Astron. Astrophys. 133, 52–56 (1984)

ASTRONOMY
AND
ASTROPHYSICS

CO Aur and the double mode Cepheids

E. Antonello and L. Mantegazza

Osservatorio Astronomico di Brera, Via E. Bianchi 46, I-22055 Merate, Italy

Received August 5, accepted November 14, 1983

Summary. New photoelectric observations confirm that CO Aur is a short period, double mode Cepheid, with the period ratio indicating the excitation of first and second overtones.

The Fourier analysis of double mode Cepheid light curves confirms the importance of the phase difference ϕ_{21} for the determination of the pulsational mode of a Cepheid.

Key words: Cepheids – double mode Cepheids – RR Lyrae stars

Introduction

The double mode (or beat) Cepheids form a well known group of 11 variable stars; these stars are characterized by the simultaneous excitation of two modes, the fundamental and the first overtone modes. Recently, Mantegazza (1983, hereafter called Paper I), using previous published data, has shown that the irregular supergiant variable CO Aur is really another double mode Cepheid; it pulsates in the first and second overtones, and it is hotter than the other double mode Cepheids. Because of these unique characteristics, we needed to confirm its Cepheids-like nature by other observations.

The double mode Cepheids are an important test for the pulsation theory: for example, it is well known that, up to now, no theoretical model of a Cepheid has been found to pulsate simultaneously in more than one mode (non-linear theory). Recently, Simon and Davis (1983) have shown that the Fourier analysis of Cepheid light and radial velocity curves is a good means for a useful comparison of theory with observations; moreover, Gieren (1982) has pointed out the possible importance of the parameter ϕ_{21} (Simon and Lee, 1981) for the determination of the pulsational mode of a single mode Cepheid. In the present paper, we attempt to show that the Fourier decomposition of double mode Cepheid light curves confirms this importance.

Photometric observations

CO Aur has been observed with the photometer attached to the 102 cm reflector of the Merate Observatory from January to March, 1983. For the instrumentation and the reduction procedures see Antonello and Mantegazza (1982).

In order to test the accuracy of our measurements two comparison stars were adopted: BD+35.1308 (C1, $V=8.12$, $B-V=0.35$, $U-B=0.05$) and BD+35.1311 (C2, $V=8.69$, $B-V=0.31$, $U-B=0.25$). The sequence of the measurements was $C1-V-C1-C2-C1-V$..., so that C2 was observed with the same frequency as the variable (V). Since we are interested less in high temporal resolution than in improving the efficiency of detecting periodicities (Scargle, 1982), we have assembled the single measurements into normal points, each consisting of about five measurements.

The U, B, V observations are reported in Table 1, where the subsequent columns contain the Heliocentric Julian Day, the difference of magnitudes between C1 and V ($C1-V$) and the standard error.

Because we needed a large set of data for the period search, we favoured the B colour. The analysis of the ΔB data was accomplished by means of the least-squares power spectra method proposed by Vaniček (1971) and with the same modalities as in Paper I. The present data confirm the presence of two periodicities: $P_1=1.783$ d and $P_2=1.430$ d, which, within the uncertainty (± 0.003 d), coincide with those found in Paper I.

A least-squares solution was then computed assuming that the observed light curve is the superposition of seven sinusoids with the following frequencies: f_1 $(=1/P_1)$, $2f_1$, $3f_1$, f_2 $(=1/P_2)$, $2f_2$, f_1+f_2, f_1-f_2.

The rms residual of this solution is of 0.009 mag, which is coincident with that of the (C1–C2) data. A synthetic light curve may be computed with the formula

$$\Delta B=\Delta B_0+\sum_{i=1}^{7} A_i\cos[2\pi\nu_i(t-T_0)-\varphi_i]\,, \qquad (1)$$

where the values of ΔB, T_0, A_i, ν_i, φ_i are those reported in Table 2. In Fig. 1 we show, from top to bottom, the ΔB magnitudes of ($C1-V$) phased with P_1, the ΔB magnitudes of (C1–C2) phased with P_1, the ΔB magnitudes of ($C1-V$) rectified for the presence of the secondary wave and the first order coupling terms and phased with P_1, the ΔB magnitudes of ($C1-V$) rectified for the presence of the primary wave and the first order coupling terms and phased with P_2.

