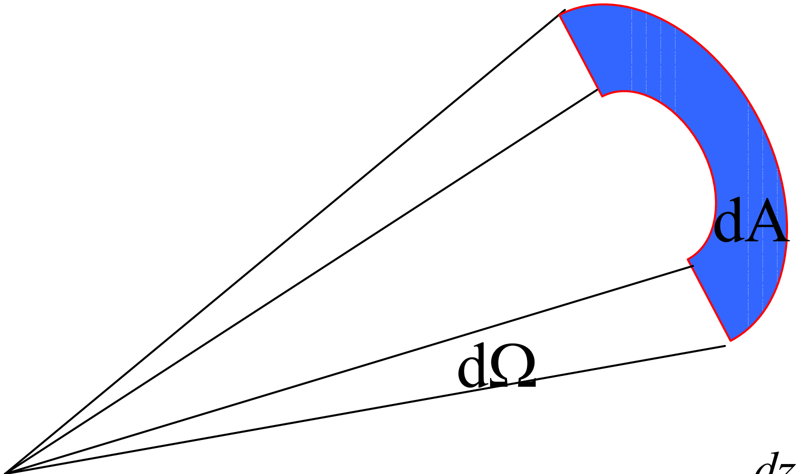
$$\Omega = \Omega_{0,R} + \Omega_{0,B} + \Omega_{0,DM} + \Omega_\Lambda \quad \Rightarrow \quad \Omega_K = 1 - \Omega$$

Approximations

$$R_H = \frac{c}{H_0\sqrt{\left[\Omega_{0,R}\left(1+z\right)^4+\Omega_{0,B}\left(1+z\right)^3+\Omega_{0,DM}\left(1+z\right)^3+\Omega_K\left(1+z\right)^2+\Omega_\Lambda\right]}}$$

$$R_H = \begin{cases} \dfrac{c}{H_0\sqrt{\left[\Omega_{0,R}\left(1+z\right)^4\right]}} & \left(Radiation\ dominated\right) \\[2ex] \dfrac{c}{H_0\sqrt{\left[\Omega_{0,B}\left(1+z\right)^3+\Omega_{0,DM}\left(1+z\right)^3\right]}} & \left(\Omega_\Lambda = 0\ ;\ \Omega_K = 0\right) \\[2ex] \dfrac{c}{H_0\sqrt{\left[\Omega_{0,B}\left(1+z\right)^3+\Omega_{0,DM}\left(1+z\right)^3+\Omega_K\left(1+z\right)^2\right]}}\ or & \\ \quad with\quad \Omega_K = \left(1-\Omega\right);\quad \Omega = \Omega_{0,B}+\Omega_{0,DM} & \\[2ex] \dfrac{c}{H_0\left(1+z\right)\sqrt{\left[1+\Omega\, z\right]}} & \text{Matter Dominated} \\[2ex] \dfrac{c}{H_0\sqrt{\left[\Omega_{0,B}\left(1+z\right)^3+\Omega_{0,DM}\left(1+z\right)^3+\Omega_\Lambda\right]}} & \end{cases}$$

Counts of Galaxies



$$dA = a^2(t)\, r_{em}^2\, d\Omega = \frac{a_0^2(t)}{(1+z)^2}\, r_{em}^2\, d\Omega$$

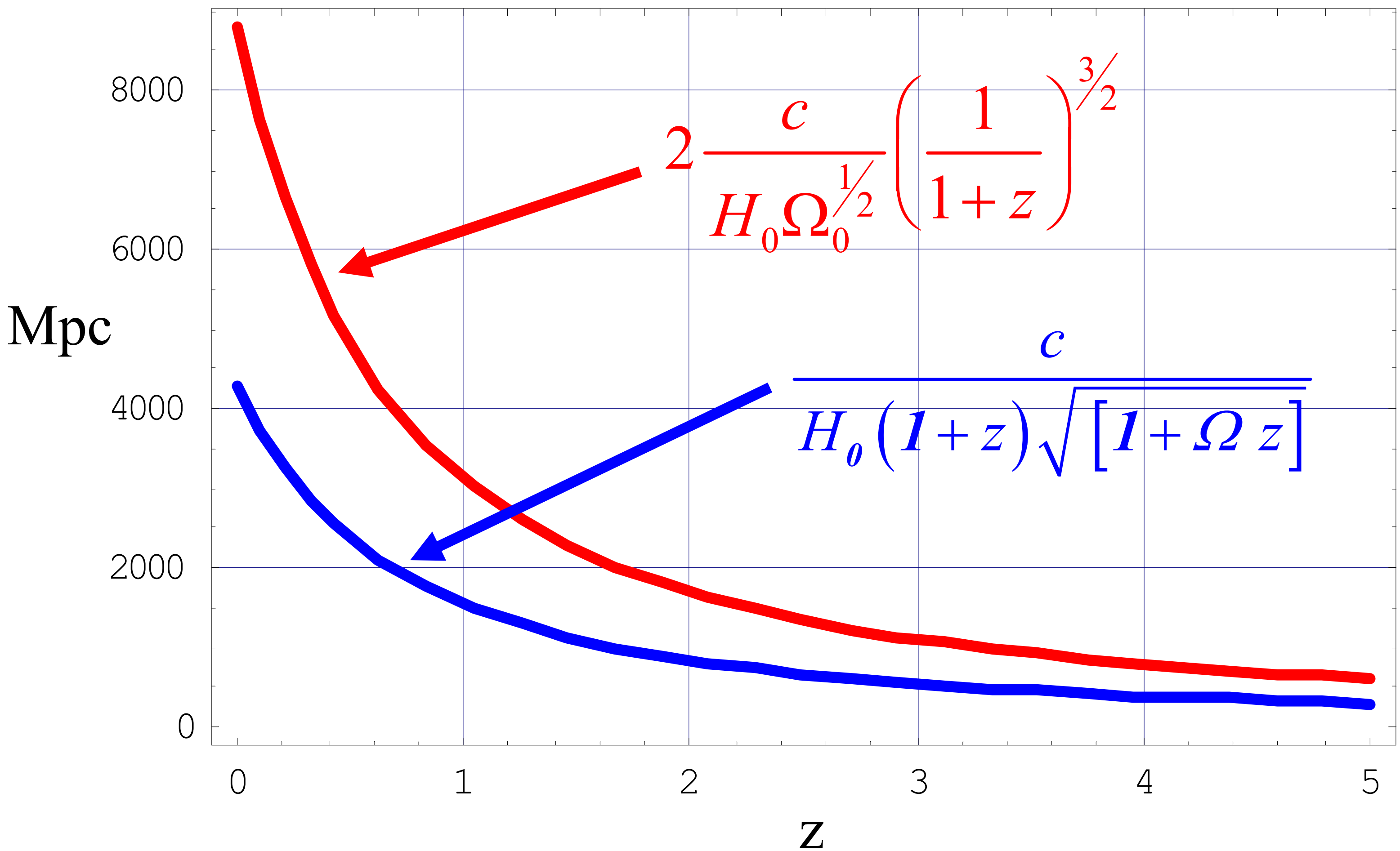
$$dV = dl * dA = R_H \frac{dz}{(1+z)} \frac{a_0^2(t)}{(1+z)^2}\, r_{em}^2\, d\Omega = \frac{a_0^2(t)\, r_{em}^2\, R_H(z)}{(1+z)^3}\, dz\, d\Omega$$

$$\frac{dN}{dz\, d\Omega} = n(z)\, dV = n(z) \frac{a_0^2(t)\, r_{em}^2\, R_H(z)}{(1+z)^3}$$

$$r = \frac{2c}{H_0 a_0 \Omega_0^2 (1+z)} \left[\Omega_0 z + (2-\Omega_0)\left(1-\sqrt{\Omega_0 z+1}\right) \right]$$

See F_Models

$$\frac{dN}{dz\, d\Omega} = n(z) \frac{a_0^2(t)\, r_{em}^2\, R_H(z)}{(1+z)^3} = n(z) \frac{4c^2 \left[\Omega_0 z + (2-\Omega_0)\left(1-\sqrt{\Omega_0 z+1}\right) \right]^2}{H_0^2 \Omega_0^4 (1+z)^2 (1+z)^3}\, 2 \frac{c}{H_0 \Omega_0^{1/2}} \left(\frac{1}{1+z} \right)^{3/2}$$



8000
6000
4000
2000
0
Mpc
0
1
2
3
4
5
z
$2\frac{c}{H_0\Omega_0^{1/2}}\left(\frac{1}{1+z}\right)^{3/2}$
$\frac{c}{H_0(1+z)\sqrt{[1+\Omega z]}}$

$$\frac{dN}{dz\, d\Omega} = n(z) \frac{a_0^2(t)\, r_{em}^2\ R_H(z)}{(1+z)^3}$$

$$\frac{dN}{d\Omega} = \int_0^z n(z) \frac{a_0^2(t)\, r_{em}^2\ R_H(z)}{(1+z)^3} dz$$

$$n(z) = n_0\,(1+z)^3$$

Make some plots eventually

Emission by class of objects

Particles and photons behave in a very similar way in an expanding Universe as far as momentum is concerned, see previous lectures exercises.

Consider now a stream of particle propagating freely. A co-moving observer will observe in a volume dV dN particles with momentum between **p** and **p+d^{3}p**. We can define for the set of particle a distribution function in the space of phase f (**x, p,** t) in the following way: dN = f (**x, p,** t) dV d^{3}p.

At a later time t+dt the proper Volume occupied by the particles is increased by the amount

$$\left[\frac{a(t+dt)}{a(t)}\right]^3$$

And the Volume in the momentum space is redshifted and decreases of the amount

$$\left[\frac{a(t)}{a(t+dt)}\right]^3$$

As a consequence the volume in the space phase does not change during the propagation of the particles.

dN is conserved because I conserve the number of particles and as a consequence the distribution function f (**x, p,** t) is conserved.

