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LIGHT CURVES AND ELEMENTS OF EM LACERTAE

Nota di P. BROGLIA e P. CONCONI (*)
(Osservatorio Astronomico di Milano-Merate)

RIASSUNTO. — Durante gli anni 1963-73 sono state ottenute circa 1500 misure fotoelettriche della variabile ad eclisse *EM Lac.* Sono presentate le curve normali *B* e *V*. Il periodo appare sostanzialmente costante. Per mezzo di un Univac 1106 è stata calcolata una serie di soluzioni, per varie combinazioni degli elementi. Il confronto delle curve di luce calcolate con le osservate permette di fissare il range delle soluzioni accettabili. Gli elementi risultano alquanto indeterminati, dato che il sistema è di tipo *W UMa* con eclissi parziali.

SUMMARY. — *B* and *V* photoelectric light curves of the eclipsing system *EM Lac* are given. New light elements are derived. The range of acceptable solutions for this partially eclipsing binary was determined computing a grid of solutions with different combinations of elements, by means of an Univac 1106.

1. - INTRODUCTION

The eclipsing binary *EM Lac*, a W Ursae Majoris system, was discovered to be a variable by KUROCKIN (1945) on Moscow plates. Then on the basis of 202 plates belonging to the same collection KUKARKINA (1953) derived a normal light curve and the light elements, reported in the General Catalogue of Variable Stars (Moscow 1970):

$$\text{Min}\, I = J.\, D.\, 2432797.285 + 0.388924\, n$$

The minimum dated *J. D.* 2417793.438 was not represented satisfactorily with the above ephemerides, so Kukarkina suspected a variation of the period. The minima were nearly of the same depth and the magnitude difference Min *I* - Max gave the result $0^{m}.68$. Moreover the phase of the secondary minimum was $0^{P}.52$.

ROMANO and PERISSINOTTO (1966) in the course of a photographic survey of variable stars determined three instants of minimum and found they are satisfactorily represented by means of the above elements. According to these Authors

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(*) Ricevuta il 7 Novembre 1973.

the light excursion Min *I* - Max was $0^m.6$ and the phase of the secondary minimum $0^P.5$.

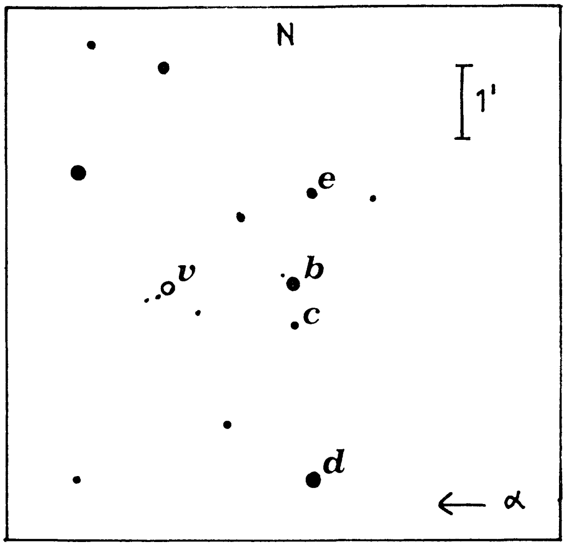
Miller and Wachmann (1971) gave a new photographic light curve. The depths of the minima were $0^m.72$ and $0^m.71$. Five new moments of minimum allowed the period to improve which then gave the result: $P = 0^d.38913278$.

2. - OBSERVATIONS AND LIGHT ELEMENTS

Two colour light curves of *EM Lac* were obtained at the Merate Observatory from 1963 to 1973 with a Lallemand photomultiplier put in the Cassegrain focus of the 40" Zeiss reflector. The effective wavelengths of the phototube + filters combinations are close to the *B* and *V* of the *UBV* system. The output from the photocell was fed into a high impedence Speedomax and during the last few years into a Gardiner-type integrator.

The comparison was the star *b*, similar in colour to the variable, the check stars *e*, *d* and *c* (Fig. 1). The latter appeared to be variable with an amplitude of little more than $0^m.1$ around the mean value $B = 13^m.9$, $V = 13^m.6$. The standard deviation of a Δm between *b* and *d* or *c* was about $0^m.02$.

