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DELL'OSSERVATORIO ASTRONOMICO DI MILANO-MERATE
a cura del Direttore
Prof. FRANCESCO ZAGAR

NUOVA SERIE **N. 258**

EDOARDO PROVERBIO

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(Estratto dalle « Memorie della Società Astronomica Italiana »
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PRELIMINARY DECLINATION CORRECTIONS OF THE MILAN LATITUDE ZENITH PROGRAM

Nota di Edoardo PROVERBIO (*)
(*Osservatorio Astronomico di Milano, Brera*)

Riassunto. — Viene studiato il problema della determinazione delle correzioni in declinazione delle stelle costituenti il catalogo di stelle zenitali utilizzate nelle osservazioni di latitudini. Le correzioni sono ottenute con due diversi procedimenti utilizzando le osservazioni di latitudine effettuate dal 1961 al 1965.

In primo luogo sono state calcolate le riduzioni al gruppo medio $\Delta\delta_c^{(i)}$ e le correzioni di gruppo $\Delta\delta_g^{(k)}$ con il metodo a catena. In tal caso la correzione per ciascuna stella assume, com'è noto, la forma

$$\Delta\delta^{(i)} = \Delta\delta_c^{(i)} + \Delta\delta_g^{(k)} \quad .$$

In seguito le correzioni delle declinazioni $\Delta\delta_1^{(i)}$ sono calcolate considerando ciascuna stella come costituente un singolo gruppo e mostrando come questo procedimento sia equivalente al metodo di Fergola. L'analisi delle differenze sistematiche $\Delta\delta^{(i)} - \Delta\delta_1^{(i)}$ mette in evidenza l'esistenza di errori sistematici di gruppo. Questi errori possono essere causati da errori sistematici strumentali (e personali).

Summary. — In this paper the proceeding utilised for the correction of the star declinations of the Milan (Brera) Astronomical Observatory zenithal latitude Catalog is examined. The corrections are directly obtained from latitude observations carried out from 1961 to 1965.

The correction for the reduction to Group Mean $\Delta\delta_c^{(i)}$ and the group correction by the chain method $\Delta\delta_g^{(k)}$ have been calculated. Consequently the total correction for each star is

$$\Delta\delta^{(i)} = \Delta\delta_c^{(i)} + \Delta\delta_g^{(k)} \quad .$$

Successively the declination corrections $\Delta\delta_1^{(i)}$ are calculated by single star method (Fergola method). The systematic differences $\Delta\delta^{(i)} - \Delta\delta_1^{(i)}$ show the existence of systematic group errors. These errors may be caused by systematic instrumental (and personal) errors.

(*) Ricevuta il 1° Agosto 1966.

STAR PROGRAM AND LATITUDE OBSERVATIONS

1. Astronomical observations for determining latitude and its variations were begun at the Brera Observatory during the course of the International Geophysical Year 1957-58 (1).

During 1960 and until the spring of 1961 the author carried out a new series of latitude determinations following a programme different from the one used during the I.G.Y, though still based on the observation of Talcott pairs, and for the mean latitude found a relatively higher value than that determined by J. O. Fleckenstein (2).

Finally, in August 1961, observations of latitude based on a programme of absolute zenithal stars were started, and are still proceeding.

The main purpose of these observations is the determination of latitude variation and the study of the long and short period terms of polar motion.

However, one of the aspects, which cannot be considered secondary, of these observations is to show the high degree of accuracy of the method used, in relation to other methods.

This research assumes particular interest at the present time. The problem of the study of polar motion and of its secular and periodical variations, in regard to the phenomena of a geophysical and astronomical nature which cause these variations does in fact at present require an appreciable improvement in the accuracy of astronomical observations.

As a result of this, it is also necessary to bring observation techniques, programmes and reduction methods up to date.

Today, the following questions are particularly important and urgent:

a) a direct or indirect comparison of the various methods used for the precise determination of latitude;

b) the use of catalogues and of observation programmes in which the mean declinations and stellar proper motions are known with a high degree of accuracy.