Furthermore for photons E = hν = p and $p \propto a^{-1}$ (redshift)

$$f\left(t,\bar{x},\bar{p}\right)=\frac{dN_\gamma}{dx^3dp^3}=\frac{dN_\gamma}{c\,dt_e dA_e}\frac{1}{dp^3\propto p^2dpd\Omega\propto \nu_e^2 d\nu_e d\Omega_e}=$$

$$\frac{dN_\gamma}{c\,dt_{obs}dA_{obs}}\frac{1}{\nu_{obs}^2 d\nu_{obs} d\Omega_{obs}} \quad consequence$$

$$\frac{dN_\gamma}{dt\,dA\,d\nu\,d\Omega}\frac{1}{\nu^2}=invariant$$

The Energy of dN photons is:

$$dE = h\,\nu\,dN \quad and\ the\ \boldsymbol{int}ensity\ erg\ s^{-1}cm^{-2}Hz^{-1}Sr^{-1}$$

$$I_\nu = \frac{dE}{dt\,dA\,d\nu\,d\Omega} \quad \Rightarrow \quad I_{obs} = \left(\frac{\nu_{obs}}{\nu_{em}}\right)^3 I_{em} \Rightarrow \frac{I_\nu}{\nu^3}\ In\boldsymbol{var}iant$$

$$Energy\ density\ U_\nu = \frac{4\pi}{c} I_\nu\ erg\ cm^{-3}\ H_z^{-1}$$

$$Therefore\ U_\nu \propto a^{-3}$$

$$I\left[\nu\left(1+z\right), z\right] = I\left[\nu, 0\right]\left(1+z\right)^3$$

$$F_{Total} = \int_0^\infty I_\nu d\nu \propto \left(1+z\right)^4 \quad Planck\ Spectrum\ T \propto a^{-1}$$

We assume that at some redshift z we have an emission $\varepsilon_\nu(z)$ in ergs cm^{-3} s^{-1} Hz^{-1}. During the interval t t+dt this emission produces a density of energy $\varepsilon_\nu(z)\ dt = dU_\nu$ with units ergs cm^{-3} Hz^{-1}. The density of energy as we have seen runs as $(1+z)^3$.

Assuming that the sources are distributed in the range of redshift between z_1 and z_2 we can easily derive the contribution to the density due to all sources.

From the density of energy I can derive the intensity and the flux I would observe.

Again the derivation is very simple if I use R_H for a matter dominated Universe with cosmological constant zero while I must integrate numerically in case I use a ΛCDM Universe.

$$dU_{\frac{\nu}{1+z}}\left(z=0\right)=\frac{\varepsilon_{\nu}\left(z\right)dt}{\left(1+z\right)^{3}};\quad dt=\frac{R_H}{c}\frac{1}{1+z}$$

$$U_{[\nu_0,z=0]}=\int_{z_1}^{z_2}\frac{\varepsilon\left(\nu_0\left(1+z\right),z\right)}{\left(1+z\right)^{3}}\frac{dt}{dz}dz=\int_{z_1}^{z_2}\frac{\varepsilon\left(\nu_0\left(1+z\right),z\right)}{\left(1+z\right)^{4}}\frac{R_H\left(z\right)}{c}dz$$

$$for\quad R_H\left(z\right)=\frac{c}{H_0\left(1+z\right)\sqrt{\left[1+\Omega z\right]}}$$

$$U_{[\nu_0,z=0]}=\int_{z_1}^{z_2}\frac{\varepsilon\left(\nu_0\left(1+z\right),z\right)}{\left(1+z\right)^{4}}\frac{1}{H_0\left(1+z\right)\sqrt{\left[1+\Omega z\right]}}dz=\int_{z_1}^{z_2}\frac{\varepsilon\left(\nu_0\left(1+z\right),z\right)}{\left(1+z\right)^{5}}\frac{1}{H_0\sqrt{\left[1+\Omega z\right]}}dz$$

$$since\quad I_{\nu}=\frac{c}{4\pi}U_{\nu}$$

$$I_{[\nu_0,z=0]}=\frac{c}{4\pi H_0}\int_{z_1}^{z_2}\frac{\varepsilon\left(\nu_0\left(1+z\right),z\right)}{\left(1+z\right)^{5}}\frac{dz}{\sqrt{\left[1+\Omega z\right]}}\ erg\ cm^{-2}\ s^{-1}\ Hz^{-1}\ sr^{-1}$$

Assume I have n sources with Luminosity L

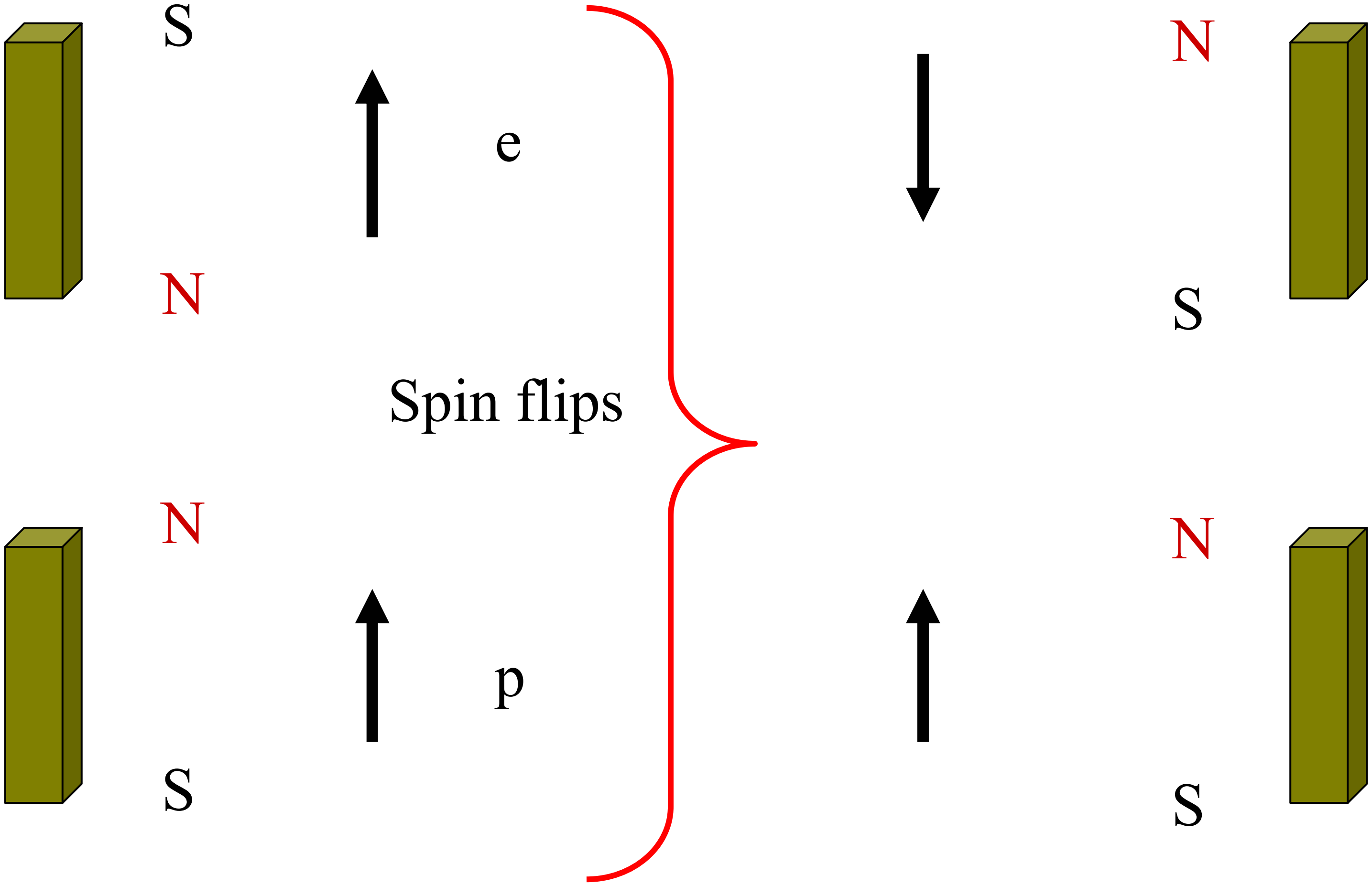
$$\varepsilon_\nu(z) = n(z)\frac{dL_{em}}{d\nu_{em}} = n_0(1+z)^3\frac{dL_{em}}{d\nu_{em}}$$

$$I_{[\nu_0, z=0]} = \frac{c}{4\pi H_0}\int_{z_1}^{z_2} n_0(1+z)^3 \frac{dL_{em}}{d\nu_{em}} \frac{1}{(1+z)^5} \frac{dz}{\sqrt{[1+\Omega z]}}$$

$$I_{[\nu_0, z=0]} = \frac{c\,n_0}{4\pi H_0}\int_{z_1}^{z_2} \frac{dL_{em}}{d\nu_{em}} \frac{1}{(1+z)^2} \frac{dz}{\sqrt{[1+\Omega z]}}$$

$$\nu_{em} = \nu_{obs}(1+z)$$

21 cm 1420.4 MHz



Characteristics

- High Energy State 1/2+1/2 F=1
- # of states 2 F +1 = 3 g=3

- Low Energy State 1/2 + -1/2 =0 F=0
- # of states 2 F+1 = 1 g=1

- Separation between levels:
- $6\ 10^{-6}$ eV ≡ T=0.07 K ≡ 1420.4 MHz ≡ 21.105 cm
- Transition probability
- $2.869\ 10^{-15}\ s^{-1}$

$$e^{-\frac{h\nu}{kT_s}} \approx 1 - \frac{h\nu}{kT_s}$$

$$\tau_\nu = \int \left(\frac{h\nu}{4\pi}\right) n_1 B_{12} \left(1 - e^{-\frac{h\nu}{kT_s}}\right) I_\nu \, \varphi(\nu) \, ds = \int \left(\frac{h\nu}{4\pi}\right) n_1 B_{12} \frac{h\nu}{kT_s} I_\nu \, \varphi(\nu) \, ds =$$

$$= \int_0^\infty n_1 \frac{B_{12} h^2 \nu^2}{4\pi k T_s} \varphi(\nu) \, ds = \left(\frac{B_{12} h^2 \nu^2}{4\pi k T_s}\right) N_1 \varphi(\nu)$$

