(81)

CONTRIBUTI

DELL'OSSERVATORIO ASTRONOMICO DI MILANO-MERATE

a cura del Direttore

Prof. FRANCESCO ZAGAR

NUOVA SERIE N. 267

EDOARDO PROVERBIO

TIME AND LONGITUDE DETERMINATION WITH THE DÖLLEN METHOD

(Estratto dal « Bulletin Géodésique » N° 83, 1967)

MILANO

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TIME AND LONGITUDE DETERMINATIONS WITH THE DÖLLEN METHOD

1. Characteristic of Longitude observations

Many systematic and accidental errors occur in time determination.

For this reason, when observation and reduction methods are being established, the specific purpose for which the time determinations are being made must be borne in mind.

The various systematic and accidental errors do in fact assume importance and even a different character according to whether the time observations are made for :

(a)- research in fundamental astronomy,

(b)- for time services,

(c)- for the determination of longitudes.

At the present time precise and homogenous determinations of differences in longitude are required :

(a)- for the réorientation of the national geodetic networks in preparation for the constitution of a single European geodetic network (Longitude of Laplace);

(b)- in research on the variations of longitude, especially on secular variation;

(c)- for working out a uniform network of vertical deviation.

When determining longitudes particular consideration must be given not only to personal errors and to those due to local causes, but also to catalogue and instrumental errors (inclination, azimuth, collimation).

These errors are in fact transformed in their entiroty to the longitudes calculated if certain collateral operations are neglected and if the time observations effected at the various Observatories are not sufficiently coordinated [1].

Personal differential errors, the only ones that are important in longitude difference operations, can be determined with sufficient accuracy by the use of astronomical methods [2].

On the other hand it is difficult to reduce systematic catalogue errors of the $\Delta\alpha_\alpha$ type, and above all, those most to be feared in determinating longitude differences between places at different latitudes, of the $\Delta\alpha_\delta$ type; such errors may reach values of the order of from $0^s.01$ to $0^s.02$ even if the FK4 catalogue is used.

In order to reduce these errors to the minimum, the same observation programme would have to be used by all observation stations.

The instrumental errors which are most to be feared are those due to inaccuracy in the determination of the instrumental azimuth.

These inaccuracies are attributable to :

(1) errors in the determination of the real instrumental azimuth (observation errors, catalogue errors);

(2) errors of extrapolation due to the real variations of the instrumental azimuth;

(3) apparent variations of the instrumental azimuth due to local causes (lateral refraction).

Errors in the determination of the instrumental azimuth may be reduced by choosing suitable groups of stars observed.

In particular it may be deduced from the theoretical analysis of observations, that it is inadvisable to observe circumpolar stars ($\delta > 80°$) as "reference" stars in determining the instrumental azimuth.

At European latitudes, on the contrary, it is found advantageous to observe equatorial stars or stars in lower culmination, for the determination of the latter quantity [3].

The errors due to the real variations of the instrumental azimuth during the period of observation (1 to 2 hours) may be reduced as follows :

(1) by the observation of numerous "reference" stars and therefore by frequent determinations of the instrumental azimuth;

(2) by trying to fulfil as far as possible the well-known condition of $\Sigma K_i = 0$ for the clock stars (K_i = the azimuth coefficient in Mayer's equation).

Bearing in mind what has been said above, and the fact that not all the Observatories concerned with the determination of two or more longitude differences may have suitable meridien marks available, it follows that the determination of precise longitude differences must fulfil the following contradictory conditions :

(a)- numerous "reference" stars must be observed to avoid errors of extrapolation of the azimuth and to compensate for observational errors;

(b)- a programme common to all Observatories must be chosen;

(c)- it is consequently impossible for all Observatories to fulfil the condition $\Sigma K_i = 0$.

2. Longitude determination using the Döllen method

One method of observation which can satisfactorily overcome the many and contradictory characteristics of observation, which arise when determining differences of longitude, in such a way as to reduce instrumental and catalogue errors to a minimum, is the so-called Dollen method.