A transfer to some *UBV* standards gave the following values for the comparison *b* and, by means of the above Δm's, for the check stars:



	V	*B-V*
b	$12^m.28$ $\pm .01$ m.e.	$+ 0^m.69$ .01
e	13.15	+ 1.08
d	11.65	+ 0.60

Fig. 1

Since the variable is less than two arc minutes far from the comparison and of the same colour, the measures were not corrected for differential extinction. Owing to the proximity to the variable of some stars, of which the nearest is a $V = 14^m.6$, it was necessary to put a diaphragm twelve arc seconds in diameter in the focal plane. Altogether we obtained 947 yellow and 594 blue measures which will be published in detail elsewhere. *EM Lac* was observed in *B* and *V* during the years 1963 and '64. Two further sets of *V* measures were obtained in 1968 and 1973 to check the period and to verify if the light curve undergoes long term variations which are not unusual for *W UMa* systems.

Fitting by least squares a second degree parabola to the lower portions of each eclipse curve eight instants of primary and eleven of secondary minimum were obtained. From these we calculated by least squares the ephemerides:

$$\begin{array}{lll} \text{Min}\, I = \text{Helioc. } J.\,D.\; 2438259.5444 & + \; 0.38913342 & n \\ \qquad\qquad\qquad\qquad\qquad\quad \pm \quad 6 & \qquad\qquad\quad 15 & m.e. \end{array}$$

The corresponding *0-C*, computed also for the earlier photographic minima are given in Table I together with the times of minimum and the number of cycles *n*. We note a small systematic trend for the residuals of the photoelectric times; moreover the mean value of the *0-C* is $0^d.0017$, whilst the incertitude of an epoch estimated by comparing the instants determined with both filters is only $0^d.0010$. The effect is small and since the scatter of the photographic times can be attributed to observational inaccuracy, it is not compelling evidence of period variation.

The phase for each measure was then computed according to the above light elements. An examination of all the observations plotted in order of phase revealed some scatter. Probably this is not due to a non repetition of the light curve from night to night, but to a natural dispersion of the observations, because this star is rather faint under our observing conditions. We note however a much greater difference between the depths of the minima derived by us and those obtained by Miller and Wachmann and by Kukarkina and reported in the introduction. As the depth is practically the same in the spectral regions considered and as it is a photometrically well defined quantity, we must suppose that the light curve has undergone a long term variation.

Normal points were subsequently derived; they are given in the Tables II and III, where *n* is the number of measures sharing in a normal. The corresponding mean light and colour curves are represented in the Fig. 2. The results for *EM Lac* are as follows:

	Max	Min *I*	Min *II*
V	$12^m.50$	13.09	13.09
B-V	+ 0.68	0.695	0.685

The colour index point for both the components to a spectral type *G* 5 or somewhat earlier in account of the low galactic latitude ($b = +\ 3°$).

TABLE I. - Observed epochs of minimum light

Observer		n	Helioc J.D. 24...	O-C
Kukarkina	ph	-52594	17793.438	-0$^{\rm d}$.023
"	"	14037	32797.285	+ .006
Miller, Wachmann	"	13826	32879.405	+ .019
" "	"	12991	33204.324	+ .012
" "	"	11464	33798.529	+ .010
" "	"	9134	34705.213	+ .013
" "	"	8419	34983.442	+ .012
Romano, Perissinotto	"	1678.5	37606.355	- .029
" "	"	1041	37854.464	+ .007
" "	"	- 923	37900.359	- .015
Broglia, Conconi	pe	+ 2.5	38260.5212	+ .0040
" "	"	10	38263.4382	+ .0025
" "	"	12.5	38264.4103	+ .0017
" "	"	143.5	38315.3866	+ .0016
" "	"	146	38316.3602	+ .0023
" "	"	909.5	38613.4590	- .0022
" "	"	920	38617.5466	- .0005
" "	"	922.5	38618.5187	- .0013
" "	"	994	38646.3434	+ .0004
" "	"	994.5	38646.5365	- .0011
" "	"	997	38647.5116	+ .0012
" "	"	999.5	38648.4818	- .0015
" "	"	1222.5	38735.2583	- .0017
" "	"	1225.5	38736.4247	- .0027
" "	"	4700	40088.4695	- .0020
" "	"	4702.5	40089.4415	- .0028
" "	"	9331	41890.5492	+ .0009
" "	"	9336	41892.4957	+ .0017
" "	"	9343.5	41895.413	+ .0005

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3. - RECTIFICATIONS AND SOLUTIONS