The aim of this study is to work out the preliminary corrections to the declinations of the stars in the zenithal star catalogue (3), so as to have available sufficiently accurate mean positions in declination for calculating the mean latitude and its variations.

The corrections worked out here have been obtained by using the same latitude observations as made during the period from August 1961 to June 1965.

These corrections have been calculated by proceeding in two different ways:

a) considering the zenithal star catalogue as being divided into 10 groups and working out the correction $\Delta\delta_c^{(i)}$ for reduction to the mean group of each star. Subsequently, correction $\Delta\delta_g^{(k)}$ is calculated for the reduction of each group to the system of all the groups together.

The correction of the declination of each star will therefore be given by

(1) $$\Delta\delta^{(i)} = \Delta\delta_c^{(i)} + \Delta\delta_g^{(k)}$$

Col metodo a catena non si determina $\Delta\delta_g^{(k)}$ ma $(\Delta\delta_g^{(k)} - z_g^{(k)})$ come dalla (7) per cui con tale metodo $\Delta\delta^{(i)} = \Delta\delta_c^{(i)} + (\Delta\delta_g^{(k)} - z_g^{(k)})$ (1')

b) considering each single star as a constituent of a single group. On this assumption the reduction to the mean group and the group reduction constitute a single correction and the number of the groups is the same as the number of the stars in the catalogue.

DECLINATION CORRECTION BY THE CHAIN METHOD

2. The following approximate relation was used for determining the preliminary corrections of the declination of each star in the catalogue

(2) $$\varphi = \varphi_{c\,t}^{(i)} + \Delta\delta_c^{(i)} + \Delta\delta_g^{(k)} - \Delta\varphi_t - z_g^{(k)} \quad ,$$

in which

$$\varphi_{c,t}^{(i)} = \frac{1}{n} \sum_{k=1}^{n} \varphi_{c,k}^{(i)}$$

n = number of observations in a month (epoch t) for the i-th pair

is the monthly mean latitude observed for pair i belonging to group k for epoch t, not corrected for the progressive inequality of the micrometer screw and for inclination of the moving wire;

$\Delta\varphi_t$ = polar variation of the Latitude for epoch t;
$z_g^{(k)}$ = mean non-polar variation of the latitude relating to group k;
φ = value of the mean Latitude.

However, the following may be written for one same group of stars subsequently observed as morning group (m), intermediate group (i) and evening group (e):

(3) $$\varphi = [\varphi_{c,t}^{(i)}] + [\Delta\delta_c^{(i)}] + \Delta\delta_g^{(k)} - \Delta\varphi_t - z_g^{(k)} \quad ,$$

where the terms between the square brackets represent the sum of the single values divided by the number n_k of stars contained in the group.

On the assumption that

$$(3') \qquad [\Delta\delta_c^{(i)}] = 0$$

and from (2) and (3) we immediately obtain

$$(4) \qquad \Delta\delta_c^{(i)} = [\varphi_{c,t}^{(i)}] - \varphi_{c,t}^{(i)}$$

for the reduction to Group Mean.

This relation should be used solely considering the observations of complete groups. In practice, however, availability of incomplete groups is very frequent. In order to get overcome this serious difficulty, therefore, the mean of the single values $\varphi_c^{(i)}$ must be worked out using the data of the three subsequent months of observation, in which case epoch t will be given by the baricentre of the dates of observation, that is, on an average, in the middle of each single three-month period. This epoch remains unaltered from one year to another so that, in place of the single $\varphi_{c,t}^{(i)}$ (mean three-month values) it is permissible to use the mean of several years of observation. On this assumption, naturally, allowance must be made for the variation of $\Delta\varphi_t$ as a function of time, that is presuming that the corrected values

$$\overline{\varphi_{c,t}}^{(i)} = \varphi_{c,t}^{(i)} - \Delta\varphi_t \quad ,$$

replace the single monthly means $\varphi_{c,t}^{(i)}$ in (4).

Table I shows the annual means (three-monthly) for the individual latitudes observed in relation to each pair, published in ([3]), corrected with the monthly variations for the latitude given in Table II.

These latter corrections have been calculated on the basis of the Coordinates of the Instantaneous Pole given in the revised system ([4]).