3

1

$$\frac{N_2}{N_1} = \frac{n_2}{n_1} \sim \frac{g_2}{g_1} = \frac{3}{1} \Rightarrow N_H = (3+1) N_1$$

$$B_{12} = \frac{g_2}{g_1} B_{21} \qquad B_{21} = A_{21} \frac{c^2}{2h\nu^3}$$

21 cm Field

$$I_{[\nu_0, z=0]} = \frac{c}{4\pi H_0}\int_{z_1}^{z_2} \frac{\varepsilon(\nu_0(1+z), z)}{(1+z)^5}\frac{dz}{\sqrt{[1+\Omega z]}}\ erg\ cm^{-2}\ s^{-1}\ Hz^{-1}\ sr^{-1}$$

$$\varepsilon(\nu_H = 1420\ MHz) = \frac{3}{4} A\, n_H\ h\, \nu\ \delta(\nu - \nu_H) \quad with\ A = 2.85\ 10^{-15}\ s^{-1}$$

$$n_H = n_0\,(1+z)^3\,[1 - x(z)] \quad and\ \ for\ \nu_0 < \nu_H\ and\ 0 < z < \infty$$

$$I_{[\nu_0, z=0]} = \frac{c}{4\pi H_0}\int_0^\infty \frac{\frac{3}{4} A\, n_0\,(1+z)^3\,[1-x(z)]\, h\,\nu_0\,(1+z)\,\delta[\nu_0(1+z)-\nu_H]}{(1+z)^5}\frac{dz}{\sqrt{[1+\Omega z]}} =$$

$$= \frac{3\, c\, A\, n_0\, h\, \nu_0}{16\pi H_0}\int_0^\infty \frac{[1-x(z)]\,\delta[\nu_0(1+z)-\nu_H]}{(1+z)}\frac{dz}{\sqrt{[1+\Omega z]}} =$$

Zero except when

$$for\ (1+z_c) = \frac{\nu_H}{\nu_0} \Rightarrow I_{[\nu_0, z=0]} = \frac{3\, c\, A\, n_0\, h\, \nu_0}{16\pi H_0}\frac{[1-x(z_c)]}{(1+z_c)\sqrt{[1+\Omega z_c]}}$$

$$And\ \Delta I = \frac{3\, c\, A\, n_0\, h\, \nu_0}{16\pi H_0}[1 - x(z_c = 0.0)] = \frac{3\, c\, A\, h\, \nu_0}{16\pi H_0} n_{HI}(z=0)$$

$Erg\ cm^{-2}\ s^{-1}$

Olber – Why the Night Sky is dark?

$$Flux = \int_0^\infty 4\pi r^2 dr\, n \frac{L}{4\pi r^2} = \infty \quad However\ for\ dL = L_0 \nu^{-\alpha} d\nu\ \ for\ \nu_{Min} \le \nu \le \nu_{Max}$$

$$\&\ for\ \Omega = 1$$

$$I_{[\nu_0, z=0]} = \frac{c\, n_0}{4\pi H_0} \int_{z_1}^{z_2} \frac{dL_{em}}{d\nu_{em}} \frac{1}{(1+z)^2} \frac{dz}{\sqrt{[1+\Omega\, z]}} = \frac{c\, n_0 L_0}{4\pi H_0} \int_{z_1}^{z_2} \nu^{-\alpha} \frac{dz}{(1+z)^{\frac{5}{2}}}$$

$$I_{[\nu_0, z=0]} = \frac{c\, n_0 L_0}{4\pi H_0} \int_0^z \nu^{-\alpha} \frac{dz}{(1+z)^{\frac{5}{2}}} = \frac{c\, n_0 L_0 \nu_{Obs}^{-\alpha}}{4\pi H_0} \frac{1}{\frac{3}{2}+\alpha} \left[1 - (1+z)^{-\left(\frac{3}{2}+\alpha\right)} \right] =$$

$$= \frac{c\, n_0 L_0 \nu_{Obs}^{-\alpha}}{4\pi H_0} \frac{1}{\frac{3}{2}+\alpha} \left[1 - \left(\frac{\nu_{Max}}{\nu_{Obs}} \right)^{-\left(\frac{3}{2}+\alpha\right)} \right] = \left(\nu_{Max} -> \infty \right) = \frac{c\, n_0 L_0 \nu_{Obs}^{-\alpha}}{4\pi H_0} \frac{1}{\frac{3}{2}+\alpha}$$

Obviously I must have $\nu_{Min} / (1+z) < \nu_{Obs} < \nu_{Max} / (1+z)$

Background due to high z sources

$$L(\nu) = A\,\nu^{-\alpha}$$

$$L(\nu_1,\nu_2) = \int_{\nu_1}^{\nu_2} A\,\nu^{-\alpha} = (\alpha \neq 1)\,\frac{A}{1-\alpha}\left(\nu_2^{1-\alpha} - \nu_1^{1-\alpha}\right)$$

$$L(\nu_1,\nu_2) = A\,Log\left(\frac{\nu_2}{\nu_1}\right)\ for\ \alpha = 1$$

$$for\quad \alpha = 1\quad \Delta(Log\,\nu) = 1\quad \Rightarrow L(\nu, 10\,\nu) = A = \nu L(\nu)$$

And even when $\alpha \neq 1$ it is very close to the Luminosity in the interval (ν 10ν) so that it is a convenient way to Depicting the continuum.

Also the dimensions give erg cm^{-3} rather than erg $cm^{-3}Hz^{-1}$

$$\Delta L(\nu_{em}, z) = \nu_{em} B(\nu_{em}) G(z) \frac{\Delta V}{\Delta z} \Delta z$$

Square arcsec
In a sphere

$$dA(\mathbf{1}\ \text{square arc}\ \mathbf{sec}) = a_0^2(t)\, r_{em}^2\, \frac{4\,\pi}{5.346\ 10^{11}}$$

$$dV = \frac{c\,dz}{H} * \left(dA = a_0^2(t)\, r_{em}^2\, \frac{4\,\pi}{5.346\ 10^{11}} \right) =$$

Time History of
The source

$$= \frac{a_0^2(t)\, r_{em}^2}{H_0(1+z)\sqrt{[1+\Omega\, z]}}\, c dz\ a_0^2(t)\, r_{em}^2\, \frac{4\,\pi}{5.346\ 10^{11}}$$

$$r_{em} a_0 = \frac{2\,c}{H_0 \Omega_0^2 (1+z)} \left[\Omega_0 z + (2-\Omega_0)\left(1-\sqrt{\Omega_0 z + 1}\right) \right]$$

$$\Delta V = \left(\frac{2\,c}{H_0 \Omega_0^2 (1+z)} \left[\Omega_0 z + (2-\Omega_0)\left(1-\sqrt{\Omega_0 z + 1}\right) \right] \right)^2$$

$$* \frac{c}{H_0(1+z)\sqrt{[1+\Omega\, z]}}\, dz\, \frac{4\,\pi}{5.346\ 10^{11}}$$

Example

The Energy emitted at a given frequency will be observed at a redshifted (smaller by 1+z) frequency.

I will collect the emitted Energy over a sphere $4 \pi a_0^2 r_{em}^2$ and However;

The photons will be redshifted by a factor (1+z)

The rate of arrival will be redshifted by a factor (1+z)

And working in a contrary way

The band pass of the observer will be smaller by a factor 1+z concentrating the Energy, Therefore:

$$\Delta I_\nu(\nu_{Obs}, z) = \frac{\Delta L(\nu_{em} = \nu_{Obs}(1+z), z)}{4 \pi a_0^2 r_{em}^2} \frac{1+z}{(1+z)^2} = \frac{\Delta L(\nu_{em} = \nu_{Obs}(1+z), z)}{4 \pi a_0^2 r_{em}^2 (1+z)} \quad and$$

$$I_\nu = \int_{z_{min}}^{z_{max}} \frac{\nu_{em} B(\nu_{em}) G(z) \frac{\Delta V}{\Delta z}}{4 \pi a_0^2 r_{em}^2 (1+z)} dz$$

From Slide 16

$$I_{[\nu_0, z=0]} = \frac{c\, n_0}{4\pi H_0} \int_{z_1}^{z_2} \frac{dL_{em}}{d\nu_{em}} \frac{1}{(1+z)^2} \frac{dz}{\sqrt{[1+\Omega z]}}$$

$$with\ \frac{dL_{em}}{d\nu_{em}} = B(\nu) \propto \nu^{-\alpha} \Rightarrow \frac{dL_{em}}{d\nu_{em}} \propto = \left[\nu_0^{-\alpha} (1+z)^{-\alpha} \right]$$

$$\frac{dL_{em}}{d\nu_{em}} \propto \nu_0^{-\alpha} (1+z)^{-\alpha} \quad and\ with\ z_1 = 0\ ,\ z_2 = \infty$$

$$I_{[\nu_0, z=0]} = \frac{c\, n_0}{4\pi H_0} \int_0^{\infty} \frac{\nu_0^{-\alpha} \propto L(\nu_0)}{(1+z)^{\alpha}} \frac{1}{(1+z)^2} \frac{dz}{\sqrt{[1+\Omega z]}} =$$

$$I_{[\nu_0, z=0]} = \frac{c\, n_0}{4\pi H_0} \int_0^{\infty} \frac{\nu_0^{-\alpha} \propto L(\nu_0)}{(1+z)^{2+\alpha}} \frac{dz}{\sqrt{[1+\Omega z]}}$$

See Math Counts_Bck_ppt for Page 535 details

Without going to complicate plots we can estimate the contribution to the background light by estimating the above equation for $\Omega = 1$ and $\alpha \sim 1$.