According to the usual expression a least-squares representation was calculated for the blue and yellow light curves outside eclipse, considering Fourier harmonics up to the 4-th. The computer program then compares the coefficients with the corresponding mean errors and rejects the insignificant terms and it iterates the solution until a given prefixed consistence for the solution is obtained. After a preliminary rectification, the phase angle of external tangency was found to be 43° and 42° in *B* and *V* respectively. Therefore the interval for the Fourier analysis was extended to these limits and the following coefficients were obtained

TABLE II. - Mean *B* points

Phase	B	n	Phase	B	n	Phase	B	n
$1\overset{\circ}{.}9$	$13\overset{m}{.}779$	7	$131\overset{\circ}{.}7$	$13\overset{m}{.}319$	7	$222\overset{\circ}{.}5$	$13\overset{m}{.}349$	7
5.4	13.788	7	134.2	13.329	7	226.0	13.317	7
9.6	13.734	7	136.8	13.356	7	229.9	13.295	7
14.6	13.672	7	139.5	13.382	7	233.5	13.269	7
19.4	13.631	7	142.8	13.385	7	236.7	13.278	7
24.1	13.565	7	146.4	13.415	7	239.2	13.245	7
28.5	13.488	7	149.6	13.459	7	243.9	13.252	7
32.2	13.456	7	153.1	13.489	7	248.2	13.236	7
35.8	13.411	7	156.1	13.512	7	252.4	13.201	7
39.5	13.374	7	158.8	13.550	7	259.0	13.181	7
43.6	13.329	7	162.2	13.596	7	264.8	13.186	7
47.3	13.336	7	164.8	13.634	7	270.2	13.181	7
50.7	13.285	7	167.9	13.650	7	277.4	13.201	7
55.5	13.271	7	170.8	13.699	7	286.8	13.197	7
59.5	13.236	7	173.7	13.740	7	294.9	13.233	7
64.0	13.222	7	177.2	13.761	7	299.7	13.272	7
68.5	13.229	7	180.4	13.773	7	304.3	13.268	7
74.3	13.207	7	184.0	13.746	7	312.1	13.311	7
81.0	13.188	7	187.7	13.712	7	316.6	13.345	7
86.7	13.190	7	191.1	13.682	7	322.2	13.399	7
91.9	13.178	7	195.2	13.643	7	326.8	13.428	7
97.6	13:173	7	198.7	13.608	7	331.4	13.460	7
102.6	13.185	7	201.8	13.563	7	335.3	13.524	7
109.1	13.193	7	205.4	13.496	7	340.1	13.584	7
113.5	13.222	7	208.6	13.473	7	344.2	13.638	7
118.2	13.246	7	212.5	13.421	7	348.8	13.718	7
121.9	13.242	7	215.7	13.386	7	353.4	13.729	7
125.1	13.273	7	218.9	13.376	7	358.3	13.764	6
128.3	13.277	7						

TABLE III. - Mean *V* points

Phase	V	n	Phase	V	n	Phase	V	n
$1^{\circ}.9$	$13^{m}.070$	7	$143^{\circ}.3$	$12^{m}.734$	9	$229^{\circ}.5$	$12^{m}.630$	9
4.4	13.066	9	146.4	12.765	9	232.3	12.615	9
7.8	13.059	9	148.8	12.770	9	235.9	12.588	9
11.9	13.023	9	151.4	12.810	9	238.6	12.574	9
15.6	12.982	9	154.2	12.838	9	242.4	12.571	9
19.4	12.927	9	156.4	12.872	9	246.2	12.549	9
22.6	12.887	9	158.9	12.883	9	249.9	12.534	9
26.0	12.864	9	161.6	12.929	9	254.1	12.525	9
29.3	12.817	9	164.2	12.955	9	258.8	12.515	9
32.6	12.782	9	166.7	12.986	9	263.2	12.513	9
36.2	12.729	9	168.5	12.996	9	268.1	12.499	9
38.9	12.707	9	171.0	13.028	9	273.4	12.496	9
42.0	12.582	9	173.4	13.060	9	278.5	12.505	9
45.3	12.653	9	175.2	13.054	9	284.3	12.520	9
48.0	12.643	9	177.7	13.065	9	291.8	12.529	9
51.8	12,625	9	180.0	13.094	9	296.9	12.560	9
55.7	12.593	9	182.5	13.087	9	299.7	12.579	9
60.1	12.565	9	184.9	13.068	9	302.6	12.584	9
64.4	12.565	9	187.1	13.039	9	306.5	12.601	9
68.7	12.549	9	189.5	13.029	9	310.7	12.631	9
73.9	12.540	9	191.4	12.984	9	314.4	12.665	9
80.2	12.502	9	194.1	12.979	9	318.1	12.672	9
84.8	12.509	9	197.1	12.931	9	321.6	12.699	9
90.0	12.493	9	199.0	12.903	9	324.6	12.715	9
97.5	12.512	9	200.8	12.884	9	327.8	12.762	9
98.7	12.503	9	202.8	12.856	9	330.5	12.789	9
104.3	12.515	9	205.1	12.823	9	334.0	12.809	9
109.6	12.525	9	207.5	12.806	9	337.0	12.862	9
115.7	12.565	9	210.0	12.780	9	340.4	12.912	9
120.9	12.579	9	212.7	12.761	9	344.2	12.957	9
124.8	12.599	9	215.4	12.724	9	347.5	12.996	9
128.1	12.611	9	218.0	12.721	9	350.6	13.025	9
130.8	12.639	9	220.5	12.704	9	354.1	13.055	9
134.3	12.652	9	223.2	12.670	9	356.8	13.070	6
137.8	12.680	9	226.4	12.641	9	359.3	13.086	7
140.6	12.709	9						