Subsequently the corrections $\Delta\delta_c^{(i)}$ have been calculated from the general averages of all the years of observation by means of (4), for the reduction to Group Mean. These corrections are given in Table III.

The group correction for each group due to the declination error of a group has been determined by chain method.

The value of the monthly mean latitude for each group observed is given in Table IV. Using these values the mean group latitudes have been calculated for morning (m), intermediate (i) and evening (e) observations respectively. By means of the individual differences ($e - i$) and ($i - m$) in which the closing sum was distributed equally among the group combinations (Table V), group corrections $\Delta\delta_g^{(k)}$ may be determined.

These latter are shown in Table VI.

TABLE I

Group I

	1	2	3	4	5	5a	6
1961	61.933 3	60.057 3	60.024 3	59.784 3	—	—	—
1962	61.980 3	59.303 4	60.007 7	60.223 7	59.707 7	60.165 9	58.990 9
1963	62.014 4	59.128 4	59.756 5	59.818 5	60.070 6	59.827 2	58.202 6
1964	62.067 5	58.643 3	59.988 6	59.727 8	59.871 9	60.807 9	58.573 5

Group II

	7	8	9	10
1963	59.812 8	61.211 6	57.252 10	60.036 9
1964	59.522 9	61.152 7	57.425 3	59.625 3
1965	59.435 7	61.389 5	57.496 10	59.603 9

Group III

	11	12	13	14	15	16
1962	58.417 12	59.141 11	60.203 9	60.701 13	60.368 14	59.953 13
1963	58.500 7	58.944 7	59.710 7	60.528 7	60.495 6	60.024 8
1964	58.226 9	59.407 8	60.022 9	60.360 9	60.461 3	60.377 3
1965	58.314 9	59.362 11	60.041 13	60.700 12	60.428 4	60.142 5

Group IV

	17	18	19	20	21	22
1962	59.169 5	58.884 8	59.379 8	60.515 7	59.644 9	59.032 5
1963	58.260 7	59.124 7	59.137 7	60.594 6	60.047 5	59.292 7
1964	57.704 1	60.074 1	—	—	59.734 2	59.514 1
1965	58.220 8	59.100 9	59.522 9	60.607 8	59.553 11	59.541 10

Table I (continued)

Group V

	23	24	25	26	27	28
1962	59.326	59.594	60.931	58.768	59.646	59.214
	12	10	11	10	11	10
1963	—	59.883	60.203	58.133	60.062	59.448
		1	2	4	7	4
1964	59.654	59.784	60.778	58.202	60.098	60.016
	4	5	5	5	5	5
1965	59.449	59.723	60.522	58.043	59.901	59.882
	8	6	6	8	13	13

Group VI

	29	30	31	32
1962	60.096	59.633	59.763	59.426
	1	2	2	2
1963	59.616	59.416	59.890	60.395
	6	6	6	13
1964	59.885	59.546	60.006	59.804
	3	5	2	4
1965	59.788	58.885	59.803	60.520
	12	10	7	3

Group VII

	33	34	35	36	37	38	39	40
1962	59.707	58.154	58.664	57.407	59.559	58.604	59.330	59.549
	10	13	11	15	20	21	15	20
1963	59.660	58.335	58.945	56.972	59.247	58.785	59.204	59.654
	12	12	10	9	16	7	9	8
1964	59.529	58.642	59.327	58.091	59.177	58.624	59.614	59.665
	9	10	10	9	9	7	7	7
1965	59.250	58.981	59.733	58.395	59.561	—	—	—
	3	5	5	4	5			

Group VIII

	41	42	43	44	45	46	47
1961	—	—	59.445	58.595	—	58.760	62.002
			1	1		6	14
1962	60.315	59.844	59.325	59.678	59.532	58.034	60.345
	15	19	19	17	3	18	9
1963	60.202	59.983	59.003	59.075	59.012	58.973	60.045
	4	5	5	5	4	5	4
1964	59.786	60.081	59.031	59.621	58.416	58.313	59.956
	1	2	2	2	2	3	6