As explained in classical literature, this method is founded, as we know, on the observation of the time where the pole star is in the same vertical with a northern star in upper culmination [4].

However, the author has shown how this method, in meridian, is theoretically equivalent (in accuracy as well) to the usual meridian methods used for the determination of time and, secondly, that the most suitable combination of pairs of stars observed at the same vertical (meridian) must exclude the observation of polar stars ($\delta > 80°$) [5].

In this way even those reasons of a practical nature which might be invoked against the use of the Dollen method, are refuted.

By using a transit instrument for the observations, the $\underline{i}$ and $\underline{j}$ stars may both be observed successively before and after inversion of the instrument.

The calculation of the Δt clock correction thus follows from the relation

$$\Delta t = \alpha_i - T_i - i^* \sec\varphi + A_D\,[(T_i - \alpha_i) - (T_j - \alpha_j)]\,.$$

where $T_{i,j}$ represents the instants observed in the passage of stars $\underline{i}$ and $\underline{j}$ at the instrumental meridian, $\underline{i^*}$ represents the instrumental inclination found from reading the levels in the two positions of the instrument, while for the coefficient A_D we have :

$$A_D = \pm \frac{tg\,\varphi - tg\,\delta_i}{tg\,\delta_j \mp tg\,\delta_i} \qquad \text{(for the upper and lower culminations)}$$

In these two relations the indices $\underline{i}$ and $\underline{j}$ of each pair refer respectively to the reference star (the Pole Star in the classical Dollen method) and to the clock star (zenithal or equatorial).

Although this method represents relatively greater observation difficulties, it is however particularly suitable in determining differences in Longitude, as :

(a)- it practically eliminates the effect of the instrumental azimuth for each single pair of stars, and therefore eliminates the problem of extrapolation of the azimuth, making possible the direct determination of the time which is the only quantity of interest in this problem;

(b)- more effective compensation can be made for any errors of observation there may be;

(c)- it becomes possible for the various Observatories to follow the same observation programme (independence from condition $\Sigma K_i = 0$) within a wide area (from 20° to 30° in latitude);

(d)- as the groups are independent of condition $\Sigma K_i = 0$, they may be taken into consideration even if observation of the stars in the group has not been completed.

3. Observation results

During the course of the campaign for the determination of the differences in Longitude between a number of Italian astronomical Observatories, the Dollen method was used for the observation of the time in meridian.

The results of the first six months of observations carried out at the Brera Observatory in Milan are given here.

During the first half of 1966 in one single evening observation was made of a group of stars in the "Fundamental catalogue", normally used for calculating the time, and following this, at least one group of stars in the "Dollen catalogue".

Both catalogues are based on the FK4. One complete group of stars in the "Fundamental catalogue" (2 hours of observation) comprises from 20 to 25 clock stars, while one complete group in the Döllen catalogue (1 hour of observation) comprises from 6 to 7 pairs.

During the fundamental group, calculation of the instrumental azimuth was made using equatorial stars. .

A synthesis of the results of the calculation of the Δt correction is given in the diagrams in Fig. 1.

The data (expressed in m. sec.) given in tables 1 and 2 are based on these diagrams; these tables contain respectively :

a) the external type errors ε_E calculated monthly on the basis of the deviations from all the individual Δt observed in relation to the interpolated monthly mean and obtained from the following relations

$$\varepsilon_E = |\Delta t_{obs.} - \Delta t_{obs.m.m.}|/n \; ; \quad \Delta t_{obs.m.m.} = \Sigma \Delta t_{obs.m.m.}/n.$$

(n = number of monthly observations)

b) the mean monthly deviations between the Δt means observed in one single evening, using the Dollen method and the classical method of observation in meridian.

Finally, table 3 shows the mean values observed of the mean internal errors of each group ε^*, calculated on the basis of the deviations of each Δt observed in relation to the group mean (also expressed in m. sec.).

These latter values confirm the practical equivalence between the two methods of observation from the point of view of the internal errors.

It is also reasonable to accept the same conclusion for the external errors ε_E, bearing in mind that observation practice with this method may even improve its performance.