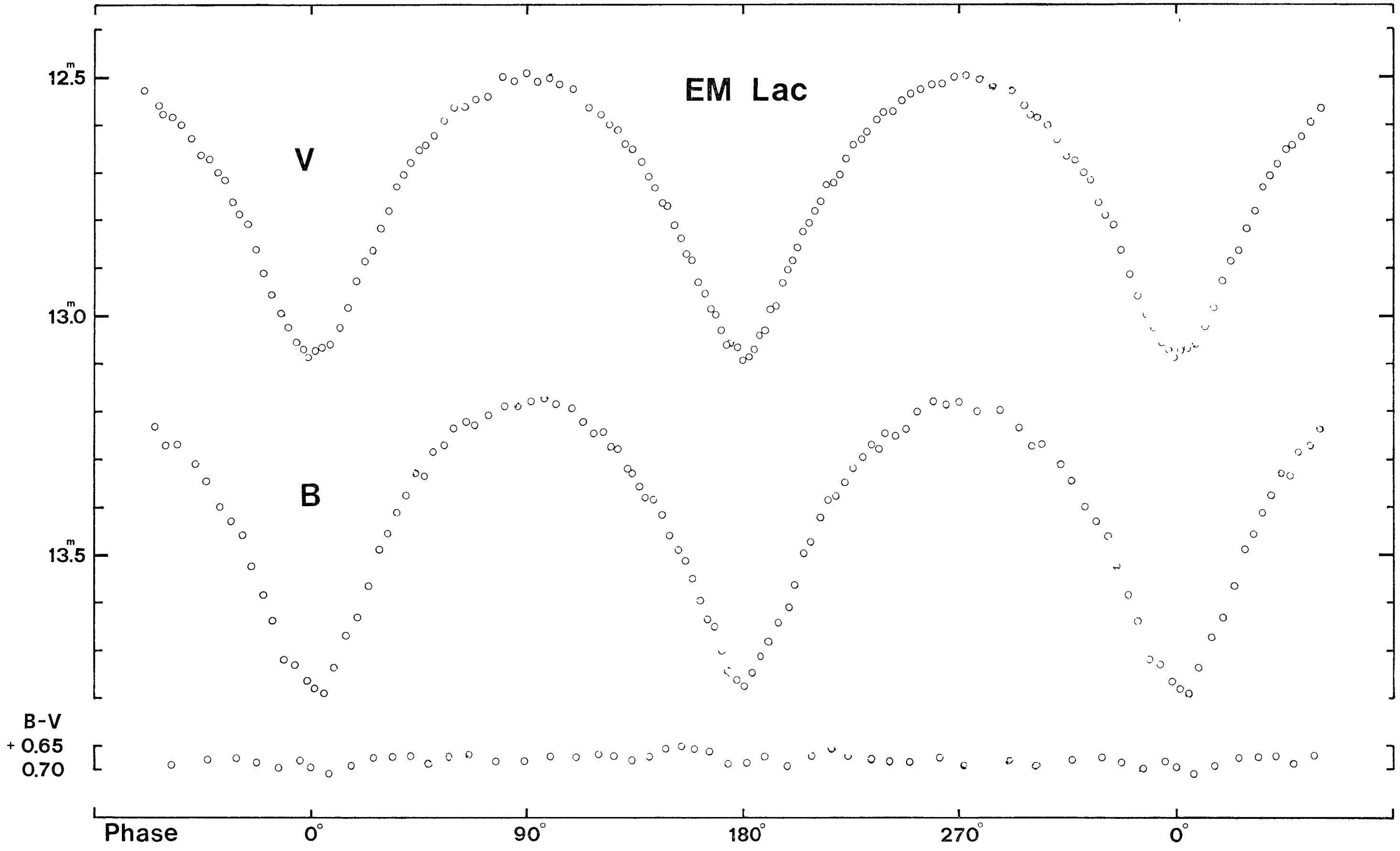

FIG. 2

The mean light and colour curves of *EM Lacertae*.

with the corresponding mean errors:

	A_0	A_1	A_2	B_1	σ	N
V	.8195	− .0009	− .1270	− .0010	.0044	45
	± 15	. 17	24	9	*m.e.*	
B	.8227	− .0018	− .1214	.0023	.0075	40
	± 27	30	42	17		

N is the number of equations solved and σ the mean deviation in light units of a normal in comparison with the Fourier representation. We note that the reflection effect is insubstantial whilst the ellipticity is strong, as expected.

The rectifications of the light and of the phases were performed according to the Russel and Merrill (1952) method, assuming $x = 0.6$.

Since the eclipse is partial and the depths of the minima are equal both for the B and for the V light curves, some uncertainty regarding the orbital elements is inevitable. Moreover it is known that if the eclipses are shallow, and for *EM Lac* the results are: $1 - l_{rect} = 0.2$, the shape of the minima varies little for a wide range of the ratio of radii k. To these intrinsic reasons of indeterminacy is added the fact that, on account of the observational difficulties previously mentioned, the light curves are of moderate precision, as appears from Fig. 1. We preferred therefore to disregard the grafical solutions and to determine numerically the range of acceptable elements by computing a set of solutions with different eclipse parameters and to calculate for each trial the corresponding mean residual of a normal. The set of parameters which minimize the mean residual is assumed to represent the best solution. An automatic search