INTENSITIES, POLARIZATION AND ELECTRON DENSITY OF THE SOLAR CORONA FROM PHOTOGRAPHS TAKEN DURING THE TOTAL SOLAR ECLIPSE OF 1961, FEBRUARY 15

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RIASSUNTO. — Si riportano i risultati delle osservazioni in luce polarizzata della corona solare durante l'eclisse del 15 febbraio 1961. Lo strumento impiegato è una camera tripla, f = 62 cm, D/f = 1/6.9. Si danno le isofote in luce polarizzata ottenute coi tre polaroidi: *a* parallelo all'asse polare, *b* con angolo di posizione di 120°, *c* a 240° e si determina il grado e l'angolo di polarizzazione della corona K+F lungo 12 semidiametri. Si determina quindi la variazione di brillanza della corona in funzione della distanza dal lembo, e si riducono i nostri valori a valori assoluti tramite confronto con la luna piena. Seguendo quindi il metodo indicato da van de Hulst si determina la densità elettronica, l'intensità e la polarizzazione della sola corona K, e si dà un dettagliato schema della successione di calcoli necessari. Si confrontano i nostri risultati col modello coronale di van de Hulst e con i risultati di precedenti eclissi.

ABSTRACT. — We give the results of the observations of the polarization of the solar corona during the eclipse of 1961, Feb. 15. The equipment consisted of a triple camera, f = 62 cm, D/f = 1/6.9. The isophotes in polarized light are given: *a* with polaroid axis parallel to the polar axis of the sun, *b* with axis at position angle 120° and *c* at 240°. The degree and angle of polarization of the K+F corona are computed along 12 radii. The coronal brightness versus the distance from the limb is given and our relative values are reduced to absolute values by comparison with the full moon. Following the method suggested by van de Hulst we compute further the electron density, the intensity and the polarization of the K corona and we give the detailed list of the steps involved in the computations. Our results are compared with the coronal model of van de Hulst and with the results of previous eclipses.

INTRODUCTION

The purpose of the present investigation is to measure the degree and the angle of polarization of the solar corona, and to compute the electron density. The instrument, which has been described in a previous paper (1) is a triple camera, with the following characteristics: f = 62 cm; D/f = 1/6.9.

(*) Ricevuta il 26 aprile 1962.

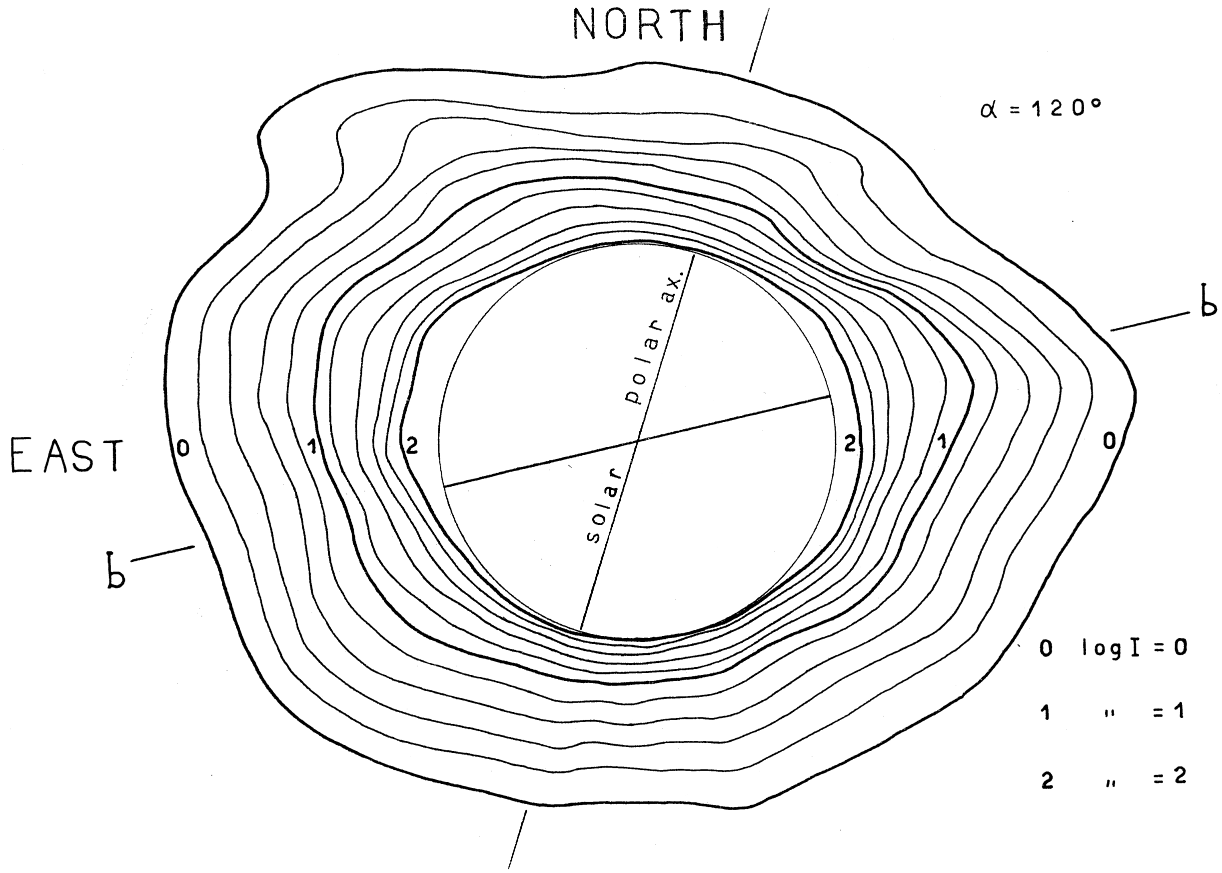


Fig. 2. — Coronal isophotes: position angle of the polaroid 120°.

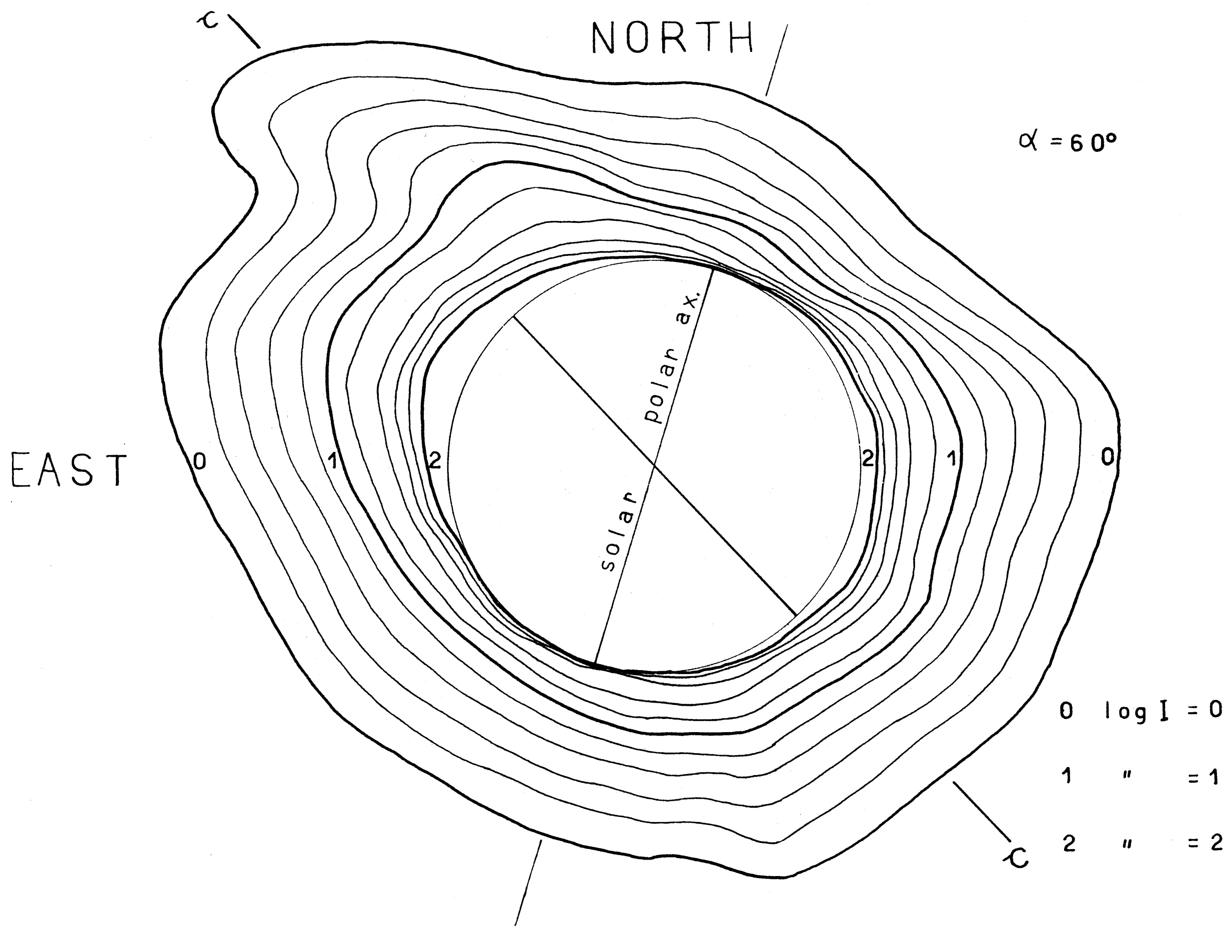
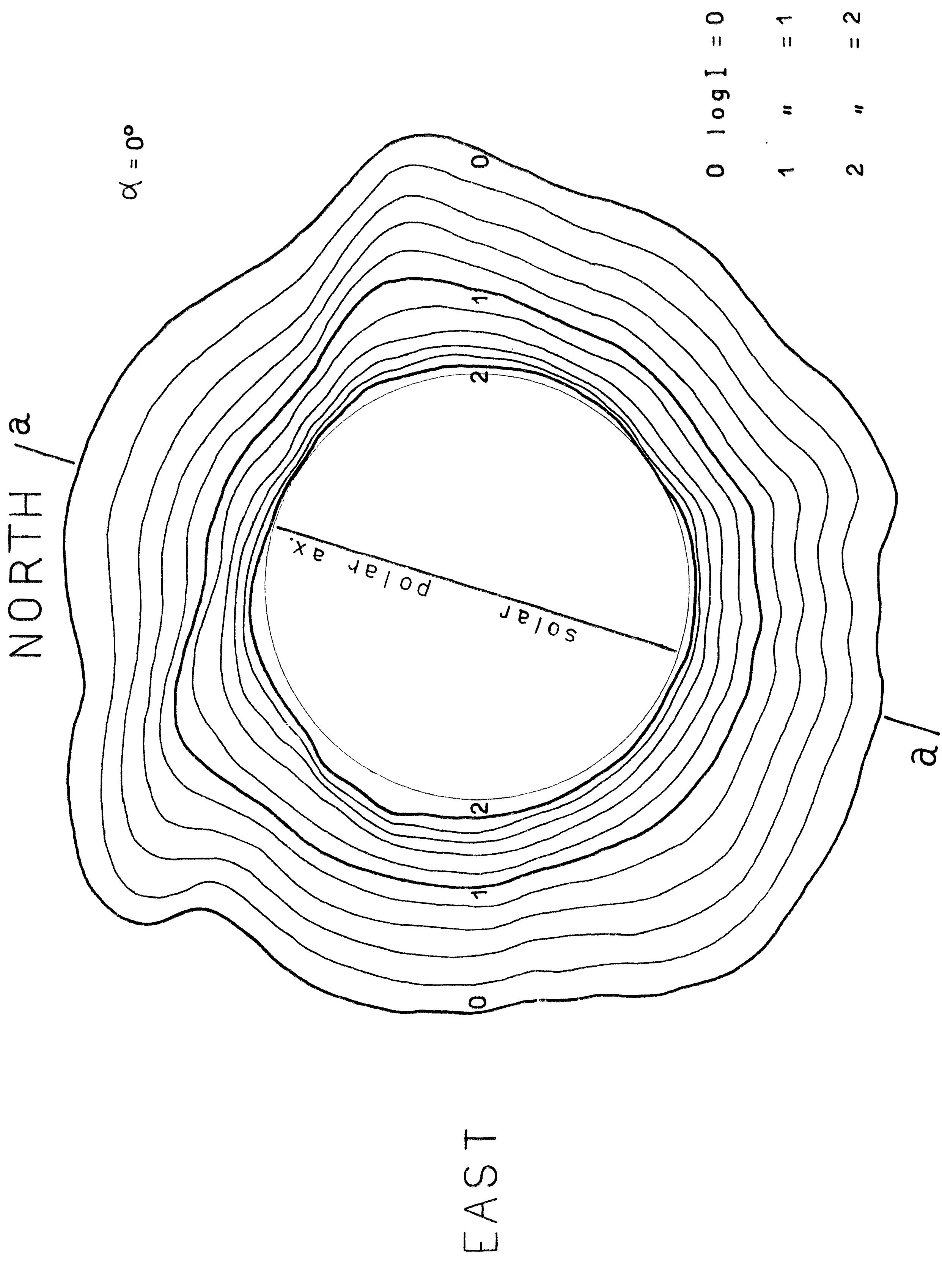