We find that 50 % of the light is emitted within z = 0.32 (see Math).

Clearly high z sources will contribute a lot to the background, it will be however difficult to disentangle the two effects.

We can however integrate for various wavelength the counts multiplied by the emitted flux and estimate the background radiation.

For AGN we may have an other problem however. In addition to the observed AGN a population of obscured dusty AGN seems to exist. This population is hard to be detected at optical wavelengths or soft X ray. It could be detected however on the hard X ray since this radiation is not absorbed by Dust (See emission from the Galactic Center at different wavelengths).

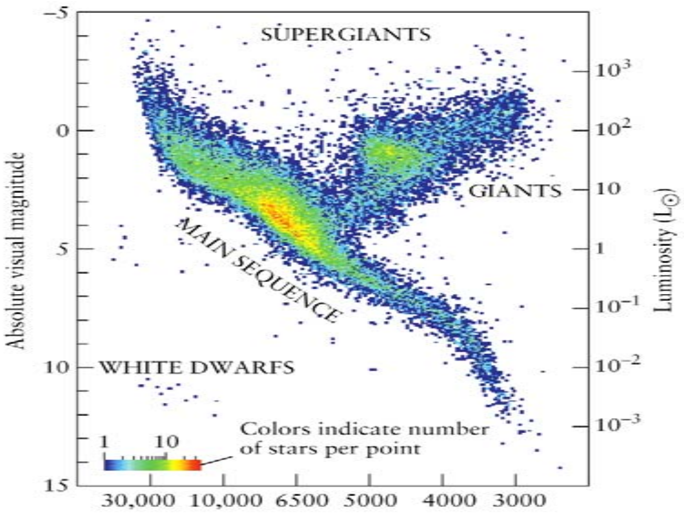
The other way is to look at the correlation between spheroid Galaxies and BH and see if we can estimate the expected contribution to the Background due to accretion.

Star Formation Rate - & - Galaxies

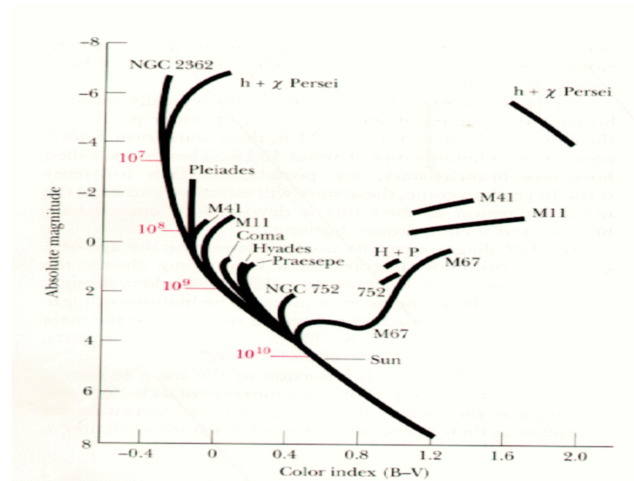
We try now to estimate the Background light due to high z sources, try to estimate the contribution of AGN in an indirect way, that is estimating the Number of Black Hole we may have and the Luminosity due to the accretion process.

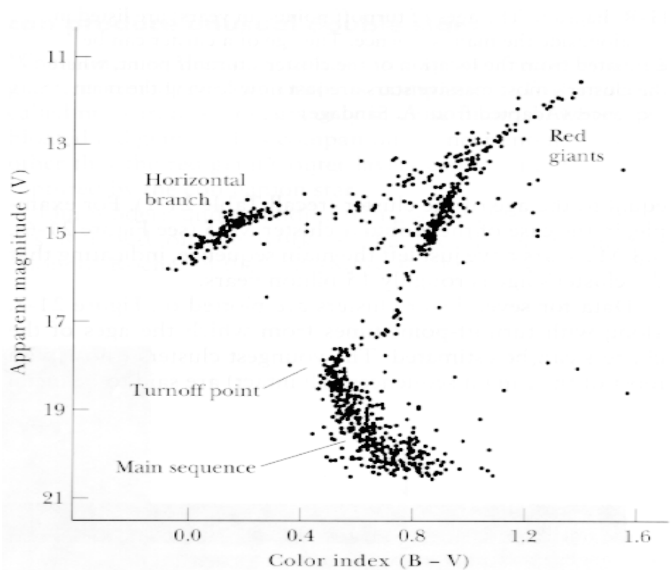
In this process however and in relation to the radiation emitted by young galaxies we need to add a few general consideration about the evolution of galaxies and the distribution of star forming galaxies as a function of redshift.

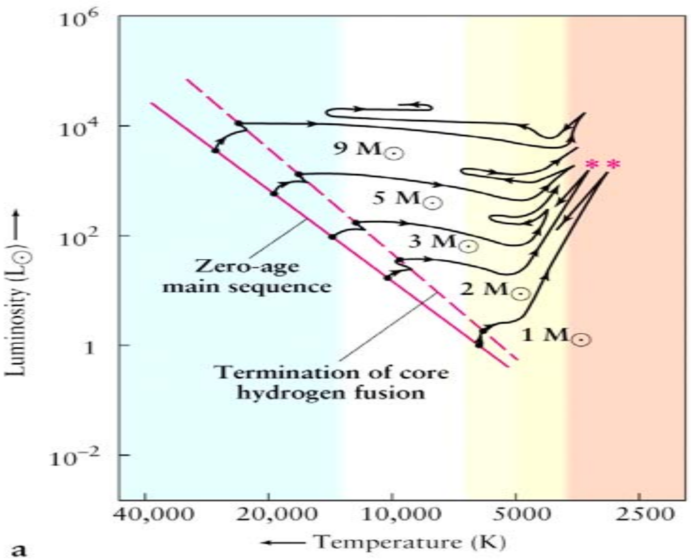
Finally we may also find ways to estimate the distribution of metals as a function of redshift and somehow come to a complete picture of how capable of explaining the radiation we observe.


SUPERGIANTS
GIANTS
MAIN SEQUENCE
WHITE DWARFS
Colors indicate number of stars per point
1 10
Absolute visual magnitude
Luminosity ($L_\odot$)
Temperature (K)
30,000 10,000 6500 5000 4000 3000
-5 0 5 10 15
10^3 10^2 10 1 10^{-1} 10^{-2} 10^{-3}

b

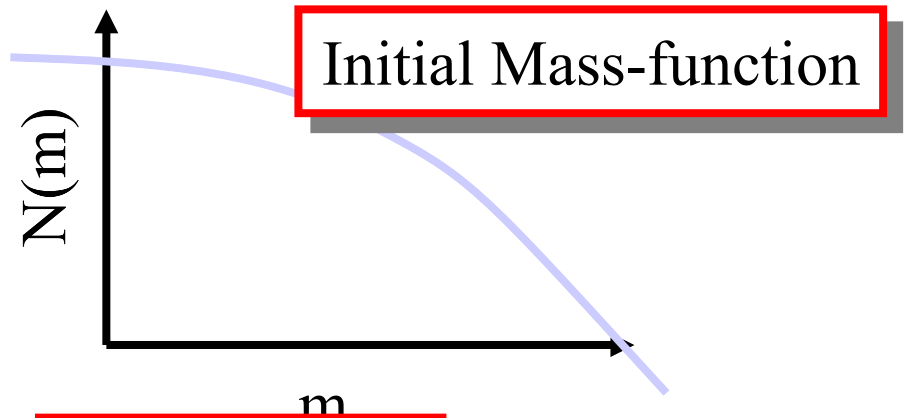

NGC 2362
h + χ Persei
h + χ Persei
10^7
Pleiades
M41
M11
10^8
Coma
Hyades
Praesepe
10^9
NGC 752
752
H + P
M67
M67
10^{10}
Sun
M41
M11
Absolute magnitude
Color index (B–V)
-0.4 0 0.4 0.8 1.2 1.6 2.0
-8 -6 -4 -2 0 2 4 6 8

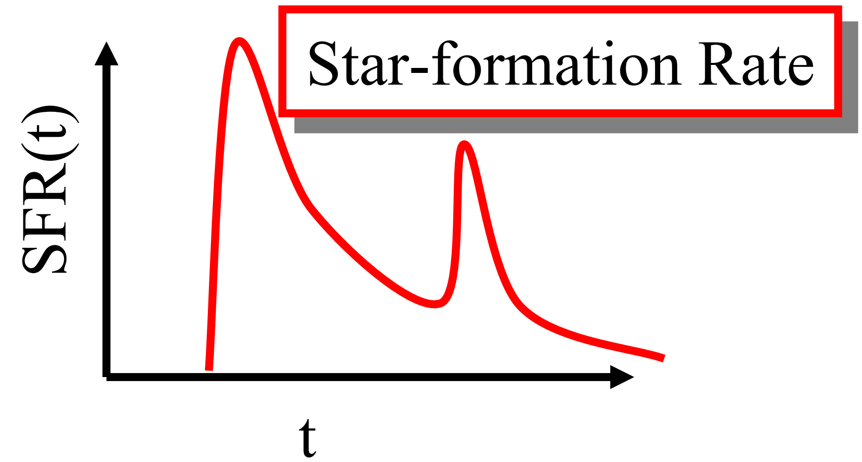

Horizontal branch
Red giants
Turnoff point
Main sequence
Apparent magnitude (V)
Color index (B – V)
0.0 0.4 0.8 1.2 1.6
11 13 15 17 19 21