for the minimum by means of a method like that of Horak (1970) disclosed that it is easy to drop in a secondary minimum and moreover that the range of indeterminacy cannot be clearly displayed. To avoid these difficulties we adopted the following method.

The rectified light curves permit us to fix the angle of external tangency θ'_{rect} with an incertitude of a few degrees at the most. As second and third parameters we chose k and i. For an assumed set of parameters the luminosity of the components was calculated by least squares from all the normals of both minima by means of the relations:

$$l^{oc} = 1 - \alpha^{occ} L_s \qquad l^{tr} = 1 - \tau\alpha^{tr}(1 - L_s)$$

Then the residuals of the normals of both minima in comparison with the light curve defined by θ', k, i, L_s were calculated.

The functions α^{occ} and α^{tr} were directly computed according to the method given by Jurkevich (1970). The Univac 1106 machine of the Milano Universiy gives about 8000 values per minute for the functions; a perfect coincidence with the Merrill (1950) tables was obtained. For an assumed value of θ' a table is printed, which for the prefixed combinations of i and k gives the mean deviation of a normal in comparison with the corresponding computed light curve and the ratio J_s/J_g. A 1° increment to θ'_{rect} is then given and the whole process repeated. After the region of acceptable solutions is defined the steps and the range of parameters are conveniently reduced. Part of the range permitted by the solutions was ruled out by the condition that the computed value of J_s/J_g is