Fig. 3. — Coronal isophotes: position angle of the polaroid 60°.


α = 0°
NORTH
EAST
a
a
solar polar ax.
0 log I = 0
1 " = 1
2 " = 2
0
1
2
2
1
0

The isophotes have been derived from four triplets of plates (Tables I and II), which were exposed with 1,2,4,8 seconds respectively. The one second exposure was taken on very slow emulsion, Ferrania Normale Orto, 11/10 DIN, the other three group on Ferrania Fine Orto, 15/10 DIN. For further details concerning exposure and calibration see reference (1).

Each plate was scanned at the Moll recording microphotometer of the Merate Observatory along directions parallel to the east-west diameter. For each plate we made 20 scannings at steps of half millimeter, and on each scanning the photographic transmissions were measured at 20 points on the average. Finally by means of the calibration curves we have derived from the transmission the logaritm of the intensity, log I, in arbitrary units. With such values of log I we have constructed three maps of the coronal isophotes in polarized light, for the three polaroids *a, b, c*. We have indicated with *a* the polaroid with axis parallel to the polar axis of the sun, with *b* the polaroid with axis having position angle of 120°, and with *c* the polaroid having position angle of 240°. The three isophotes are given in the figures 1, 2 and 3. The scale is 20 times the scale of the image on the plate.

THE POLARIZATION

Let us call α the position angle of the polaroid with respect to a given direction, where the amount of light transmitted is maximum; this angle is the position angle of the plane of vibration of the electric vector of the polarized light, with respect to this given direction.

Let us now call I_{max} and I_{min} the maximum and minimum intensities transmitted through the polaroid, and I_p and I_o the intensities of the polarized and unpolarized light incident on the polaroid. By the Malus' law we have:

$$I_{max} = I_p + \left[\frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta \, d\, \theta \right] I_o = I_p + \frac{I_o}{2}$$

$$I_{min} = \left[\frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta \, d\, \theta \right] I_o = \frac{I_o}{2}$$

The percentage of polarization (2) at a given point P of an extended source is defined as

$$p = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100 = \frac{I_p}{I_p + I_o} \times 100$$

i.e. p represents the ratio per cent between the polarized light and the total light coming from the point P.

When sources of partially polarized light are observed the characteristics of the polarization are defined by the two quantities p and α. Different symbols for the quantities which have been now defined are often found in the literature concerning the solar corona. For instance:

$i_1 = I_{max}$ $i = I_{min}$ are used by Fessenkoff,(3)

$i_t = i_{max}$ $i_r = I_{min}$ are used by Baumbach(4) and by von Klüber,(5)

$i_1 = I_{max}$ $i_2 = I_{min}$ are used by Öhman;(6)

instead of p the decimal number p/100 is often used, which is called degree of polarization or simply polarization and which is indicated by x (3) or by p_{K+F} (5).

The equipment for measuring the coronal polarization during the total eclipses was planned by Fessenkoff (3) and applied later by Cimino (7) and by Öhman (6) during the total eclipse of 1945, July 9 and then by us (1).

Let a, b, c be the intensities of a same point P emerging from the three polaroids a, b, c. If we apply the Malus' law to the two components I_{max} and I_{min} along the three directions a, b, c, we have

$$a = \left[I_{max} \cos^2 \alpha + I_{min} \sin^2 \alpha \right] k$$

$$b = \left[I_{max} \cos^2 (120 - \alpha) + I_{min} \sin^2 (120 - \alpha) \right] k$$

$$c = \left[I_{max} \cos^2 (60 - \alpha) + I_{min} \sin^2 (60 - \alpha) \right] k$$

where k is a constant depending upon the instrumental absorption and scattering. We find easily

$$a + b + c = \frac{3\,k}{2} (I_{max} + I_{min})$$

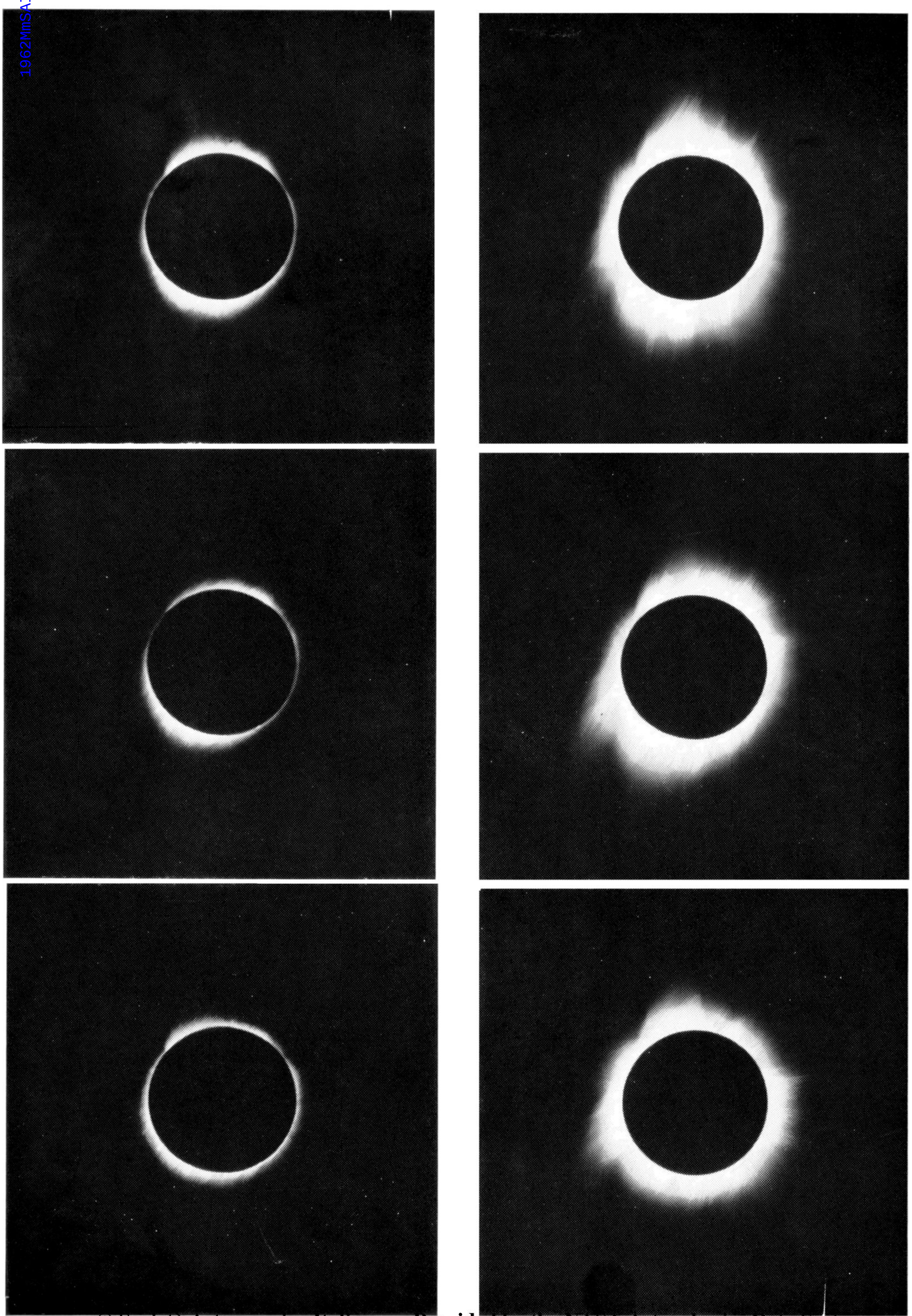
$$4 \sqrt{a (a - b) + b (b - c) + c (c - a)} = 3k (I_{max} - I_{min})$$

and then

$$1) \qquad x = \frac{I_{max} - I_{min}}{I_{max} - I_{min}} = \frac{2 \sqrt{a (a - b) + b (b - c) + c (c - a)}}{a + b + c}$$