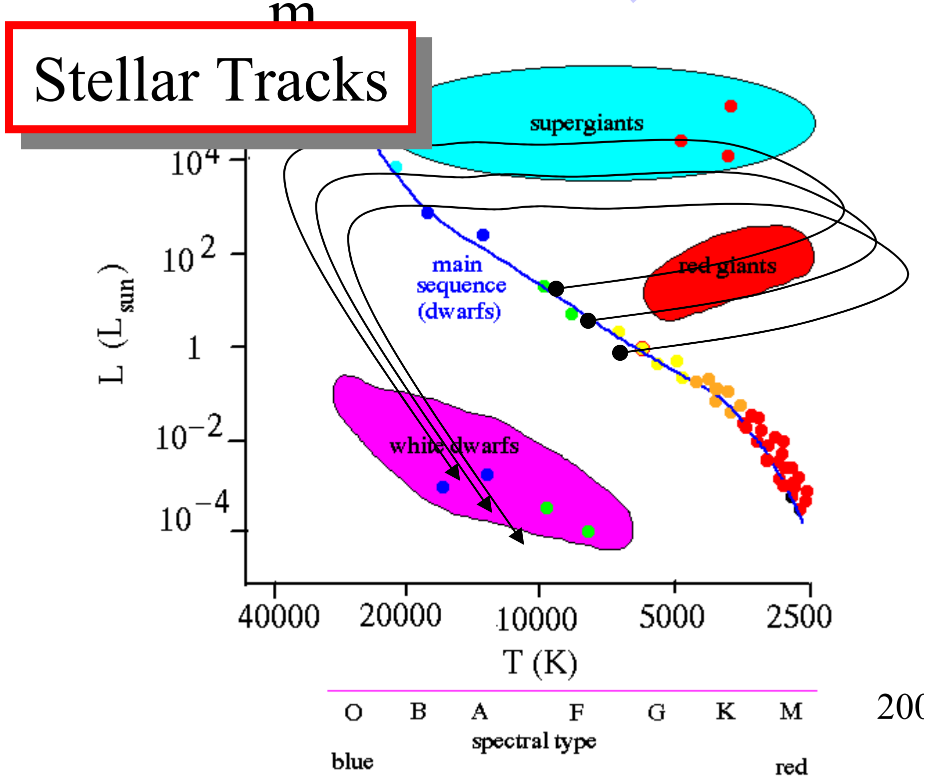

9 $M_\odot$
5 $M_\odot$
3 $M_\odot$
2 $M_\odot$
1 $M_\odot$
Zero-age main sequence
Termination of core hydrogen fusion
Luminosity ($L_\odot$)
Temperature (K)
40,000 20,000 10,000 5000 2500
10^6 10^4 10^2 1 10^{-2}

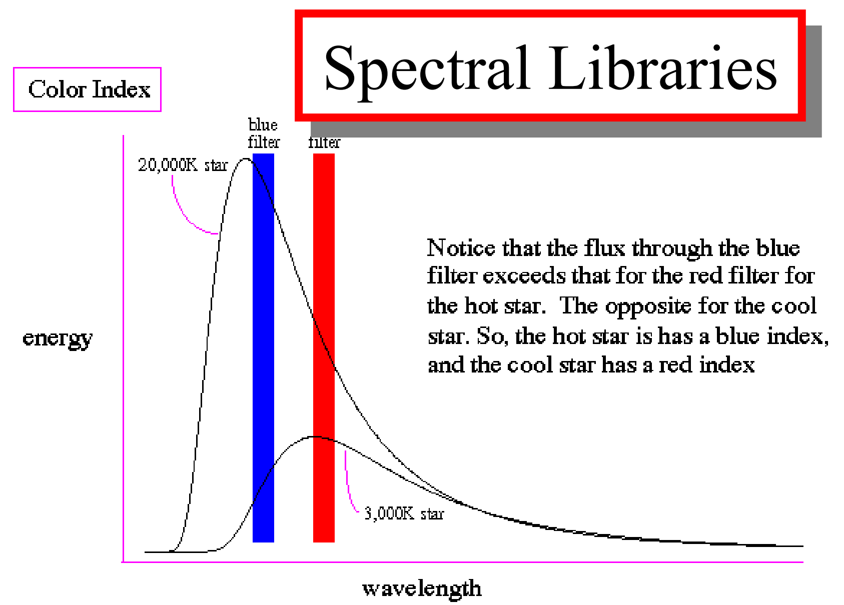
a

Passive Evolution - Stellar Synthesis

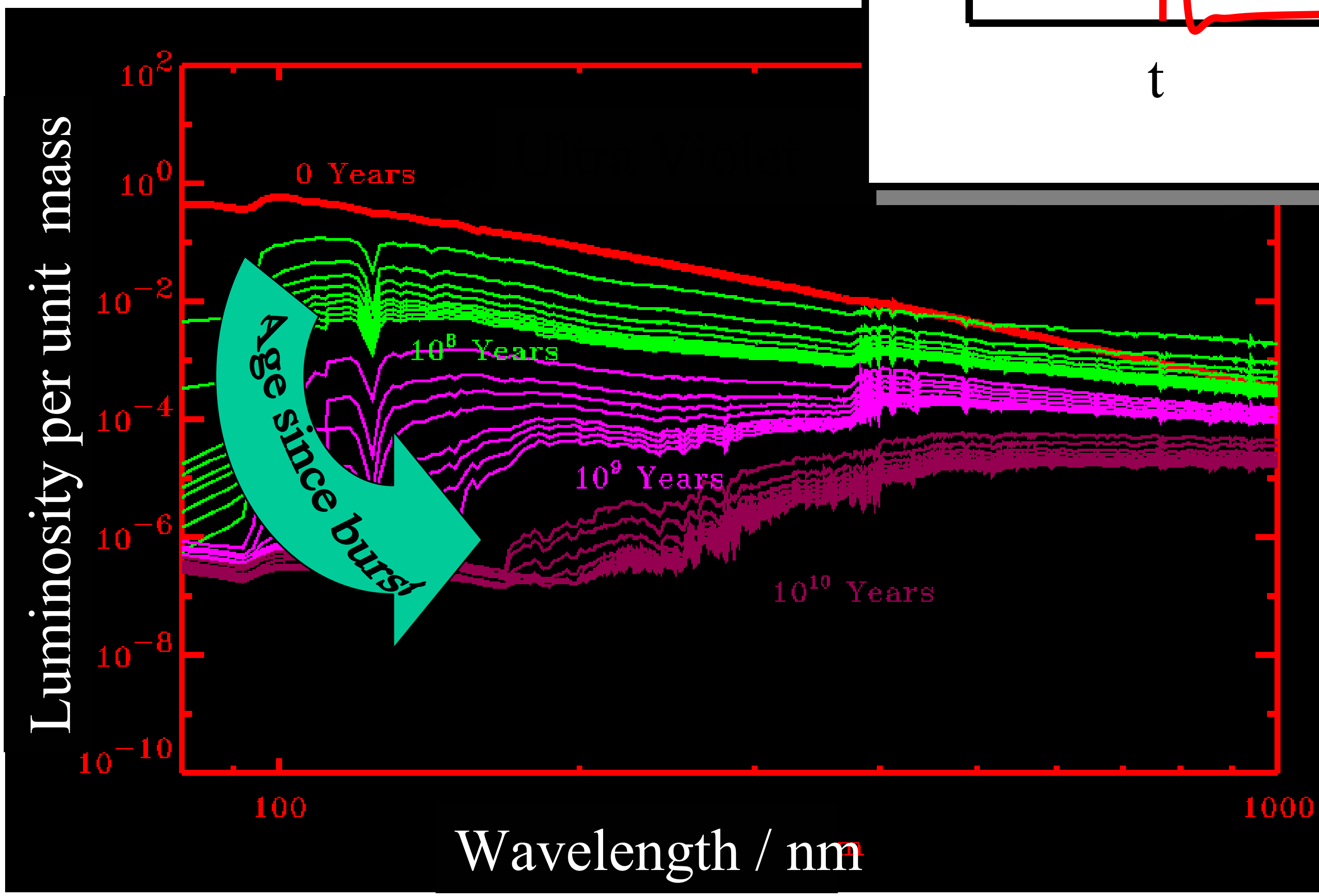


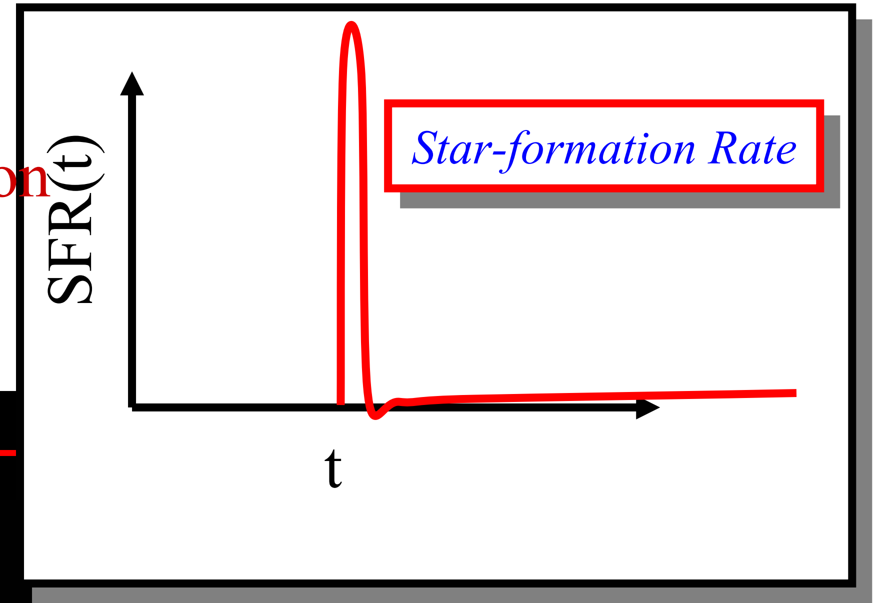






Evolution of light from a collection of stars born in a single burst

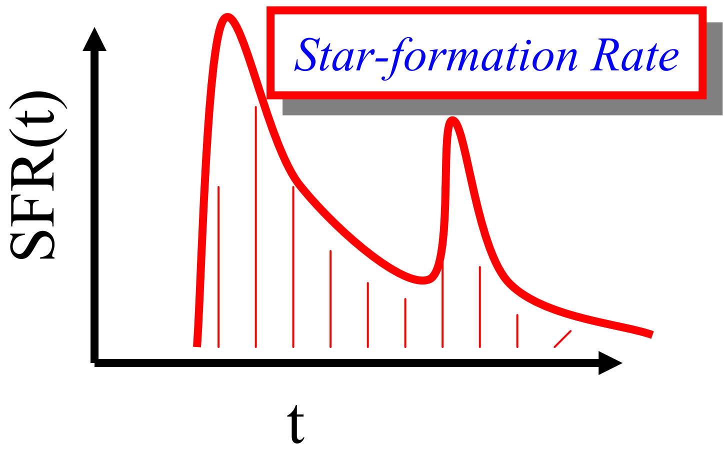




Passive Evolution

- More complicated star formation histories can be imagined as a series of instantanious bursts

$$\frac{\partial M}{\partial t} \propto \exp\left(-\frac{t}{\tau}\right)$$



- An exponential star-formation rate fits many galaxies well
- t is time since start of star-formation
- τ is time-scale

Stellar evolution – (see Padmanabhan)

Any stellar population that is originated from a burst of star formation for t > 5 Gyr will have only stars with masses

$$M \leq \mathbf{1.25} M_{\odot}$$

Since stars with $M \geq \mathbf{1.25} M_{\odot}$ evolve in a time smaller than 5 Gyr.