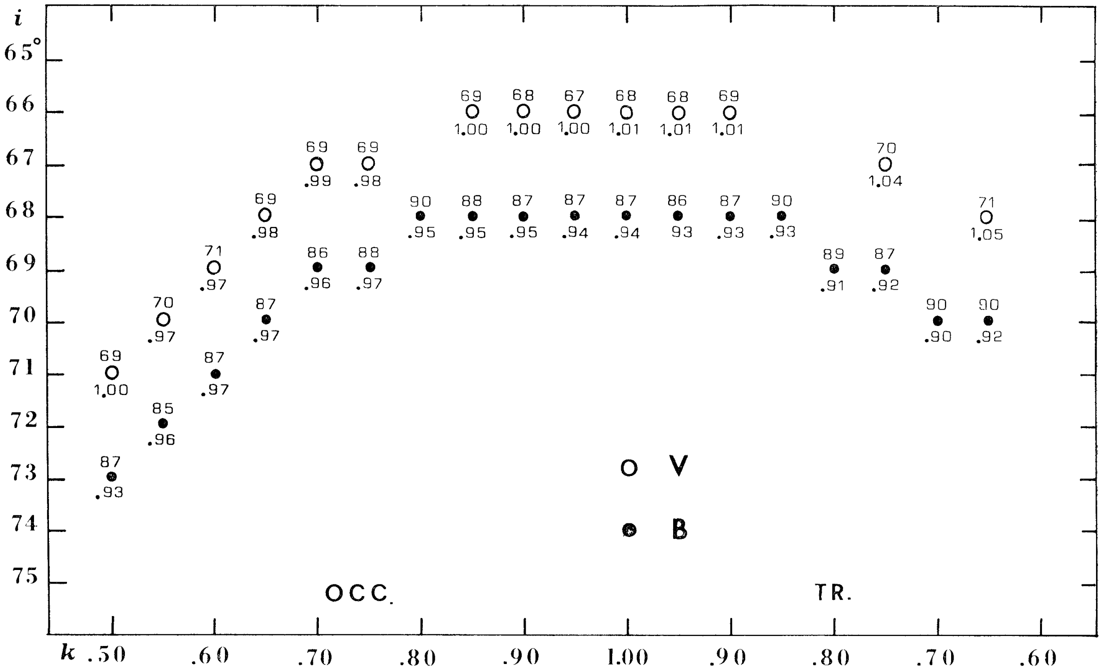

Fig. 3

Representation in the plain *i-k* of the best solutions obtained, assuming θ' = 46°. The upper number indicates the mean deviation of a normal in comparison with the computed light curve in units of 0.0001, the lower one is the value of the ratio J_s/J_g.

equal to the ratio of the observed rectified depths of the minima (1.00 and 0.95 for the *V* and the *B* light curves). The possibilities of primary eclipse being either a transit or an occultation were each investigated.

The Fig. 3 represents the solutions obtained with $\theta'_{rect} = 46°$ which we judged to be acceptable. We see that it is difficult to assess if the primary minimum is a transit or an occultation, even if in the latter hypothesis we have a better agreement with the observed ratio J_s/J_g. The system is poorly determined: in Table IV are given the estimated ranges for the elements. They represent only preliminary solutions because the computation include no allowance for perturbations which in contact systems are noteworthy.

TABLE IV

V	B
$0.50 \leq L_g \leq 0.80$	$0.48 \leq L_g \leq 0.79$
$.39 \leq r_g \leq .50$	$.38 \leq r_g \leq .50$
$.25 \leq r_s \leq .39$	$.25 \leq r_s \leq .38$
$66^\circ \leq i_{rect} \leq 71^\circ$	$68^\circ \leq i_{rect} \leq 74^\circ$
$.5 \leq k \leq 1.0$	$.5 \leq k \leq 1.0$

Recently new methods in solving the light curves of the *W UMa* systems have been proposed on the basis of the LUCY (1968) contact model. A summary of the characteristics of the model and a reference to the computing methods of the light curves is given for instance by RUCINSKI (1973). The common envelope of the binary system is described by means of three parameters: the mass ratio q, the inclination i, and the fill-out, or degree of contact, parameter $f = (C\text{-}C_2)/(C_1\text{-}C_2)$, where C_2, C_1 and C mean respectively the Jacobi constants for the outer, the inner critical common envelope and for the envelope effectively occupied by the stars. Moreover, according to Rucinski, the parameter f can be determined also if the mass ratio is unknown, as in the case of *EM Lac*. The Fourier analysis of the whole unrectified light curve gives an evaluation of f, through the coefficients A_2 and A_4. For *EM Lac* we obtain in light units:

V	$A_0 = 0.787$	$A_2 = -0.181$	$A_4 = -0.035$	
	$\pm\ 1$	2	2	*m.e.*
B	0.786	-0.182	-0.039	
	1	2	2	

According to Rucinski to these values corresponds $f = 0.8$. In other words the common envelope is rather close to the inner critical surface.

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