Table I (continued)

Group IX

	48	49	50	51	52	53	54
1961	59.861	59.319	59.519	58.501	59.102	58.937	59.430
	21	11	11	16	30	28	29
1962	58.983	59.477	58.604	59.093	59.320	59.418	59.386
	7	11	8	6	10	8	9
1963	59.899	60.202	58.791	58.550	59.459	59.022	59.695
	4	8	14	15	17	7	8
1964	59.647	59.462	58.601	58.668	59.383	59.812	59.693
	7	7	7	8	8	7	7

Group IX

	55	56	57	58	59	60
1961	59.208	59.389	59.341	59.882	59.565	59.993
	19	15	15	13	13	2
1962	59.344	59.194	59.241	59.506	59.744	60.029
	1	2	6	5	5	6
1963	59.310	59.204	59.649	59.448	59.578	59.709
	6	8	13	5	12	8
1964	59.448	59.408	58.942	59.670	59.769	59.789
	6	4	5	5	6	6

Group X

	61	62	63	64	65	66	67
1961	60.045	59.442	59.743	—	—	—	—
	7	5	6				
1962	59.734	59.564	59.519	59.459	59.406	60.492	58.555
	5	5	4	6	8	7	4
1963	60.248	59.524	59.352	59.378	58.518	60.274	57.943
	4	6	6	5	4	6	4
1964	59.855	59.110	59.205	59.685	58.235	59.968	57.993
	3	2	2	1	1	2	3

Group X

	68	69	70	71
1961	—	—	59.060	57.920
			3	3
1962	58.032	59.674	59.540	57.812
	5	1	5	5
1963	58.611	59.803	60.021	58.001
	4	2	5	5
1964	58.210	59.732	59.532	57.655
	3	1	2	1

TABLE II

Month	1961	1962	1963	1964	1965
	$\Delta\varphi$	$\Delta\varphi$	$\Delta\varphi$	$\Delta\varphi$	$\Delta\varphi$
	(0″.001)	(0″.001)	(0″.001)	(0″.001)	(0″.001)
JAN	+23	—38	—175	—236	—86
FEB	+49	—22	—140	—213	—180
MAR	+37	+39	—86	—186	—221
APR	+10	+76	+16	—124	—255
MAY	+11	+124	+127	—26	—196
JUN	+7	+139	+199	+127	—117
JUL	+30	+149	+273	+191	+15
AUG	+9	+113	+233	+194	+100
SEP	+15	+31	+140	+234	+166
OCT	—20	—34	+46	+205	+189
NOV	—84	—105	—62	+88	+185
DEC	—50	—151	—164	+21	+48

TABLE III

(i)	$\Delta\delta_c^{(i)}$	(i)	$\Delta\delta_c^{(i)}$	(i)	$\Delta\delta_c^{(i)}$	(i)	$\Delta\delta_c^{(i)}$
1	—1.998	18	+0.345	36	+1.279	54	—0.123
2	+0.735	19	+0.059	37	—0.417	55	+0.098
3	+0.067	20	—1.139	38	+0.330	56	+0.047
4	+0.106	21	—0.267	39	—0.383	57	—0.003
5	+0.138	22	+0.068	40	—0.622	58	—0.325
5a	—0.409	23	+0.198	41	—0.659	59	—0.254
6	+1.362	24	—0.065	42	—0.282	60	—0.470
7	—0.093	25	—1.116	43	+0.359	61	—0.752
8	—1.737	26	+1.266	44	+0.098	62	—0.242
9	+2.119	27	—0.262	45	+0.554	63	—0.279
10	—0.292	28	—0.020	46	+1.264	64	—0.220
11	+1.416	29	—0.001	47	—1.334	65	+0.181
12	+0.550	30	+0.535	48	—0.295	66	—1.111
13	—0.237	31	—0.086	49	—0.210	67	+1.045
14	—0.819	32	—0.449	50	+0.454	68	+0.955
15	—0.638	33	—0.629	51	+0.748	69	—0.529
16	—0.272	34	+0.545	52	+0.111	70	—0.379
17	+0.936	35	—0.102	53	+0.226	71	+1.333