However, the most significant result from the point of view of the systematic errors remains the value of the quantity $\eta_{\text{Fund - Döl.}}$ which provides more than satisfactory results, both individually (Fig. 1) and for the mean monthly values (Table 3). The values $\eta_{\text{Döl. - Döl. (Rid.2)}}$ represent the monthly mean of the differences between the Δt obtained by the observation of the Döllen pairs calculated by means of the Döllen formula, and by means of the pole star method (using Mayer's formula).

The appreciable amount of difference must be attributed to the dissymmetry of the Döllen pairs in relation to the zenith ($\Sigma K_i \neq 0$).

This result thus gives an idea of the errors that are committed when, in a difference of longitudes, on single programme must be used by all the stations.

Finally, as a conclusion to this discussion, the results are given which are obtained by calculating the mean error in the instant of meridian passagge of each single star observed, using the Fundamental programme and Döllen's programme.

The errors were calculated using Albrecht's well-known relation,

$$d\,T_n^2 = a^2 + b^2 \times \sec^2 \delta_n \,,$$

The calculation of the coefficients of this relation for the stars of the two catalogues has given the following results relating to the instant of meridian Transit T_n, using 25 contacts before and after the inversion of the instrument :

$$d\,T_i^2 = (0^s.\,0057)^2 + (0^s.\,0036 \sec \delta_i)^2 \quad \text{(Fundamental catalogue)}$$
$$\pm .0011 \qquad\qquad \pm .0002$$

$$d\,T_{i,j}^2 = (0^s.\,0116)^2 + (0^s.\,0032 \sec \delta_{ij})^2 \quad \text{(Döllen's catalogue)}$$
$$\pm .0013 \qquad\qquad \pm .0002$$

It is interesting to note that the value of the constant a calculated during the I.G.Y. (1957-1958) was $a = 0^s.0096$ [3].

It should also be considered that while the stars used for calculating dT_i are mostly circumzenithal stars, many equatorial stars are also considered when calculating the $dT_{i,j}$.

As appears from some research done, the results of which have not yet been published, Albrecht's relation used here is not rigorously proved; the post significant parameter of the error dT in the two previous relations, must be considered as being the constant b which appears almost unvariable in all our observations. (In the observations carried out during the I.G.Y. it was found to be $b = 0^s.0038$).

Although only partial, these results confirm that even in practice, as regards accidental errors, the method of Döllen and the classical method of observation in meridian are found to be practically equivalent, and thus confirm the advisability of using the method of Döllen in meridian when operations concerning highly accurate differences of Longitude are being carried out.

TABLE 1

Month	Fundam.			Döllen			Fund. - Döl.
	$\Delta t_{obs.m.m.}$	ε_E	n	$\Delta t_{obs.m.m.}$	ε_E	n	
JAN	- 138	8.0	4	- 135	17.2	4	- 3
FEB	- 136	11.5	2	- 137	7.0	2	+ 1
MAR	- 123	11.4	9	- 121	9.1	8	- 2
APR	- 111	7.1	7	- 96	10.3	7	- 15
MAY	- 90	4.7	3	- 91	7.0	5	+ 1
JUN	- 103	3.5	2	- 94	16.3	4	- 9
Mean w.		8.5			10.9		

TABLE 2

	$\eta_{Fund. - Döl.}$	$\eta_{Döl. - Döl. (Rid.2)}$
JAN	+ 2.5	+ 11.8
FEB	+ 1.5	- 1.5
MAR	± 0.0	- 6.0
APR	+ 0.7	+ 4.0
MAY	- 2.0	+ 4.0
JUN	-	- 4.8
Mean	+ 0.5	+ 1.3

TABLE 3

	$\varepsilon^*_{Fund.}$	$\varepsilon^*_{Döl.}$	$\varepsilon^*_{Döl.\ (Rid.2)}$
JAN	27	35	37
FEB	27	23	27
MAR	28	27	29
APR	24	32	36
MAY	28	29	35
JUN	33	34	36
Mean	28	30	34