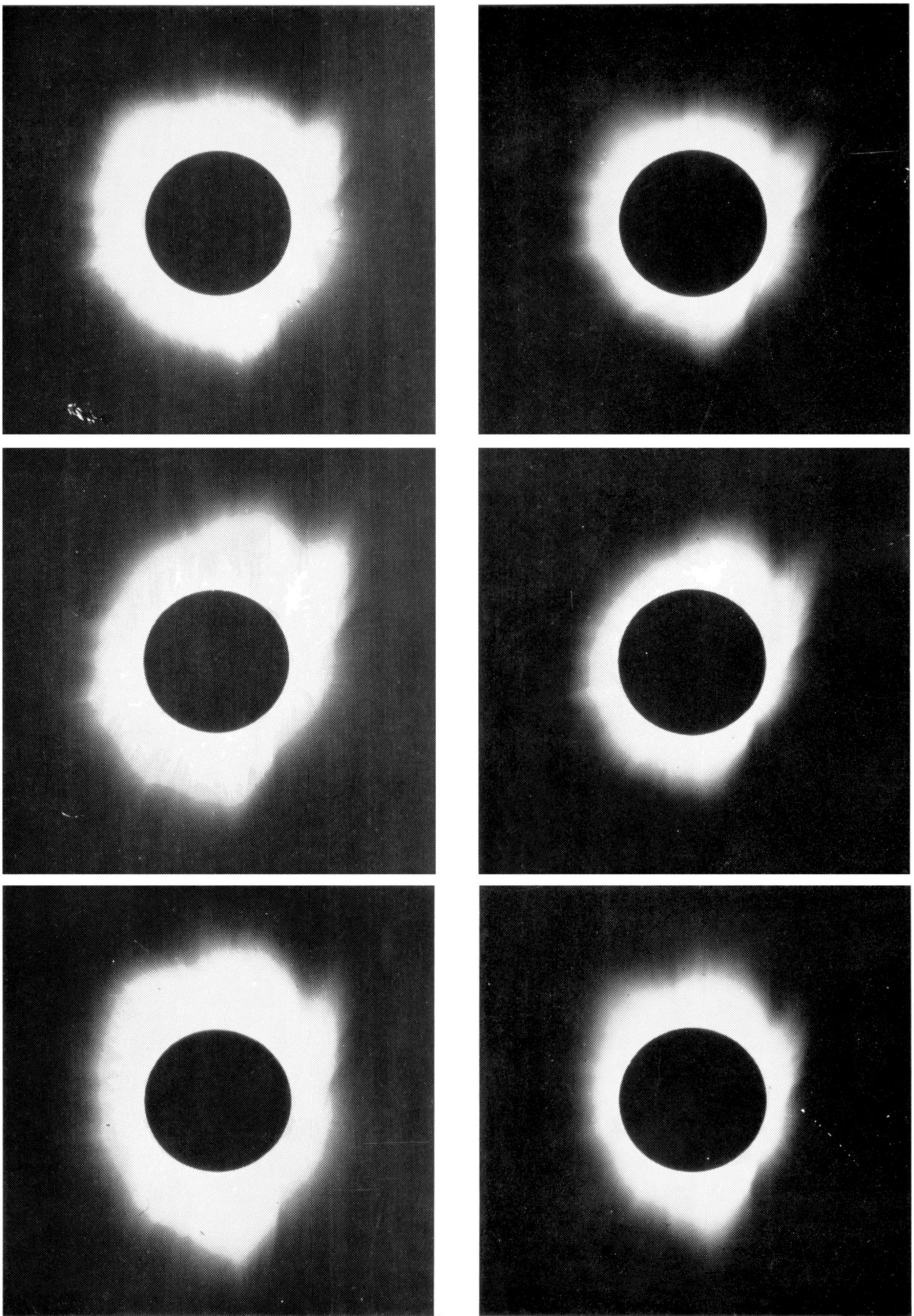
This expression is given in its correct form by Öhman (6) while in Fessenkoff's paper (3) the whole fraction is under the root sign.

TABLE I



The solar corona (total eclipse of Feb. 15, 1961). Top: 1^s exposure. Bottom: 2^s exposure. From left to right: polaroid axis parallel to the solar axis; axis at 60° East; axis at 60° West.

TABLE II



The solar corona (total eclipse of Feb. 15, 1961). Top: 4^s exposure. Bottom: 8^s exposure. From left to right: polaroid axis parallel to the solar axis; axis at 60° East; axis at 60° West.

We have also

$$a - b = \frac{k}{4} (I_{max} - I_{min}) (3 \cos^2 \alpha - 3 \sin^2 \alpha + 2 \sqrt{3} \sin \alpha \cos \alpha)$$

$$b - c = - \sqrt{3}\; k\; (I_{max} - I_{min}) \sin \alpha \cos \alpha$$

$$c - a = - \frac{k}{4} (I_{max} - I_{min}) (3 \cos^2 \alpha - 3 \sin^2 \alpha - 2 \sqrt{3} \sin \alpha \cos \alpha)$$

from which it follows as a check the relation

$$(a - b) + (c - a) = - (b - c)$$

and the relation

$$\mathrm{tg}\; 2\, \alpha = \sqrt{3}\; \frac{c - b}{2\, a - b - c}$$

given in its correct form by Öhman; in Fessenkoff's paper a instead of b is erroneously written in the numerator. We shall write this relation in the form

$$\mathrm{tg}\; 2\, \alpha = \sqrt{3}\; \frac{b - c}{(c - a) - (a - b)} \tag{2}$$

which is preferable for the computations, since the bynomial expressions (b-c), (c-a), (a-b) have been already computed in relation 1).

DETERMINATIONS OF THE CORRECTIONS FOR THE UNEQUALITY OF THE THREE TELESCOPES.

This correction can be found by two methods ([6]):

1) The full moon, which is a source of unpolarized light, was photographed with the triple camera; the differences among the photographic transmissions of the three images were measured and then, through the calibration curve, the difference in intensity derived. These corrections are indicated as follows:

$$\Delta \log b = \log a - \log b$$

$$\Delta \log c = \log a - \log c$$

These corrections were determined by scanning the plates of the full moon using both a narrow microphotometer slit, and a sufficiently broad and high slit, which integrated the details of the lunar image. The single measurements are all in good agreement.

2) Following Öhman ([6]) we make the hypothesis that the polarization of the apparent two-dimensional image of the actual three-dimensional corona is radial. In this case the following relation must be satisfied:

$\log a - \log b = \Delta \log b = 0$ along the diameters at position angles 60^0 and 150^0;

$\log a - \log c = \Delta \log c = 0$ along the diameters at position angles 30^0 and 120^0;

$\log b - \log c = \Delta_2 = 0$ along the diameters at position angles 0^0 and 90^0.

If these relations are not satisfied the differences are imputed to instrumental inequality and $\Delta \log b = \Delta \log c - \Delta_2$ and $\Delta \log c = \log b + \Delta_2$ are averaged with the corrections found in case 1).
The following results are found:

TABLE I

	Δ log b	Δ log c
Full Moon	+ 0.010	— 0.050
Diameters at 0°,60°,,120°	— 0.074	+ 0.025
» » 30°,90°,150°	— 0.028	+ 0.013
Average	— 0.031	— 0.013

We remark that the corrections given by the first method (full moon) were in very good agreement among themselves, while the correction Δ_2 along the directions 0° and 90° differed from zero by a quantity appreciably greater than the errors of measurement. This fact has convinced us that we should not make hypothesis about the radiality of the polarization. We applied therefore only the corrections derived by the first method. These corrections must be applied to the isophotes of fig. 2 and 3. But in order to have an idea of the influence of the corrections for inequality upon the value of the polarization we computed x and α for the North polar axis (where the deviations from radial polarization are large) introducing the corrections given by method 1) (full moon) and by method 2) (Öhman's method). Fig. 4 shows that x computed introducing the corrections according to Öhman becomes less by 5% with respect to our full moon method. The differences in α are given in Table II. This table shows that also the Öhman's method gives a sensitive deviation from radial polarization.

TABLE II

North polar axis

r =	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	method
α =	— 16.5	— 16.2	— 20.3	— 17.4	— 25.8	— 25.8	— 7.5	— 5.3	— 7.3	Öhman
α =	— 22.2	— 23.6	— 26.9	— 24.1	— 23.6	— 18.4	— 17.1	— 17.5	— 18.0	Full Moon

Using the corrections given by method 1) alone, we compute x and α for the diameters at the following position angles: 0°, 30°, 60°, 90°, 120°, 150°. These values are given in Table III and fig. 5 gives a graphical representation of x and α. Fig. 6 gives the theoretical values computed by van de Hulst ([8]) for $p_{K+F} = x$, for equator and poles at the epochs of maximum and minimum, and our observations for the polar north and south radii and the equatorial east and west radii. The curves are cubic paraboles fitted to the observed points by least squares method ([9]) This figure shows that the polarization along the equatorial

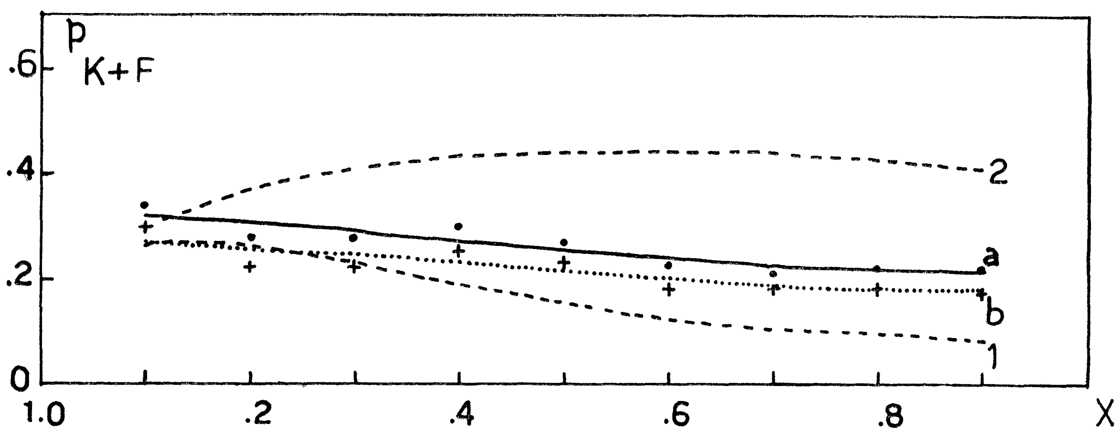


Fig. 4. — Polarization p_{K+F} for the north polar axis: the full line and dots are derived with our corrections (full moon); the dotted line and crosses are derived with the Öhman method (full moon and bisectrices). The broken lines give the values of van de Hulst, 1 minimum, 2 maximum.