TABLE IV

G.	Epoch	φ_g	G.	Epoch	φ_g	G.	Epoch	φ_g
		45°27′			45°27′			45°27′
I	JAN 1963	59.894	II	JAN 1963	59.661	III	JAN 1962	59.847
	64	60.449		64	59.383		63	59.262
	65	59.736		65	59.421		64	59.420
II	FEB 63	59.485	III	FEB 62	59.716	IV		
	64	59.544		63	59.787			
	65	59.524		64	59.828			
				65	59.838			
III	MAR 63	59.750	IV	MAR 62	59.326	V	MAR 62	59.436
	64	59.894		63	59.382		65	59.161
	65	59.807		65	59.457			
IV	APR 62	59.410	V	APR 62	59.533	VI	APR 64	59.439
	63	59.462		63	59.292		65	59.568
	64	59.257		64	59.783			
	65	59.453		65	59.616			
V	MAY 62	59.544	VI	MAY 62	59.906	VII	JUN 62	58.973
	63	59.504		63	59.820		63	58.974
	64	60.183		64	59.836		64	59.113
	65	59.481		65	59.719		65	59.184
VI	JUN 62	59.533	VII	JUL 62	58.846	VIII	JUL 62	59.000
	63	60.411		63	58.983		63	59.392
	64	59.958		64	59.053			
	65	59.427						
VII	AUG 62	59.174	VIII	AUG 62	59.450	IX	AUG 61	59.485
	63	59.560		63	59.322		63	59.114
				64	59.317		64	59.711
VIII	SEP 61	59.721	IX	SEP 61	59.503	X	AUG 61	59.873
	62	59.572		62	59.281		SEP 61	59.850
	63	60.045		63	59.412		63	59.123
	64	59.026		64	59.452			
	OCT 61	60.180						
	62	59.542						
IX	OCT 61	59.255	X	OCT 61	59.606	I	NOV 62	59.968
	62	59.334		62	59.231		63	59.743
	63	59.480		63	59.216		64	59.953
	64	59.193		64	58.873			
	NOV 63	59.482						
X	NOV 63	59.378	I	DEC 61	60.350	II	DEC 62	59.494
	64	59.089		62	60.078		63	60.106
	DEC 61	58.490		63	59.875		64	59.598
	62	59.381		64	60.018			

TABLE V

Evening		Intermediate		Morning		Observed		Corrected	
Group	$\overline{\varphi}_g^{(e)}$	Group	$\overline{\varphi}_g^{(i)}$	Group	$\overline{\varphi}_g^{(m)}$	$e - i$	$i - m$	$(e - i)$	$(i - m)$
I	60.026	II	59.454	III	59.605	+.572	—.151	+.513	—.171
II	59.509	III	59.792	IV	59.402	—.283	+.390	—.342	+.370
III	59.819	IV	59.388	V	59.299	+.431	+.089	+.372	+.069
IV	59.416	V	59.594	VI	59.603	—.178	—.009	—.237	—.029
V	59.456	VI	59.808	VII	59.046	—.352	+.762	—.411	+.742
VI	59.832	VII	58.961	VIII	59.196	+.871	—.235	+.812	—.255
VII	59.367	VIII	59.351	IX	59.437	+.016	—.086	—.043	—.107
VIII	59.681	IX	59.405	X	59.615	+.276	—.210	+.217	—.231
IX	59.365	X	59.229	I	59.888	+.136	—.659	+.077	—.679
X	59.143	I	60.043	II	59.732	—.900	+.311	—.958	+.291
						+.589	+.202	.000	.000

TABLE VI

Group	$\Delta\delta^{(e-i)}$	$\Delta\delta^{(i-m)}$	Mean $\Delta\delta_g^{(k)}$	Group	$\Delta\delta^{(e-i)}$	$\Delta\delta^{(i-m)}$	Mean $\Delta\delta_g^{(k)}$
I	+0″.464	+0″.587	—0″.525	VI	+0″.569	+0″.057	—0″.313
II	— .049	+ .296	— .123	VII	— .243	— .685	+ .464
III	+ .293	+ .467	— .380	VIII	— .200	— .430	+ .315
IV	— .079	+ .097	— .010	IX	— .417	— .323	+ .370
V	+ .158	+ .028	— .093	X	— .494	— .092	+ .293