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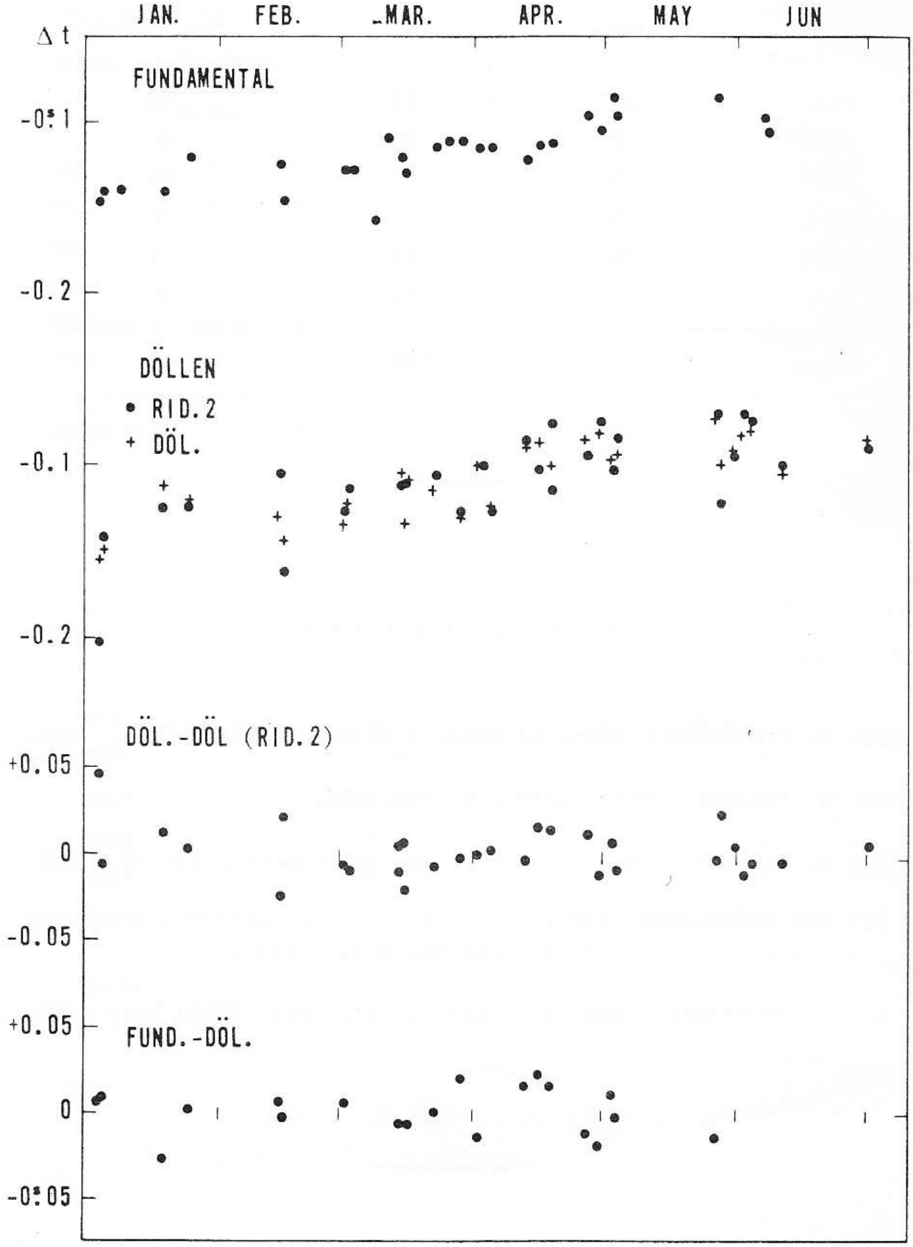
JAN.
FEB.
MAR.
APR.
MAY
JUN
Δ t
FUNDAMENTAL
-0.1
-0.2
DÖLLEN
• RID.2
+ DÖL.
-0.1
-0.2
DÖL.-DÖL (RID.2)
+0.05
0
-0.05
+0.05
FUND.-DÖL.
0
-0.05

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