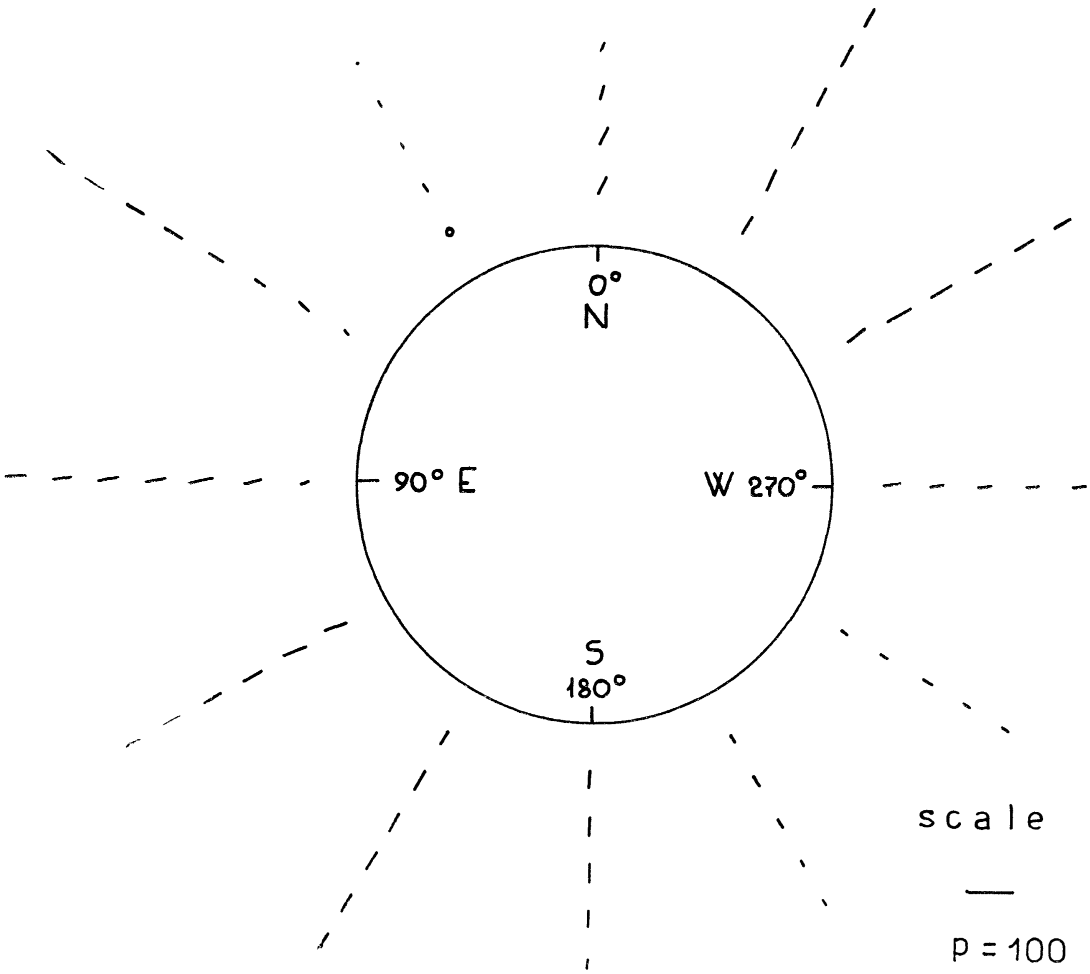


Fig. 5. — Map of the degree and direction of the polarization of the K+F corona.

east radius is definitely higher than the theoretical one. The existence of deviations $\Delta\alpha$ of the polarization from the radial directions is in disagreement with the theory for a spherical and isotrope corona ([10, 11]). However such result has been previously found by other observers ([12]). But, as observed already by von Klüber ([5]), this is not surprising because each polarized vector defined by x and α is the resultant of the projection of three vectors on the plane tangent to the celestial sphere. The intensities of these vectors depend upon the irregularities in distribution of the coronal matter and scattered light, surrounding a given point. The closer the coronal envelope is to a regular spherical shape, the closer the polarized vectors on the tangent plane will be to the radial directions. This is especially true at the epochs of solar maximum. At minimum on the contrary (and our north polar zone has just the characteristics of minimum) the irregularities (plumes, streamers, etc.) can explain large scattering from radial polarization.

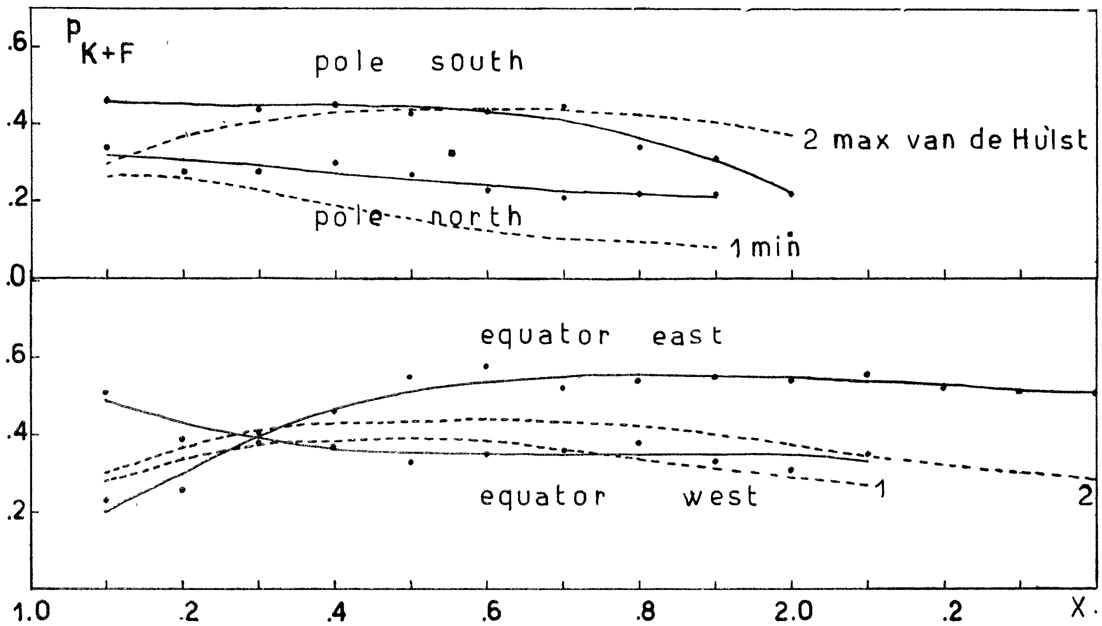


Fig. 6. — Polarization of the K+F corona for the north and south polar axis compared with the values given by van de Hulst for the poles at the epoch of maximum and minimum activity, and for the east and west equatorial axis compared with the values given by van de Hulst for the equator at maximum and minimum.

REDUCTION OF THE BRIGHTNESS OF THE CORONA K + F IN UNITS OF AVERAGE BRIGHTNESS OF THE SOLAR DISK.

It is possible to derive the brightness of the unpolarized corona K + F by the equation

$$a + b + c = \frac{3}{2} k \left(I_{max} + I_{min} \right) = \frac{3}{2} k (K + F) = h (K + F)$$

The problem is now to determine the value of the constant h and so find the reduction scale from our relative units to absolute units. This reduction was made by comparison with the full moon. Assuming for the average photovisual magnitudes of the sun and the moon the following values ([13]) :

$$m_{\odot} = - 26.86 \qquad m_{☾} = - 12.7$$

and using for the average brightness of the solar disk $I_{\odot}$ the value ([8])

$$\log I_{\odot} = 10$$

TABLE III

	α = 0°		α = 30°		α = 60°		α = 90°		α = 120°		α = 150°		α = 180°		α = 210°		α = 240°		α = 270°		α = 300°		α = 330°	
1.1	0.343	— 22°	0.254	— ()	0.331	+ 12°	0.234	+ 09°	0.469	+ 09°	0.446	00°	0.458	— 11°	0.255	()	0.224	00°	0.509	+ 19°	0.543	00°	0.666	()
1.2	.275	24	.138	()	.351	07	.260	08	.518	09	.452	00	.457	04	.300	00°	.184	00	.394	18	.593	00	.556	00°
1.3	.283	27	.126	00°	.101	00	.377	08	.441	00	.492	00	.442	06	.308	00	.306	00	.405	14	.542	00	.400	00
1.4	.300	24	.242	00	.150	00	.464	11	.542	00	.458	00	.451	09	.261	00	.163	00	.375	07	.564	+ 05°	.370	00
1.5	.274	24	.196	00	.226	00	.550	10	.479	00	.486	00	.431	08	.283	00	.226	00	.332	08	.631	06	.335	00
1.6	.226	18	.169	00	.285	00	.576	09	.508	00	.478	00	.431	08	.263	00	.206	00	.354	12	.670	04	.335	00
1.7	.210	17	.156	00	.308	00	.520	08	.455	00	.531	00	.450	09	.239	00	.162	00	.356	12	.627	04	.307	00
1.8	.216	17	.000	00	.367	00	.540	08	.456	00	.493	00	.337	11	.183	00	.163	00	.384	14	.628	04	.322	00
1.9	.217	18	.000	00	.406	00	.550	04	.421	00	.484	00	.314	10	.182	00	.216	00	.330	10	.628	11	.345	00
2.0			.000	00	.431	00	.541	04	.427	00	.471	00	.221	— 14	.182	00	.285	00	.311	10	.609	00	.340	00
2.1					.359	00	.559	06	.381	00	.428	00			.195	00			.354	+ 10	.568	00		
2.2					.399	00	.522	05	.365	00	.407	00									.543	00		
2.3					.437	07	.509	04													.561	00		
2.4					.461	08	.511	00																
2.5					.474	10																		
2.6					.476	13																		
2.7					.456	+ 17																		

Note to table III:

The first column gives the distances (in units of the solar radius) from the center of the sun. The 2nd, 4th, ... 24th columns give the polarization along the 12 radii at position angle α. The columns 3d, 5th, ... 25th give the deviations from the radial direction (positive at east, negative at west). () indicates that the values are very uncertain.

we have

$$\log I_{☾} = 4.32$$

Il we compare a picture of the full moon taken under the same conditions as for the solar corona, and we determine which isophote of the unpolarized corona K + F has an intensity equal to the average intensity of the lunar disk we have the scale for reduction to absolute units. We therefore photographed the full moon with the triple camera using the same type of plates (Ferrania Normale Orto, 11/10 DIN), the same exposure time (1^s) and the same procedure of calibration and development used for the first triplet of coronal plates. Using several microphotometric scannings of the lunar disk we derived the average transmission and hence the average intensity of the lunar disk on our arbitrary scale: $\log \frac{k}{2} I_{☾}$ (since each polaroid transmits 1/2 of the unpolarized lunar light). We than plotted the brightness values K + F of the corona along the polar axis and the equator from the relations

$$\log (a + b + c) = \log \frac{3}{2} k (K + F)$$

and afterwards computed $\log \frac{3}{2} k I_{☾}$. Then we were able to compare this value with the corona K + F in our units. This arbitrary value was made equal to $\log I_{☾} = 4.02$, which is derived from the value $\log I_{☾} = 4.32$ after two corrections:

a) the first due to the higher altitude of the moon (photographed at Merate) with respect to the sun (photographed at Monte Conero); the atmospheric extinction has been taken equal for both places to 0.30;

b) the second due to the value of the lunar diameter, which at the epoch of our picture was greater than the average (moon at perigee). We have in this way determined the contours K + F of the corona as a function of the distance from the solar limb, in units of the average brightness of the solar disk.