DECLINATION CORRECTION BY THE SINGLE STAR METHOD

3. Declination corrections of the zenithal stars have been made by a different method, considering each single star as a group in itself. In this way the groups considered number 72 and in equation (2)

$$\Delta\delta_\epsilon^{(i)} = \Delta\delta_c^{(i)} + \Delta\delta_g^{(k)}$$

must be understood, so that we may write

$$\varphi = \overline{\varphi}_{c,t}^{(i)} + \Delta\delta_\epsilon^{(i)} - z_c^{(i)} \tag{2'}$$

We shall thus have,

$$\Delta\delta_\epsilon^{(i+1)} = \Delta\delta_\epsilon^{(i)} - (\overline{\varphi}_{c,t}^{(i+1)} - \overline{\varphi}_{c,t}^{(i)}) + z_c^{(i+1)} - z_c^{(i)}$$

Proceeding now with the usual chain method, and putting

$$\sum_{i=1}^{72} \Delta\delta_c^{(i)} = 0$$

(5) $$\sum_{i=1}^{72} z_c^{(i)} = 0$$

$$\sum_{i=1}^{72} (\overline{\varphi}_{c,t}^{(i)} - \overline{\varphi}_{c,t}^{(1)}) = \Delta\overline{\varphi}$$

we obtain the relation

(6) $$\Delta\delta_1^{(i)} = \Delta\delta_c^{(i)} - z_c^{(i)} = \frac{\Delta\overline{\varphi}}{72} - (\overline{\varphi}_{c,t}^{(i)} - \overline{\varphi}_{c,t}^{(1)}) \,, \qquad (i = 1, ..., 72)$$

which supplies the individual corrections $\Delta\delta_1^{(i)}$ to be made to the declinations of the stars in the whole catalogue.

It is interesting to note how this procedure is rigorously equivalent to the co-called Fergola method which as everyone knows consists of subtracting their arithmetical average from the equations (2'). Bearing in mind the two first relations (5), in this case we do in fact obtain,

$$\Delta\delta_1^{(i)} = \Delta\delta_c^{(i)} - z_c^{(i)} = \frac{1}{72} \sum_{i=1}^{72} \overline{\varphi}_{c,t}^{(i)} - \overline{\varphi}_{c,t}^{(i)} \,,$$

which, recalling the third relation in (5) is immediately transformed into (6).

In Table VII the single mean weighed values $\overline{\varphi}^{(i)}$ are given deducted from Table I relating to each star, as well as the calculated corrections $\Delta\delta_1^{(i)}$. Table VII also shows the values $\Delta\delta^{(i)}$ of the corrections calculated by the previous method derived from (1).

COMPARISON BETWEEN THE TWO METHODS

4. The values of the residues $\Delta\delta^{(i)} - \Delta\delta_1^{(i)}$ show that the two methods differ on account of systematic group errors. This result may be explained taking account of the procedure which is used for calculating group corrections.

Allowing for (5'), the observation of two groups (morning and intermediate or intermediate and evening) is equivalent to considering the two equations

$$\varphi = \overline{\varphi}_{c,t}^{(k)} + \Delta\delta_g^{(k)} - z_g^{(k)} \,,$$

$$\varphi = \overline{\varphi}_{c,t}^{(k+1)} + \Delta\delta_g^{(k+1)} - z_g^{(k+1)} \,,$$