These low mass stars will emit most of their Energy during the brief period they will spend on the Giant Branch. The Luminosity of the population is going to be approximately.

$$L \approx \left(E_{GB}(M) \frac{dN}{dM} \right)_{M_{GB}} \left| \frac{dM_{GB}}{dt} \right|$$

$M_{GB}(t)$ Age at which stars of mass M turn off the Main Sequence

Total Energy emitted during the $E_{GB}(M)$ phase

Stars with Mass in the range M M+dM

From the stellar evolution we can write the following equations with the intent to show how important is stellar evolution in the estimate of the Galaxy evolution:

$$\frac{M_{GB}(t)}{M_{\odot}} \simeq \left(\frac{t}{10Gyr}\right)^{-0.4} \qquad \frac{dN}{dM} \simeq K\left(\frac{M}{M_{\odot}}\right)^{-\alpha} \qquad \alpha = 2.35$$

$$L \approx \frac{E_{GB}(M_{GB})\,K\left(\frac{M}{M_{\odot}}\right)^{-\alpha} 0.4\, M_{\odot}\, t^{-1.4} \equiv \left(t^{-0.4}\right)^{\frac{1.4}{.4}=3.5}}{10\ Gyr} =$$

$$= \frac{E_{GB}(M_{GB})\,K\left(\frac{M}{M_{\odot}}\right)^{-\alpha} 0.4\, M_{\odot}\, t^{-1.4} \equiv \left(t^{-0.4}\right)^{\frac{1.4}{.4}=3.5}}{10\ Gyr} = \frac{E_{GB}(M_{GB})\,K\,M_{\odot}}{(25\ Gyr)}\left(\frac{M}{M_{\odot}}\right)^{3.5-\alpha} \quad or$$

$$\ln(L) = \ln\left[K\,M_{\odot}\right] + \ln\left[E_{GB}(M_{GB})\right] + (3.5-\alpha)\ln\left(\frac{M}{M_{\odot}}\right)$$

$$\frac{d\ln(L)}{d\ln(t)} = \left[\frac{d\ln\left[E_{GB}(M_{GB})\right]}{d\ln(M_{GB})} + (3.5-\alpha)\right]\frac{d\ln(M_{GB})}{d\ln(t)} = 0.4\,\alpha - \left(1.4 + 0.4\frac{d\ln E_{GB}}{d\ln M_{GB}}\right)$$

Since $d\ln(E_{GB}) / d\ln(M_{GB})$ is in the range 0 – 1, $d\ln(E_{GB}) / d\ln(t) < 0$ and the Luminosity is a decreasing function of time. The detailed computation is in agreement with this rather course estimate.

Observationally the star formation rate is estimated using the UV continuum, Balmer lines, the [OII] doublet and Radio Observations. For a detailed analysis we refer to the literature or to an other specific lecture. Conceptually:

The correlation between star formation and UV radiation or Balmer emission lines in the surrounding regions is straight forward. Young massive stars, these are short lived stars, tend to produce a lot of UV radiation and this ionizes and excites the surrounding gas. See Osterbrock for the details. For the empirical correlations between the SFR and the intensity of the lines see Kennicutt in Annual Review of Astronomy and Astrophysics.

The Radio observations and for that matter the FIR observations are more indirect and are based on the presence of dust. The UV radiation absorbed by the dust is re-emitted at Radio wavelengths and that gives a measure of the UV radiation. In some cases the discrepancies between the optical and radio estimates is very high. The Radio estimate is very much dependent on the estimate of the Dust Temperature and it seems to me it is rather dangerous to estimate the SFR without estimating first the temperature.

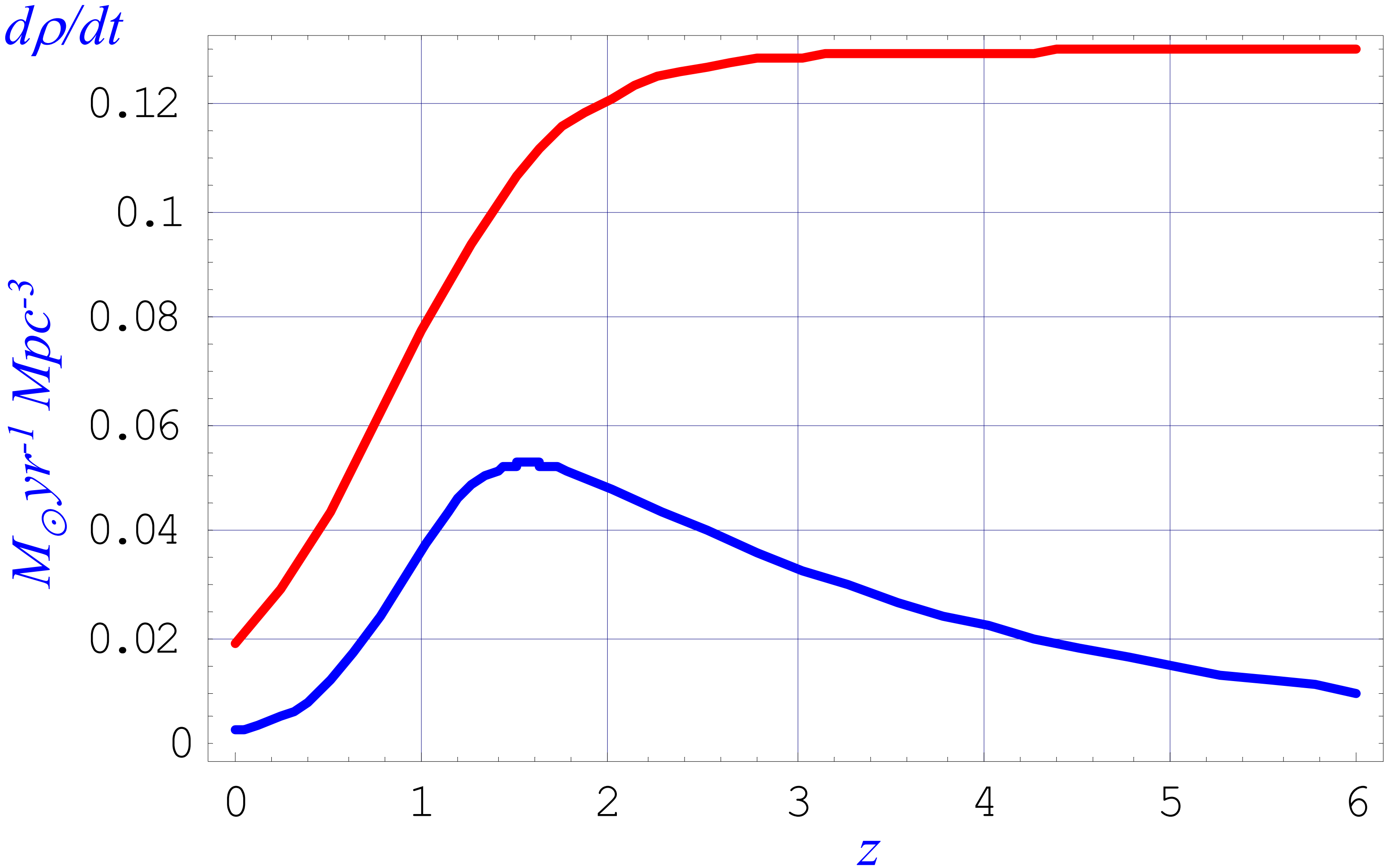
It seems quite reasonable to expect that the SFR is correlated with the amount of gas available. Indeed such correlation exists for instance on the disk of galaxies and it is known as the Schmidt law:

$$\sum_{SFR} Disk = \left(\mathbf{2.5} \pm \mathbf{0.7}\right) \mathbf{10}^{-5} \left[\frac{\sum_{Gas} Disk}{\mathbf{1}\, M_{\odot}\, pc^{-2}} \right]^{\mathbf{1.4 \pm 0.15}} M_{\odot} yr^{-1} kpc^{-2}$$

As soon as surveys of galaxies at high redshift became available thanks to the large telescopes and new detectors various scientists were able, using different techniques, to detect high z population of Galaxies. It has been possible therefore to study the global SFR as a function of z. The plot we show are generally called the Madau plots.