From experience gained from the past eclipse we remark that reducing to absolute units might be easier and more precise employing a different method: i.e. photographing the partially eclipsed sun just before and after totality with the same instrument used for the corona, and filters of known transmission. The error in the assumption of the atmospherical extintion will in this way be practically eliminated.

Fig. 7 and 8 give the intensities of the corona in unpolarized light for the equator and poles. It appears from this graph that log (K + F)

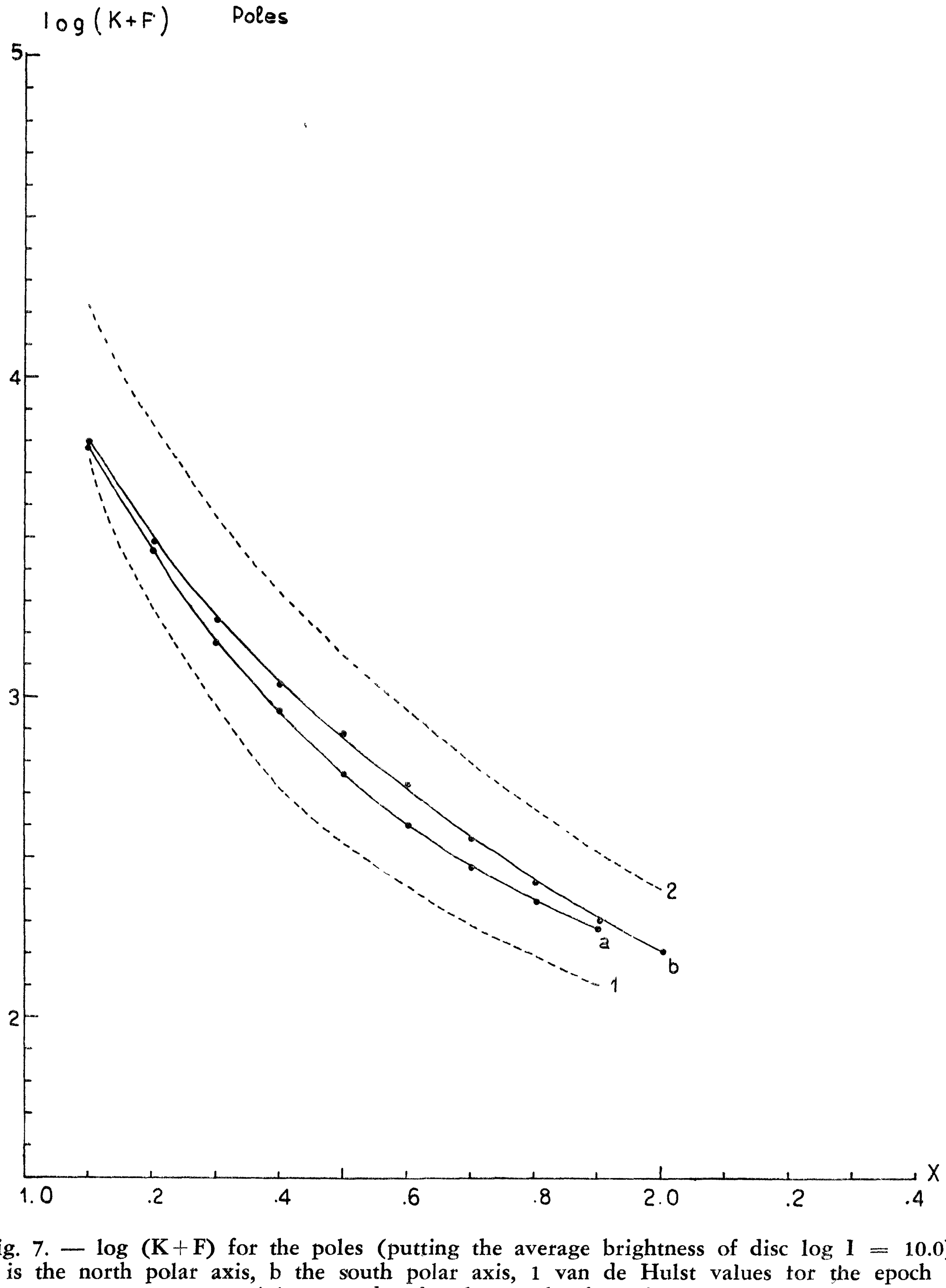


Fig. 7. — log (K+F) for the poles (putting the average brightness of disc log I = 10.0): a is the north polar axis, b the south polar axis, 1 van de Hulst values for the epoch of minimum and 2 for the epoch of maximum.

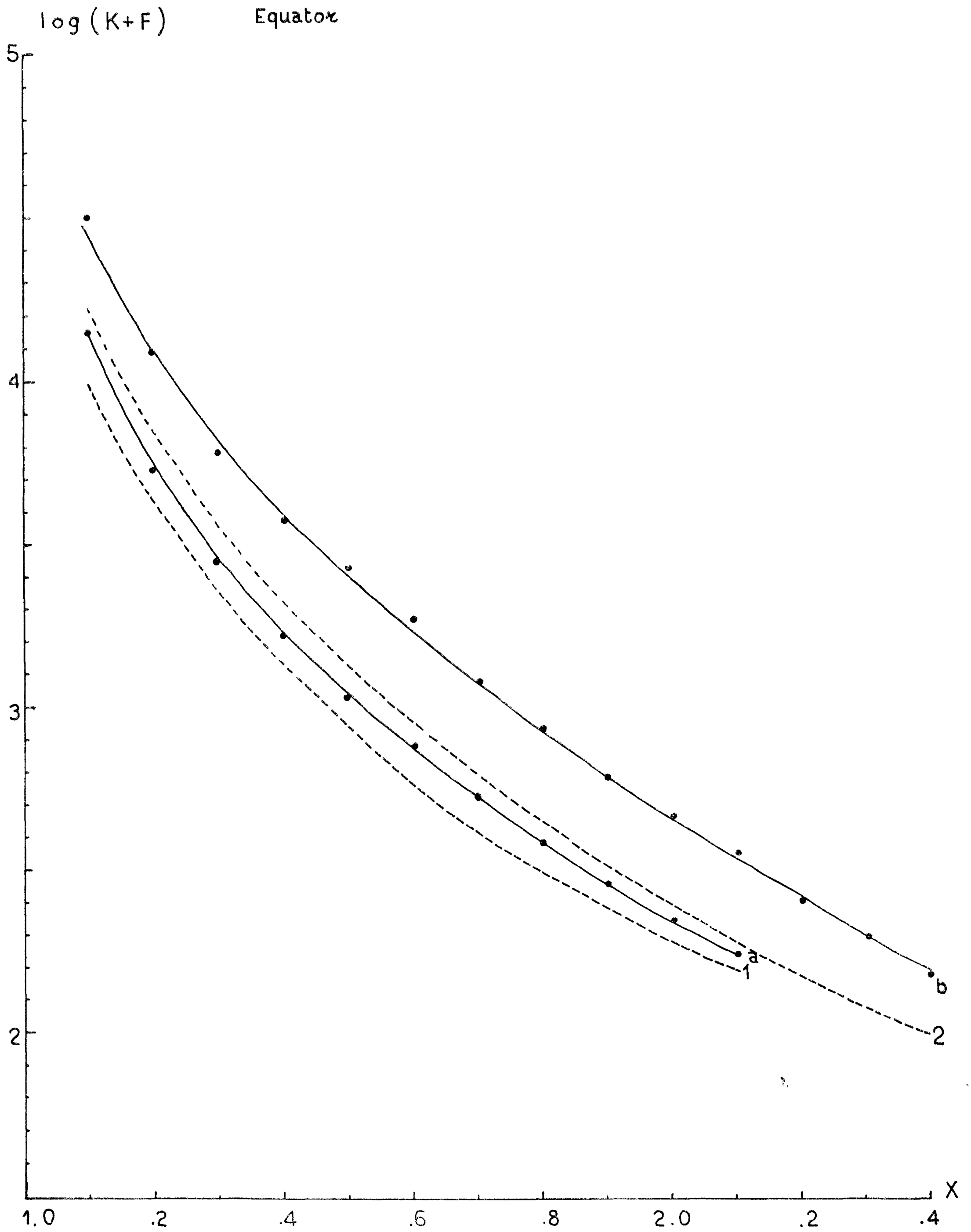


Fig. 8. — log (K+F) for the equator. Symbols and units are the same than in fig. 7; a is the west equatorial radius, b the east.

for the south pole are brighter than for the north pole, and the same occurs for the east equatorial radius with respect to the west radius. The east equatorial radius has higher values than the theoretical ones given by van de Hulst.

COMPUTATION OF THE ELECTRON DENSITY ACCORDING VAN DE HULST.