TABLE VII

i	$\overline{\varphi}^{(i)}$	$\Delta\delta_1^{(i)}$	$\Delta\delta^{(i)}$	$\Delta\delta^{(i)} - \Delta\delta_1^{(i)}$	$\overline{\Delta\delta}^{(i)}$
1	62."009	—2."531	—2."523	+0."008	—2."527
2	59.276	+0.202	+0.210	+0.008	+0.206
3	59.994	—0.466	—0.458	+0.008	—0.462
4	59.905	—0.427	—0.419	+0.008	—0.423
5	59.873	—0.395	—0.387	+0.008	—0.391
5a	60.420	—0.942	—0.934	+0.008	—0.938
6	58.549	+0.828	+0.837	+0.009	+0.833
7	59.593	—0.115	—0.216	—0.101	—0.166
8	61.237	—1.759	—1.860	—0.101	—1.810
9	57.381	+2.097	+1.996	—0.101	+2.047
10	59.792	—0.314	—0.415	—0.101	—0.365
11	58.361	+1.117	+1.036	—0.081	+1.077
12	59.227	+0.251	+0.170	—0.081	+0.211
13	60.014	—0.536	—0.617	—0.081	—0.577
14	60.596	—1.118	—1.199	—0.081	—1.159
15	60.415	—0.937	—1.018	—0.081	—0.978
16	60.049	—0.571	—0.652	—0.081	—0.612
17	58.485	+0.993	+0.926	—0.067	+0.960
18	59.076	+0.402	+0.335	—0.067	+0.369
19	59.362	+0.116	+0.049	—0.067	+0.088
20	60.560	—1.082	—1.149	—0.067	—1.116
21	59.688	—0.210	—0.277	—0.067	—0.244
22	59.353	+0.125	+0.058	—0.067	+0.092
23	59.422	+0.056	+0.105	+0.049	+0.081
24	59.685	—0.207	—0.158	+0.049	—0.183
25	60.736	—1.258	—1.209	+0.049	—1.234
26	58.354	+1.124	+1.173	+0.049	+1.149
27	59.882	—0.404	—0.355	+0.049	—0.380
28	59.640	—0.162	—0.113	+0.049	—0.138
29	59.768	—0.290	—0.314	—0.024	—0.302
30	59.232	+0.246	+0.222	—0.024	+0.234
31	59.853	—0.375	—0.399	—0.024	—0.387
32	60.216	—0.738	—0.762	—0.024	—0.750
33	59.603	—0.125	—0.165	—0.040	—0.145
34	58.429	+1.049	+1.009	—0.040	+1.029
35	59.075	+0.403	+0.362	—0.040	+0.383
36	57.695	+1.783	+1.743	—0.040	+1.763
37	59.391	+0.087	+0.047	—0.040	+0.067
38	58.644	+0.834	+0.794	—0.040	+0.814
39	59.357	+0.121	+0.081	—0.040	+0.101
40	59.596	—0.118	—0.158	—0.040	—0.138
41	60.266	—0.788	—0.344	+0.444	—0.566
42	59.889	—0.411	+0.033	+0.444	—0.189
43	59.248	+0.230	+0.674	+0.444	+0.452
44	59.509	—0.031	+0.414	+0.444	+0.191
45	59.053	+0.425	+0.869	+0.444	+0.647
46	58.343	+1.135	+1.579	+0.444	+1.357
47	60.941	—1.463	—1.019	+0.444	—1.241
48	59.669	—0.191	+0.075	+0.266	—0.058
49	59.584	—0.106	+0.160	+0.266	+0.027
50	58.920	+0.558	+0.824	+0.266	+0.691
51	58.626	+0.852	+1.118	+0.266	+0.985
52	59.263	+0.215	+0.481	+0.266	+0.348

Table VII (continued)

i	$\overline{\varphi}^{(i)}$	$\Delta\delta_1^{(i)}$	$\Delta\delta^{(i)}$	$\Delta\delta^{(i)} - \Delta\delta_1^{(i)}$	$\overline{\Delta\delta}^{(i)}$
53	59.″148	+0.″330	+0.″596	+0.″266	+0.″463
54	59.497	—0.019	+0.247	+0.266	+0.114
55	59.276	+0.202	+0.468	+0.266	+0.335
56	59.327	+0.151	+0.417	+0.266	+0.284
57	59.377	+0.101	+0.367	+0.266	+0.234
58	59.699	—0.221	+0.045	+0.266	—0.088
59	59.628	—0.150	+0.116	+0.266	—0.017
60	59.844	—0.366	—0.100	+0.266	—0.233
61	59.976	—0.498	—0.459	+0.039	—0.479
62	59.466	+0.012	+0.051	+0.039	+0.032
63	59.503	—0.025	+0.014	+0.039	—0.006
64	59.444	+0.034	+0.073	+0.039	+0.054
65	59.043	+0.435	+0.474	+0.039	+0.455
66	60.335	—0.857	—0.818	+0.039	—0.838
67	58.179	+1.299	+1.338	+0.039	+1.319
68	58.269	+1.209	+1.248	+0.039	+1.229
69	59.753	—0.275	—0.236	+0.039	—0.256
70	59.603	—0.125	—0.086	+0.039	—0.106
71	57.891	+1.587	+1.626	+0.039	+1.607