. *The observations are roughly fit by the following analytical expression:*

$$\frac{dM_*}{dtdV} = \dot{\rho}_* = A \frac{e^{a z}}{e^{b z} + c} \begin{cases} w/o\ Dust\ Correction \\ A = \mathbf{0.11}\ M_{\odot} yr^{-1} Mpc^{-3}\ a = \mathbf{3.4}\ b = \mathbf{3.8}\ c = \mathbf{44.7} \\ w\ Dust\ Correction \\ A = \mathbf{0.13}\ M_{\odot} yr^{-1} Mpc^{-3}\ a = \mathbf{2.2}\ b = \mathbf{2.2}\ c = \mathbf{6.0} \end{cases}$$


dρ/dt
M⊙yr-1 Mpc-3
0.12
0.1
0.08
0.06
0.04
0.02
0
0
1
2
3
4
5
6
z

The Amount of gas processed in stars by the epoch t is therefore

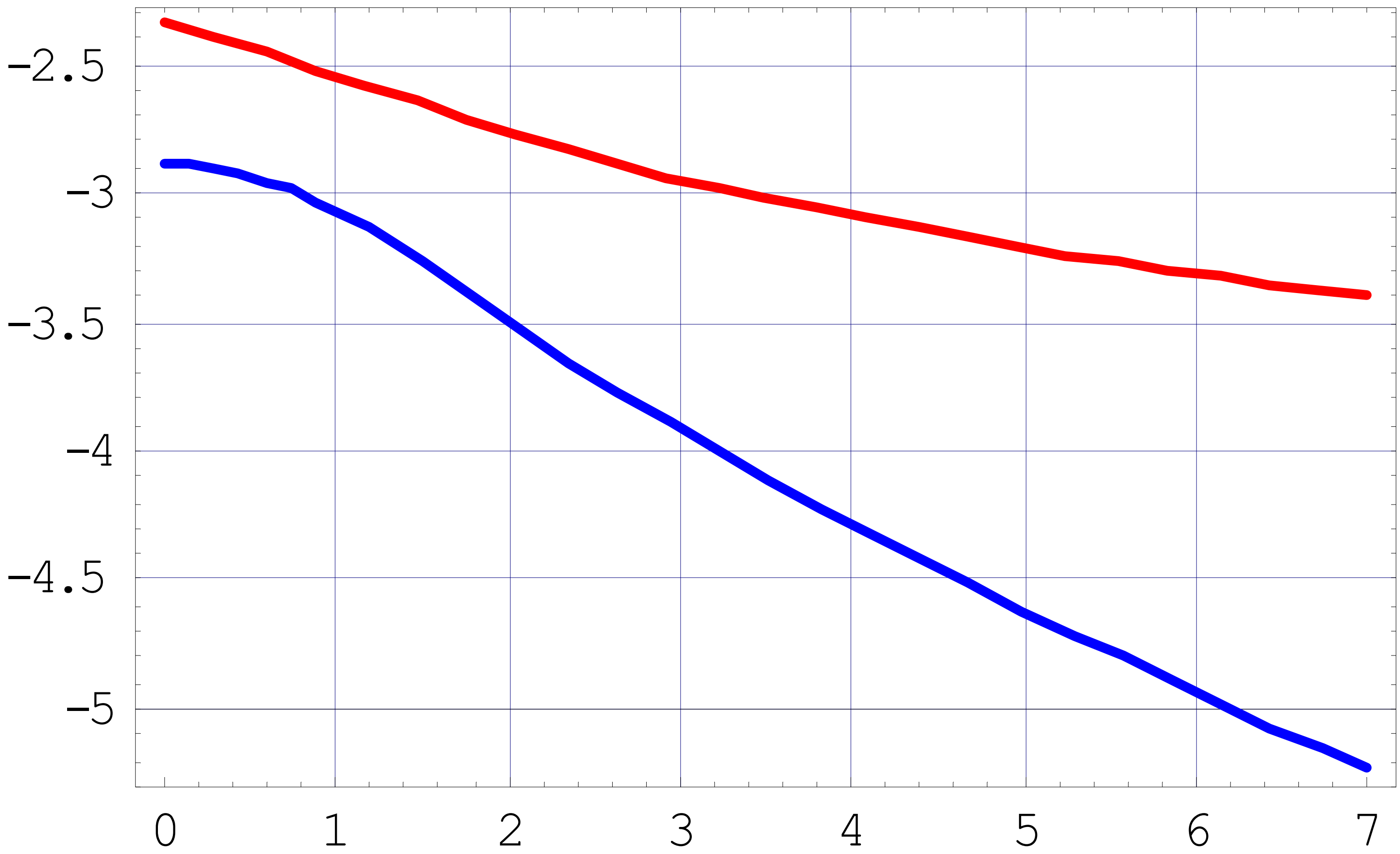
$$\rho_*(t) = \int_0^t dz \left(\frac{dt}{dz}\right) \dot{\rho}_*(z) \; or \; \Omega_*(t) = \frac{\rho_*(t)}{\rho_c(t)} \quad or \left(\frac{a}{\dot{a}}\right) \dot{\rho}_*(z)$$

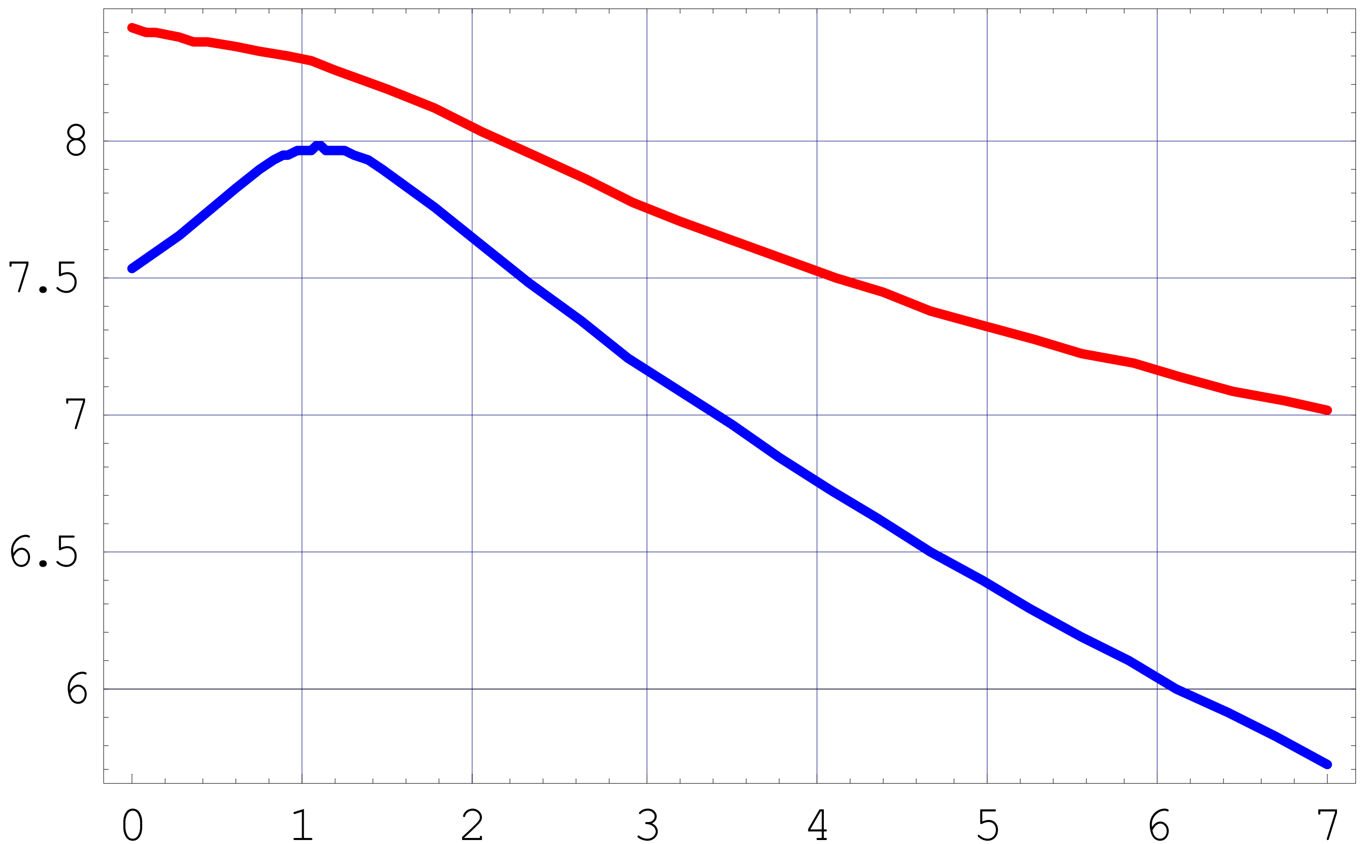
Gas converted into stars per Hubble time

Empirical relations for the Star Formation Rate

$$SFR\left(M_\odot \; yr^{-1}\right) = \mathbf{1.4 \; 10^{-28}} \; L_{FUV}\left(erg \; s^{-1} \; Hz^{-1}\right)$$

$$SFR\left(M_\odot \; yr^{-1}\right) = \mathbf{7.9 \; 10^{-42}} \; L_{H_\alpha}\left(erg \; s^{-1}\right)$$


-2.5
-3
-3.5
-4
-4.5
-5
0
1
2
3
4
5
6
7


8
7.5
7
6.5
6
0
1
2
3
4
5
6
7

Star Forming Galaxies

If we have a star formation rate of about 10 $M_\odot$ yr^{-1} in a Gyr we form stars for a Mass of about 10^{10} $M_\odot$.

The Luminosity produced by such stars can be computed easily using a stellar mass function and the mass luminosity relation.

On the other hand to avoid the detailed computation, I will give it eventually under request since I have it done, we can get and easy estimate from the mass to luminosity relation, in cgs units this translates in the relation below (see Novotni's book) for details.

The relation overestimate the luminosity produced by very massive stars. Using Cox Tables for a star of 63.1 solar masses the luminosity is overestimated by a factor 4. Better use the IMF.