The fundamental equations used by van de Hulst for computing the electron density are:

$$3)\qquad K_t\,(x) = C\int_x^{\infty} N\,(r)\; A\,(r)\,\frac{r\;\;dr}{\sqrt{r^2 - x^2}}$$

$$4)\qquad K_t\,(x) - K_r\,(x) = C\int_x^{\infty} N\,(r)\left[\,A\,(r) - B\,(r)\,\right]\frac{x^2\;\;dr}{r\,\sqrt{r^2 - x^2}}$$

where the symbols are the same used by van de Hulst ([11]).

Since he knew the observed values of $K(x) = K_t(x) + K_r(x)$ and assumed a preliminary value for p_K, the problem consisted in dividing K into its two components K_t and K_r in such way that by solution of equations 3) and 4) the same value of $N(r)$ is found. As van de Hulst remarked, it is probably safe to assume that the solution is unique. The method indicated by van de Hulst for solving the two equations 3) and 4) is a variation of the collocation and least squares method ([14]).

The value of K being known and assuming a preliminary set of values for p_K the following functions are computed for the corresponding points:

$$5_{a,b})\qquad K_t = (1/2)\,(1 + p_K)\,K \quad ; \qquad K_t - K_r = p_K\,K$$

Then the values so found are represented in the best way by least squares with sums of the type

$$6_{a,b})\qquad K_t = \sum_s h_s\,x^{-s} \quad ; \qquad K_t - K_r = \sum_s k_s\,x^{-s}$$

Let us put in equation 3) $r\;C\;N\;A = \sum_s \frac{h_{s-1}}{r^{s-1}}$; then we have

$$K = \sum_s \left[\,h_{s-1}\int_x^{\infty} \frac{dr}{r^{s-1}\,\sqrt{r^2 - x^2}}\,\right]$$

and since $r = x/\sin\,\theta$ it follows

$$K_t = \sum_s \frac{h_{s-1}}{x^{s-1}} \int_0^{\pi/2} \sin^{s-2}\,\theta\, d\,\theta$$

that is, by changing $s - 1$ in s and putting $a_{s-1} = \int_0^{\pi/2} \sin^{s-1}\,\theta\, d\theta$, it follows

$$K_t = \sum_s h_s\, x^{-s}\, a_{s-1} \quad ;$$

if we want K_t to take the form 6 a) we must have $r\,C\,N\,A = \sum_s \frac{h_s}{a_{s-1}} r^{-s}$

Similarly by equation 4) through integration we have

$$r\;C\;N\;(A - B) = \sum_s \frac{k_s}{a_{s+1}}\; r^{-s}$$

Finally we have

$$7_{a,b})\quad r\;C\;N\;(r)\;A\;(r) = \sum_s \frac{h_s}{a_{s-1}}\, r^{-s} ;\; r\;C\;N(r) \Big[A\,(r) - B\,(r) \Big] = \sum_s \frac{k_s}{a_{s+1}}\, r^{-s}$$

where the coefficients a_s are given by

$$a_n = \int_0^{\pi/2} \sin^n\,\varphi\; d\,\varphi$$

which for $n > 0$ has the form

$$a_n = \frac{\pi}{2^{n+1}} \frac{n!}{\left[\frac{n}{2}!\right]^2} = \frac{\pi}{2^{n+1}} \frac{\Gamma\,(n+1)}{\left[\Gamma\,(\frac{n}{2}+1)\right]^2}$$

Such coefficients have been computed by van de Hulst.

By means of equations $7_{a,b}$) van de Hulst computed in two ways the electron density. For the hypothesis that the solution $N(r)$ is unique, if two different values for $N(r)$ are found it must be imputed to the first uncorrect preliminary set of values assumed for p_K. Now two auxiliary functions are defined

$$c_t = \frac{K_t\,(r)}{r\;C\;N\;(r)\;A\;(r)} \quad ; \quad c_v = \frac{K_t\,(r) - K_r\,(r)}{r\;C\;N\;(r)\;[A\;(r) - B\;(r)]}$$

Equations $6_{a,b}$) nad $7_{a,b}$) show that c_t and c_v are the « effective values » of a_{s-1} and a_{s+1}. Then this equation follows

$$\frac{1}{p} + 1 = \frac{2}{A-B} \frac{A}{} \frac{c_t}{c_v}$$

which gives a second approximation for the set of values for p_K. Introducing these new values in equation $5_{a,b}$) the procedure is repeated; hence by iteration two coincident values of $N(r)$ can be found.

THE ELECTRON DENSITY $N(x)$, THE ELECTRONIC CORONA $K(x)$, THE POLARIZATION $p_K(x)$ AND THE CORONA $F(x)$ DERIVED FROM THE DATA OF OUR OBSERVATIONS.

From our observations we derived the intensity of the corona K+F and the polarization p_{K+F} along several diameters. The following relation [11] connects the two observed quantities with the unknown intensity of the K corona and its polarization p_K:

$$\frac{p_K}{p_{K+F}} = \frac{K+F}{K}$$

Then, by following the procedure used by von Klüber [5] we can compute

$$K_t - K_r = K\, p_K = (K+F)\, p_{K+F}$$

or the equivalent relation

$$K_t - K_r = 2\,h\,\sqrt{a\,(a-b) + b\,(b-c) + c\,(c-a)}$$

where h is the constant for reducing our arbitrary units to absolute units ($\log I_\odot = 10$). This constant, which has been determined as described previously, has the value

$$h = 62.09$$

The set of values for $K_t - K_r$ is represented by the sum

$$8) \qquad K_t - K_r = \sum_s K_s\, x^{-s}$$

(where the coefficients have been determined by least squares method); x is now the apparent distance of a given point from the center of the sun, in units of the solar radius. Consequently for the electron density we have

$$9) \qquad N(x) = \sum_s \frac{k_s}{a_{s+1}} x^{-(s+1)} \frac{1}{C} \frac{1}{A(x) - B(x)}$$

where the coefficients a_s are given by van de Hust ([11]) or Baumbach ([4]); $C = 3.44 \times 10^{-4}$ cm^3; $A(x)$ and $B(x)$ are given by van de Hulst. We used the set of values for A and B computed for the coefficient of limb darkening $q = 0.75$ which is the value closest to our effective wave length (at λ 5300 A, which is our effective wave length is $q = 0.66$).

This assumption for q is completely justified since A and B differ very little for values of q between 0 and 1. Using the least squares method we obtain the sum 8) by approximating the observed values $K_t - K_r$ with the polynomial

$$10) \qquad K_t - K_r = k_{1/2}\, x^{-1/2} + k_5\, x^{-5} + k_{15}\, x^{-15}$$

which follows from the expression given by Baumbach ([4]) and by Woolley and Stibbs ([15]) for $N(r)$

$$N(r) = a\, r^{-3/2} + b\, r^{-6} + c\, r^{-16}$$

From this polynomial expression 10) we computed the coefficients of the sum 9) and then $N(x)$ along the polar axis and the equator. Figures 9 and 10 show our results compared with the theoretical values given by van de Hulst for the epochs of maximum and minimum activity.

Following von Klüber and van de Hulst, since we must have

$$K_t(x) = \sum_s h_s\, x^{-s} \quad \text{and also} \quad N(x)\, A(x)\, C = \sum_s \frac{h_s}{a_{s-1}} x^{-(s+1)}$$

we can represent the values $N(x)$ by the sum

$$N(x)\, A(x)\, C = \sum_s h'_s\, r^{-(s+1)}$$

from wich it follows

$$11) \qquad K_t(x) = \sum_s h'_s\, x^{-s}\, a_{s-1}$$

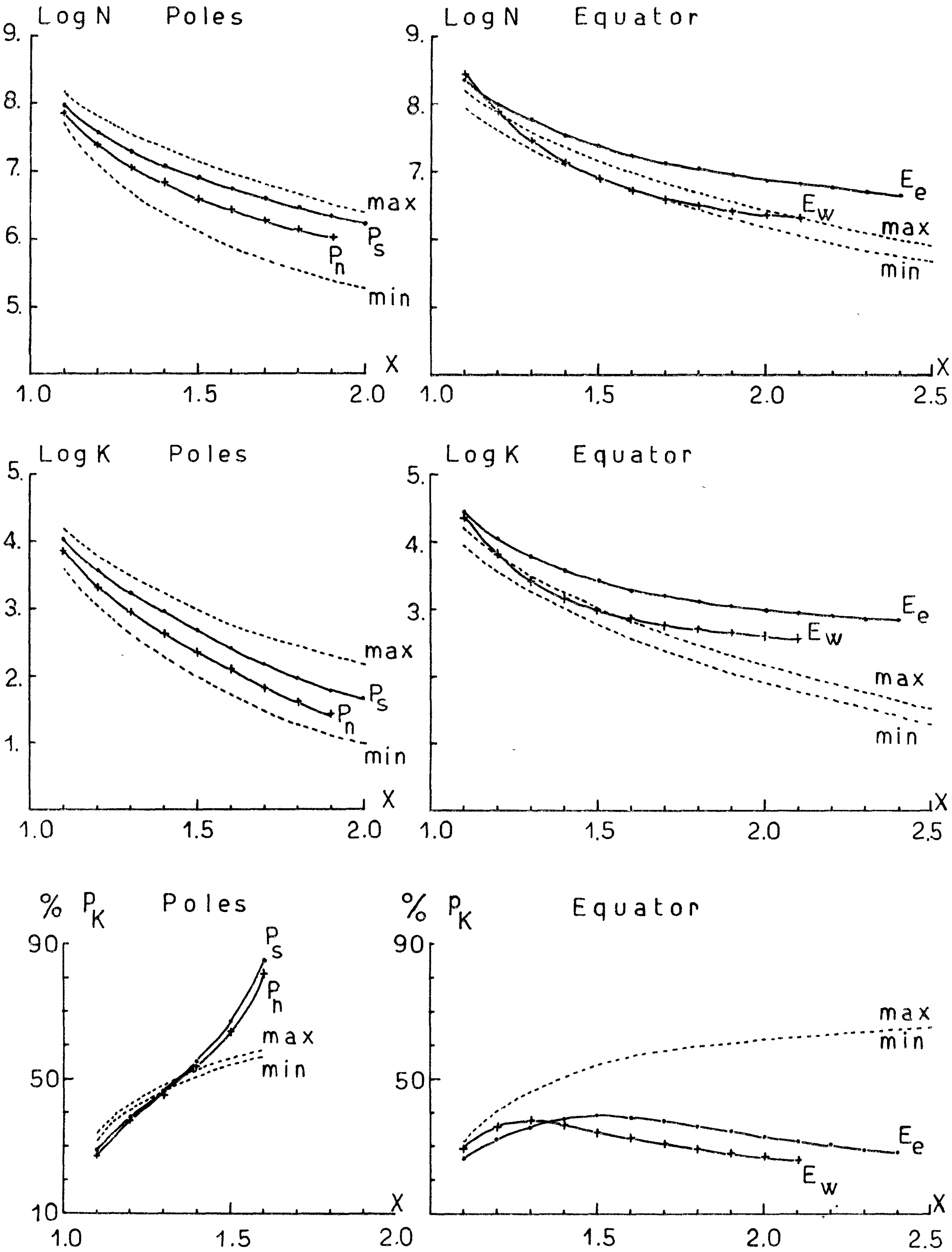


Fig. 9, 10, 11, 12, 13 and 14. — log N(x), log K(x) and log $p_K(x)$ for the north pole (P_n) and south pole (P_s) and for the east equatorial radius (E_e) and west equatorial radius (E_w). The broken lines are the values given by van de Hulst.