which give rise to the relation

$$\overline{\Delta\varphi}^{[k-(k+1)]} = \overline{\varphi}_g^{(k)} - \overline{\varphi}_g^{(k+1)} = \Delta\delta_g^{(k+1)} - z_g^{(k+1)} - (\Delta\delta_g^{(k)} - z_g^{(k)}) \quad .$$

Proceeding by the chain method and on the assumption, which is practically equivalent to the second of the (5), that

$$\sum_{k=1}^{10} z_g^{(k)} = 0 \quad , \qquad \longrightarrow \sum_{i=1}^{72} z_c^{(i)} = 0$$

this latter gives us the following recurring formula

$$(7) \qquad \Delta\delta_g^{(k+1)} - z_g^{(k+1)} = \Delta\delta_g^{(1)} - z_g^{(1)} + \sum_{k=1}^{9} \overline{\Delta\varphi}^{[k-(k+1)]} , \qquad (k = 1, ..., 9)$$

where,

$$\Delta\delta_g^{(1)} - z_g^{(1)} = -\frac{1}{10} \sum_{k=1}^{9} \sum_{k=1}^{9} \overline{\Delta\varphi}^{(k-[k+1)]} \quad .$$

Taking account of relations (1), (4), (6) and (7) we have

$$\Delta\delta^{(i)} - \Delta\delta_1^{(i)} = [(\Delta\delta_g^{(k+1)} - z_g^{(k+1)}) + $$
$$+ \Delta\delta_c^{(i)}] - (\Delta\delta^{(i)} - z_c^{(i)}) \quad , \qquad (k = 1, ..., 9)$$

and as, inside each group, we have always presumed the existence of

$$z_g^{(k)} = z_c^{(i)} \tag{8}$$

from it we should obtain the following theoretical result

$$\Delta\delta^{(i)} = \Delta\delta_1^{(i)} \quad .$$

{ Risultato che si ha direttamente, tenendo conto della (8), confrontando la (1') con la (7)

Apart from the approximate nature of the procedures used for elaborating the material, the existence of the residues $\Delta\delta^{(i)} - \Delta\delta_1^{(i)}$, given in Table VII, may thus be attributed to the fact that (8) has not been checked, that is the *local variation of latitude obtained by the chain method is not a continuous function of time.*

Nevertheless it is symptomatic to note that the greatest residue occurs for the month of August, that is for the hottest month the of year.

On the other hand, if the local variation relative to the latitude, given by the difference

$$\Delta z^{(i)} = z_g^{(k)} - z_c^{(i)} = \Delta\delta^{(i)} - \Delta\delta_1^{(i)} \quad ,$$

shows values which cannot be ignored, it may be presumed that the absolute values of the local term z must be sufficiently appreciable.

Allowing for the fact that the programme is made up of zenithal stars, and if confirmed by the analyses of the variation of the latitude, this result would allow the meaning, more of an *internal systematic error of an instrumental type* (and also personal) rather than of an error due to external causes (refractional), to be attributed to the local term.

As at present there is no reason why preference should be given to the solution $\Delta\delta^{(i)}$ and $\Delta\delta_1^{(i)}$, the preliminary mean corrections $\Delta\delta^{(i)}$ to the declinations of the stars in the zenital catalogue of Milan (Brera) have been obtained by the aritymetical means of the two solutions.

These corrections are given in the last column of Table VII.

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