Note however that the Mass function is very important so that it may be even reasonable to use the approximation 1 mass -> 1 Luminosity as in Padmanabahn

$$Lum = Mass^{4.1}\ 1.1486\ 10^{-103} - 0.1 \quad \left(erg\ s^{-1}\ \&\ gr \right)$$

Population of objects

To compute a few observable we can assume for instance that we conserve the number of galaxies (discuss merging eventually as a function of redshift) which base on the local Luminosity Function can be assumed to be of about $n_G = 0.015\ h^3\ \mathrm{Mpc}^{-3}$.

We can compute for instance the number density in the sky by the relation we wrote earlier with z_f the beginning of the luminosity process.

Eventually we can also use the expected counts and the photometric and spectroscopic characteristics of the sources to evaluate detectability.

$$\frac{dN}{dz\, d\Omega} = n(z)\frac{a_0^2(t)\, r_{em}^2\ R_H(z)}{(1+z)^3}$$

$$\frac{dN}{d\Omega} = \int_0^z n(z)\frac{a_0^2(t)\, r_{em}^2\ R_H(z)}{(1+z)^3}dz$$

$$n(z) = n_0\,(1+z)^3$$

Metals

To evaluate metal production as a function of redshift we should convolve the SFR with the production of metals that are ejected and enrich the interstellar medium. A very simple correlation however is the following:

$$(d\rho/dt)_{Metals} \sim 2.4\ 10^{-2}\ (d\rho/dt)_{Stars}$$

An other way to look at the Energy as $\nu\ I_\nu$ is to notice that this has the same dimension as $\int I_\nu\ d_\nu$ so that I can use one or the other provided the spectrum is sharply peaked or when I need the energy only in an approximated way.

Assume I convert in Energy a fraction of the Baryons I have at some redshift z. What is the total energy I measure? Indeed I can not have more than that.

ε Is the efficiency of conversion of matter into radiation and f_* the fraction of mass density.

$$\rho_B(z) = \rho_c \Omega_B = \rho_{c,0}(1+z)^3\,\Omega_B = \frac{3H_0^2(1+z)^3}{8\pi G}\Omega_B \sim (H_0 = 70) \sim$$

$$\rho_B(z) \simeq 1.35\ 10^{11}(1+z)^3\,\Omega_B\ M_\odot\ Mpc^{-3}$$

$$\rho_\gamma \approx \rho_B(z)\frac{c^2\varepsilon\ f_*}{(1+z)^4} = 8.27\ 10^{-9}\ 5.71\ 10^{-7}\left(\frac{\Omega_B}{0.02}\right)\left(\frac{f_*}{0.1}\right)\left(\frac{3.5}{1+z}\right)\left(\frac{\varepsilon}{0.001}\right)$$

$$1\, nW\ m^{-2} sr^{-1} = 10^{-6}\, ergs\ cm^{-2} s^{-1} sr^{-1}$$

$$I = \frac{c}{4\pi}\rho_\gamma = 8.27\ 10^{-9}\ 5.71\ 10^{-7} \left(\frac{\Omega_B}{0.02}\right)\left(\frac{f_*}{0.1}\right)\left(\frac{3.5}{1+z}\right)\left(\frac{\varepsilon}{0.001}\right) c^2$$

$$1.12\ 10^{-5} \left(\frac{\Omega_B}{0.02}\right)\left(\frac{f_*}{0.1}\right)\left(\frac{3.5}{1+z}\right)\left(\frac{\varepsilon}{0.001}\right) ergs\ cm^{-2} s^{-1} sr^{-1}$$

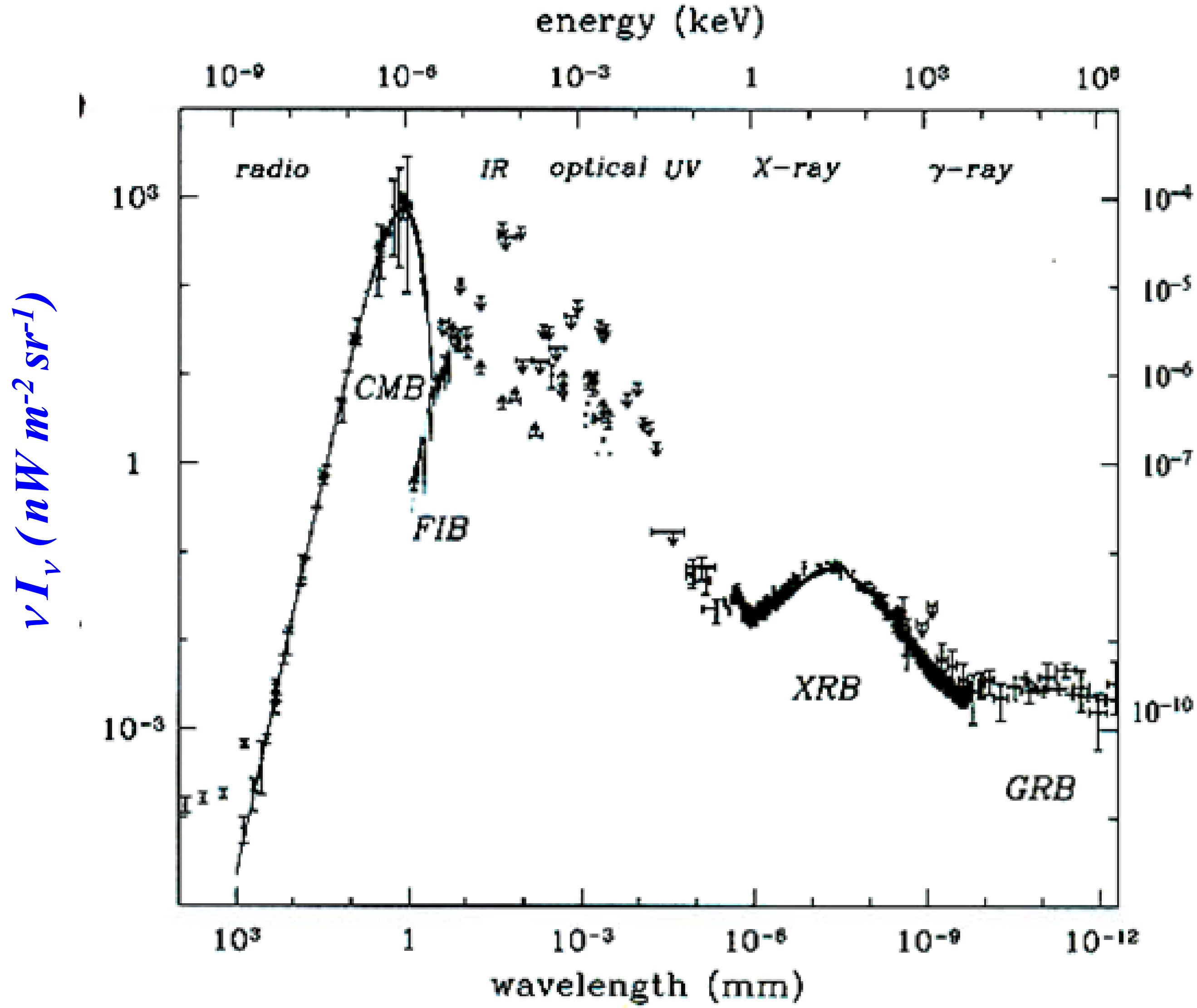
$$= 11.28 \left(\frac{\Omega_B}{0.02}\right)\left(\frac{f_*}{0.1}\right)\left(\frac{3.5}{1+z}\right)\left(\frac{\varepsilon}{0.001}\right) nW\ m^{-2} sr^{-1} \simeq \nu I_\nu$$

Observed :

$$\nu I_\nu\big|_{Opt} \approx 17 \pm 3\ nW\ m^{-2} sr^{-1}$$

$$\nu I_\nu\big|_{FIR} \approx 40\ nW\ m^{-2} sr^{-1}$$

A quiescent star formation would be enough to explain the background light in the optical band. On the other hand it is not quite enough for the FIR. This could be accounted for by obscured AGN where the light of the central source is absorbed by Dust and re emitted in the Infrared or by simply modifying the IMF. Indeed even changing of a factor two the efficiency parameter we could get the result.


energy (keV)
10⁻⁹ 10⁻⁶ 10⁻³ 1 10³ 10⁶
radio
IR
optical
UV
X-ray
γ-ray
CMB
FIB
XRB
GRB
10³
1
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10⁻¹⁰
10³ 1 10⁻³ 10⁻⁶ 10⁻⁹ 10⁻¹²
wavelength (mm)
νI_ν ($nW\, m^{-2} sr^{-1}$)

AGN Contribution

To avoid the problem of obscured AGN we estimate the number of BH that are related to AGN.

We use of the correlation between the Mass of a spheroidal bulge and the Mass of a BH. We have indeed $M_{BH} \sim 0.006\ M_{Sph.}$

Today the population of spheroidal is about Ω_{sph} h = 0.0018 ± 0.001 so that we can write (need to check however the original papers since it is not known the normalization for h:

$$\rho_{BH} = \rho_c \Omega_{BH} = \rho_{c,0} (1+z)^3 \Omega_{BH} = \frac{3H_0^2}{8\pi G}(1+z)^3\ 0.006\ \Omega_{Sph} = \frac{3H_0^2}{8\pi G}(1+z)^3\ 0.006\ 0.0018\ (g\ cm^{-3}) =$$

$$9.92\ 10^{-35}\ (1+z)^3 (g\ cm^{-3}) = 1.46\ 10^6\ (1+z)^3 (M_\odot Mpc^{-3})$$

$$I_{BH} = \frac{c}{4\pi}\frac{\eta\ 9.92\ 10^{-35}\ (1+z)^3\ c^2}{(1+z)^4} = 3.05\ 10^{-6}\left(\frac{3.5}{1+z}\right)\left(\frac{\eta}{0.05}\right) ergs\ cm^{-2} s^{-1} sr^{-1}$$

$$= 3.05\left(\frac{3.5}{1+z}\right)\left(\frac{\eta}{0.05}\right)\ nW\ m^{-2} sr^{-1}$$

Efficiency of Accretion

Not much, Not enough