In order to use this formula we were obliged to compute

$$a_{-0.5} = \int_0^{\pi/2} \frac{d\,\theta}{\sqrt{\sin\,\theta}}$$

since we can not in this case use the Γ function. By the substitution

$$\sin\,\theta = \frac{2\,t}{1 + t^2}$$

we have the elliptic integral ([16]) ([17])

$$a_{-0.5} = \sqrt{2} \int_0^1 \frac{d\,t}{\sqrt{t + t^3}} = \sqrt{2}\;F\,(30^0,\,90^0) = 2\,3841$$

When $K_t(x)$ is computed we have $K = K_t + K_r = 2\,K_t - (K_t - K_r)$, and $p_K = K_t - K_r)/K$, and $F = (K + F) - K$.

For sake of clearness we give here the sequence of computations:

1) Compute $\frac{1}{A-B}\;\frac{1}{C}\;\frac{1}{x}$

2) Compute k_s/a_{s+1}

3) Compute $(k_s/a_{s+1})\;x^{-s}$ for $s = 1, 2, 3, \ldots.\; n$ according to the number and power of the terms of the sum used for $K_t - K_r$

4) Compute $\sum_s \frac{k_s}{a_{s+1}}\;x^{-s}$

5) Compute the product: $\frac{1}{A-B}\;\frac{1}{C}\;\frac{1}{x}\;\sum_s \frac{k_s}{a_{s+1}}\;x^{-s} = N\,(x)$

6) Using the terms computed in 3) compute $\frac{k_s}{a_{s+1}}\;x^{-s}\;a_{s-1}$
for $s = 1, 2, 3, \ldots.\; n$ corresponding to the number of terms established in 3)

7) Compute $\sum_s \frac{k_s}{a_{s+1}}\;x^{-s}\;a_{s-1}$

8) Compute $A/(A\text{-}B)$

9) Compute the product $\frac{A}{A-B} \sum_s \frac{k_s}{a_{s+1}} x^{-s} a_{s-1} = \sum \frac{h'}{x_s} a_{s-1} = K_t\ (x)$

10) Compute $2K_t - (K_t - K_r) = K$, where $K_t - K_r$ are the values derived from the sum $\sum_s k_s r^{-s} = K_t - K_r$ fitted previously by least squares.

11) Compute $p_k = \frac{K_t - K_r}{K_t + K_r} = \frac{K_t - K_r}{K}$

12) With the observed values $K+F$ represented by least squares method compute $(K+F) - K = F$.

The results derived by this method are given in fig. 11 and 12 (K corona) fig. 13 and 14 (p_K) and 15 (F corona). Concerning the interpretation of these results the following must be observed:

a) all the computations are based on the hypothesis that the solar corona has a density depending only upon r according a law which can be expressed with a sum of the type $N(r) = \sum_s A_s r^{-(s+1)}$; that is radial summetry is assumed for the corona. In this assumption the integral equations 3) and 4) give the values $N(r)$ for $r = x$. From the observation that the coronal unpolarized isophotes are generally fairly regular we note that this assumption is not far from the true case.

b) The computation of the F corona for the equatorial diameter was not possible since the values for the K corona were almost equal to the $K+F$ corona. For the polar axis on the contrary north and south values agree fairly well but they are slightly higher than those given by van de Hulst. In the sums $K_t - K_r = \sum k_s x^{-s}$ for the poles a negative coefficient was present; in those relative to the equator on the contrary all the coefficients are positive. Since the sums consist of three terms only it is probable that the values of $N(x)$ and $K(x)$ relative to the polar axis are a lower limit and those for the equator a superior limit. A confirmation of this is given by the values we find for p_K (fig. 14); the equatorial values of p_K are lower than the values p_{K+F}. The values of p_K for the polar axis (fig. 13) have low weight until $x = 1.3$ (the measurements of a, b, c are difficult because the isophotes are very close to each other) and for $x > 1.3$ are too high confirming that in this case the values for K are a lower limit.

On the basis of these two points we can adopt other methods for attaining a better solution. One method consists in adopting the values for the F corona computed by van de Hulst, and with these values compute K, p_K and N. This is acceptable since the F corona is very probably invariable with the solar cycle (the F corona being light scattered by interplanetary dust). Another way consists in representing $K_t - K_r$ with a sum with a larger number of terms. We might use the formula given by van de Hulst (11)

$$K_t - K_r = k_{34}\, x^{-34} + k_{17}\, x^{-17} + k_7\, x^{-9.7} + k_7\, x^{-7} + k_{2.5}\, x^{-2.5}$$

We expect that the best approximation given by this formula indicates the direction at least in which the values computed for $N(x)$, $K(x)$, $p_K(x)$, and $F(x)$ improve. Unfortunately the solution of the system for computing the coefficients of the sum by the least squares method presents such difficulty that the method is impractical. The matrix of the

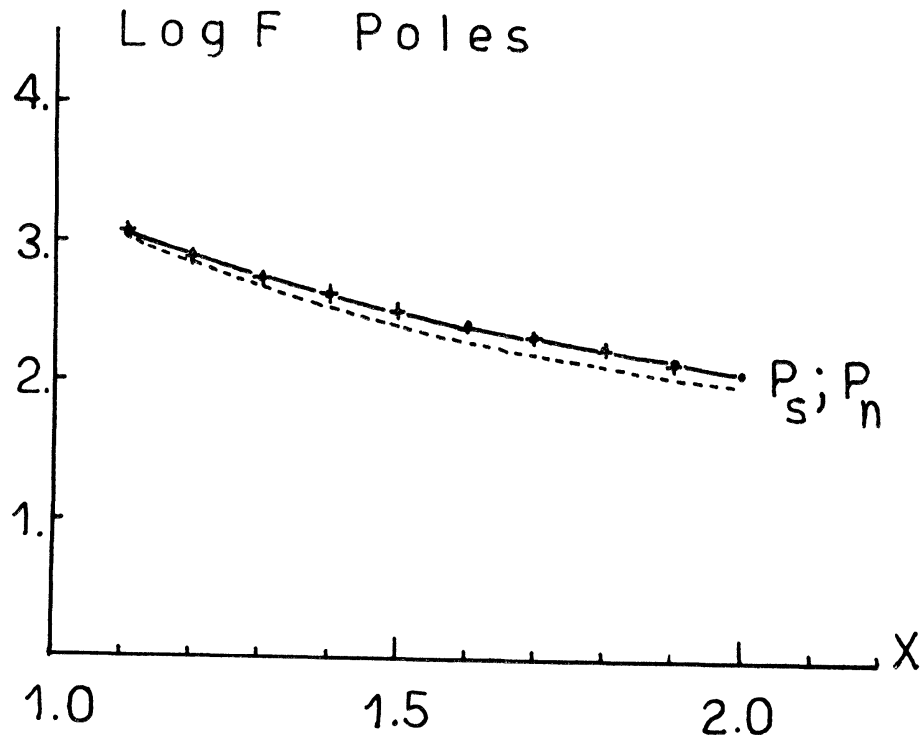


Fig. 15. — log F derived by our measurements for the polar axis (full line) and van de Hulst values for equator and poles (broken line).

normal system is very critical; attempts at solutions have been made with the electronic computer IBM 650 which indicated that the inversion of the matrix of the normal equations requires computations with a number of significant figures greater than eight. Since the observational values give on the average three significant values it is meaningless to perform such laborious computations.

We give therefore the results derived by using the values for the F corona given by van de Hulst (fig. 16-21).

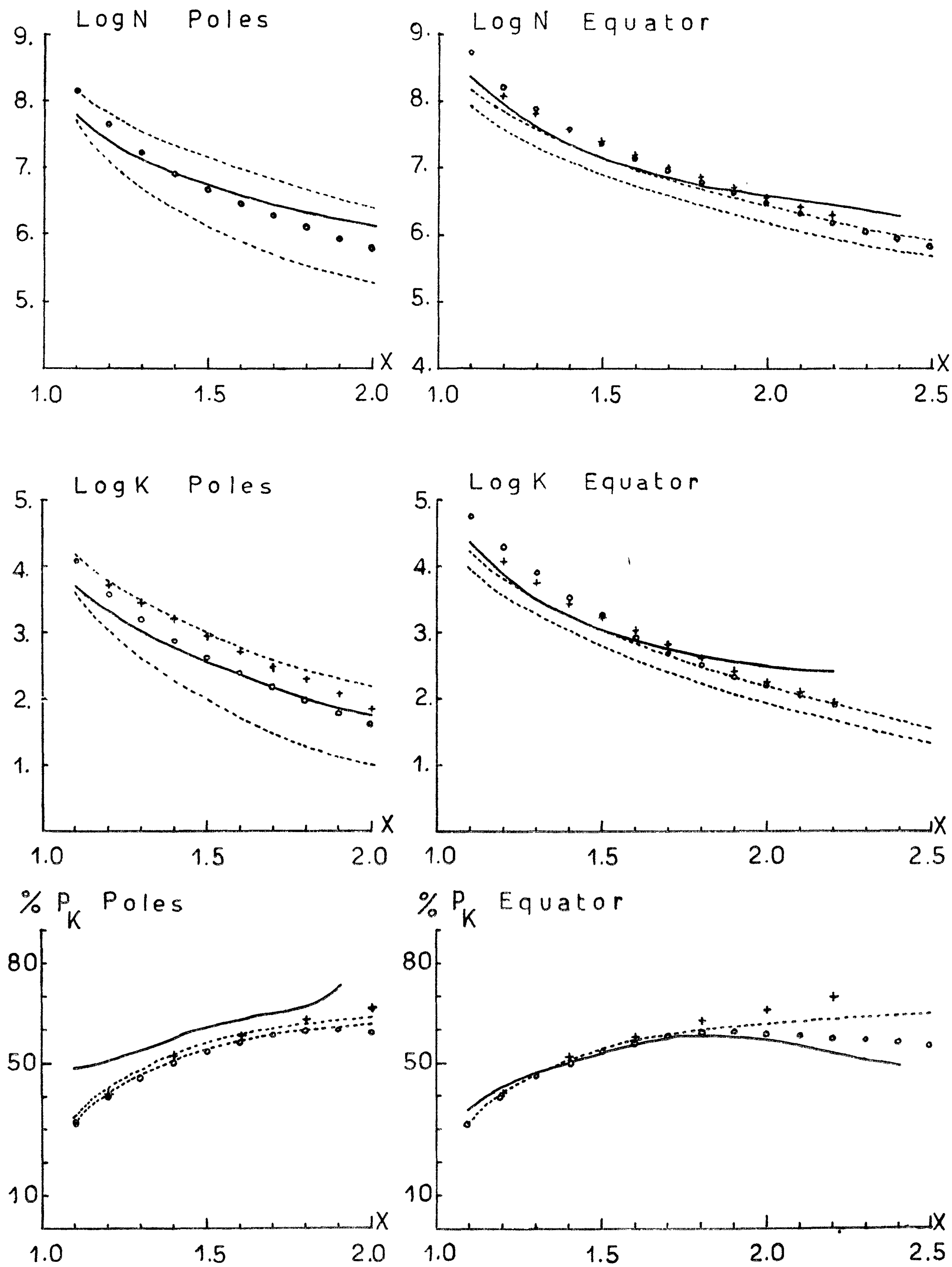


Fig. 16, 17, 18, 19, 20 and 21. — log N, log K, and log p_K for the poles (average of north and south polar radii) and for the equator (average of east and west radii) computed with the three terms sum, using the values for the F corona given by van de Hulst. The full lines are our results, the broken lines the van de Hulst results (maximum and minimum), the circles are the results of von Klüber and the crosses are the result of Néy etc.

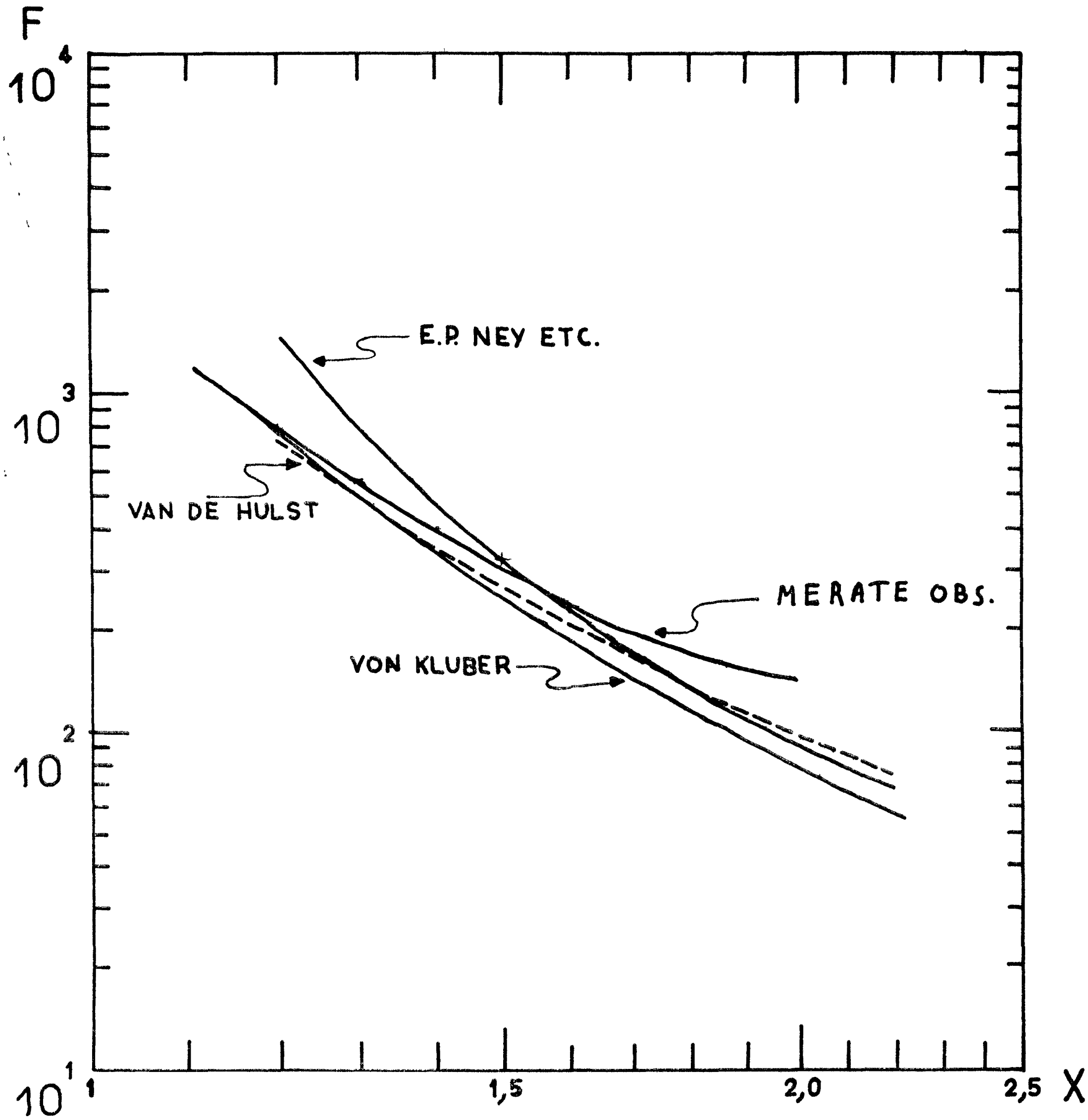


Fig. 22. — The F corona, according the observations of von Klüber and Ney etc.

While in fig. 11-15 we give the results (derived by the three term sum) separately for the north and south polar radii, and for the east and west equatorial radii, in fig. 16-21 we give the results (derived by the three term sum and the van de Hulst F corona) averaging north and south poles, and east and west equator. This procedure allows a direct comparison with the results derived by the observations of von Klüber ([5]), of Ney, Huch, Kellogg, Stein and Gillett ([18]) and with the coronal model of van de Hulst ([11]), all of them giving only the average values. Fig. 22 gives the F corona according van de Hulst, von Klüber and Ney etc. and for comparison we plotted also our values (which we know represent a superior limit for F). Fig. 23 gives the same polari-

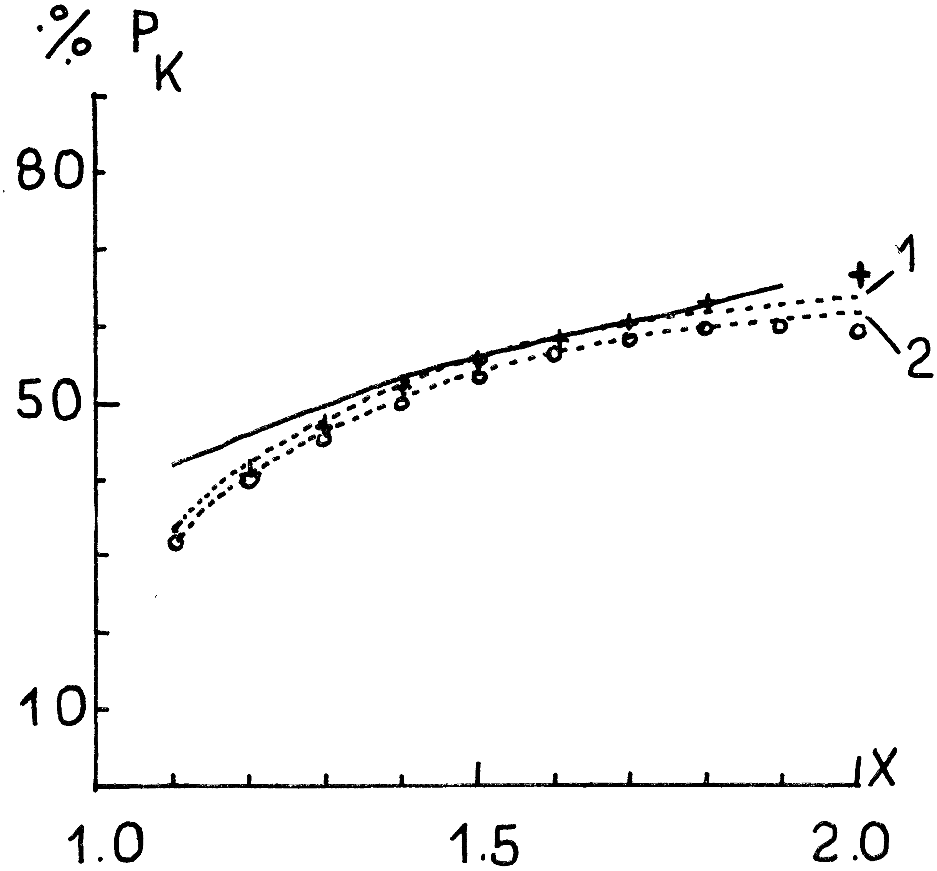


Fig. 23. — Comparison of the values of p_K (average between poles and equator) derived by our observations (full line), von Klüber observations (circles), Ney etc. observations (crosses) and the values given by van de Hulst for maximum (2) and minimum (1).

zation p_K of fig. 20 and 21, but averaged over the poles and the equator for sake of comparison with the other observers which give only an average value.

By comparing the set of figures 9-14 with the corresponding fig. 16-21 we note the following: 1) the electron densities do not change appreciably; 2) the K corona becomes appreciably more intense at the poles, and weaker at the equator, by assuming the F values given by van de Hulst. This confirms that our previous values (fig. 9-15) N, K, F give an upper limit for the equator and a lower limit for the poles. 3) p_K is more affected than the two other quantities by our assumption for the F corona, because $p_K = (K_t - K_r)/(K)$, and therefore the errors of the observed quantity $K_t - K_r$ and of the computed quantity K become greater by division.

We note that the eclipse observed by von Klüber in 1952, February 25 falls two years before minimum, that observed by Ney etc. in 1959, October 2, falls about one year after maximum, and the eclipse of 1961, February 15 falls three years after maximum. The observations of Ney etc. which are closer to the epoch of maximum give coronal isophotes more symmetrical and smaller deviations from radial polarization, than our and von Klüger's observations.

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