Fourier parameters for double mode Cepheids

Simon and Lee (1981) have represented the Cepheid light curves by means of the formula

$$m=A_0+\sum_i A_i\cos[i\omega(t-T_0)+\varphi_i]\,, \qquad i=1,2,\ldots, \qquad (2)$$

where m is the magnitude, A_i and φ_i are the amplitude and the phase of each Fourier component, ω is the pulsational frequency,

---

Send offprint requests to: E. Antonello

Table 1. Photometric observations of CO Aur

J. D. 2445000.+	ΔU	σ
355.493	0.064	0.006
0.537	0.046	0.005
357.531	−0.097	0.001
358.497	−0.315	0.004
0.552	−0.226	0.007
0.591	−0.170	0.007
359.272	−0.145	0.002
0.312	−0.189	0.008
369.264	−0.319	0.002
385.341	0.281	0.004
0.372	0.224	0.003
0.401	0.172	0.001
390.302	−0.422	0.002
0.333	−0.416	0.006

2445000.+	ΔB	σ
347.255	−0.049	0.004
0.264	−0.051	0.002
0.346	−0.111	0.003
354.559	−0.116	0.005
355.510	0.308	0.003
0.549	0.285	0.005
0.572	0.283	0.009
357.516	0.171	0.002
0.559	0.136	0.003
0.589	0.110	0.005
358.484	−0.069	0.004
0.539	0.015	0.003
0.579	0.058	0.003
0.604	0.093	0.003
359.255	0.141	0.004
0.299	0.095	0.004
0.358	0.057	0.007
0.398	0.029	0.007
369.244	−0.060	0.003
0.273	−0.036	0.003
0.590	0.281	0.003
385.320	−0.040	0.005
0.351	0.001	0.002
0.381	0.051	0.003
0.424	0.119	0.011
389.317	0.263	0.002
0.338	0.247	0.002
389.361	0.241	0.002
0.388	0.232	0.003
0.413	0.225	0.004
0.438	0.203	0.008
0.459	0.193	0.002
0.484	0.187	0.003
390.290	−0.124	0.004
0.323	−0.120	0.003
0.358	−0.121	0.003
0.373	−0.124	0.004
0.396	−0.125	0.003
0.423	−0.122	0.007
0.447	−0.114	0.004
0.478	−0.101	0.004
400.444	0.068	0.004
419.306	0.049	0.006
0.316	0.091	0.005
0.336	0.111	0.004
0.356	0.125	0.006
0.376	0.157	0.003
0.395	0.161	0.003
420.307	−0.055	0.004
0.328	−0.068	0.003

J. D. 2445000.+	ΔV	σ
354.543	0.210	0.003
355.525	0.506	0.006
0.561	0.507	0.005
357.502	0.437	0.003
0.545	0.404	0.003
0.575	0.392	0.002
358.513	0.269	0.015
0.565	0.311	0.008
0.617	0.374	0.012
359.286	0.386	0.001
0.338	0.331	0.003
0.379	0.332	0.007
369.254	0.251	0.001
385.330	0.270	0.005
0.362	0.301	0.003
0.391	0.343	0.002
389.328	0.488	0.002
0.349	0.473	0.002
389.374	0.475	0.004
0.402	0.472	0.004
0.424	0.454	0.002
0.448	0.432	0.005
0.471	0.437	0.004
390.313	0.205	0.001
0.342	0.213	0.003
0.362	0.209	0.007
0.385	0.208	0.002
0.409	0.205	0.006
0.435	0.205	0.003
0.462	0.213	0.003
419.326	0.365	0.003
0.346	0.374	0.004
0.366	0.393	0.010
0.385	0.403	0.005
420.318	0.261	0.005
0.340	0.252	0.003

and t is the time. These authors have defined the phase difference ϕ_{21} as

$$\phi_{21}=\varphi_2-2\varphi_1$$

to be independent of both time translations and the basic frequency ω. According to Gieren's suggestion (1982), the phase difference for short period Cepheids pulsating in the first overtone should be greater than that for short period Cepheids pulsating in the fundamental mode. Indeed, Simon and Teays (1982) have shown that, for field RR Lyrae stars, there is such a difference between RRab stars (fundamental mode) and RRc stars (probable first overtone).

We have considered the Fourier parameters of double mode Cepheid light curves in order to verify the difference of ϕ_{21} among the fundamental and the overtone modes; indeed, in this case the pulsation modes are well identified.

The data for eight double mode Cepheids have been taken from Stobie and Balona (1979), whereas for U TrA and TU Cas the data of Jansen (1962; *V* colour) and Worley and Eggen (1957), respectively, were analysed. Faulkner's results (1977a, b) have not been used because he analysed the brightness values instead of the magnitude values. The parameters for CO Aur have been derived from the analysis of the observations of Smak (1964; *V* colour).

For a useful comparison of the phase difference ϕ_{21} with that of Simon and Lee (1981), we have used the following formula

$$m = A_0 + \sum_{ij} A_{ij} \cos[(i\omega_0 + j\omega_1)(t - T_0) + \varphi_{ij}],$$
$$i = 0, \pm 1, \ldots \quad j = 0, 1, \ldots, \tag{3}$$

where ω_0 and ω_1 are the fundamental and first overtone mode frequencies, respectively. As Stobie and Balona (1979) have used a different formula from (3), their phase data were reduced to a form similar to (3) in order to have a homogeneous set of phase differences. The results of the analysis for CO Aur, TU Cas, and U TrA are reported in Table 3.

For each star only the results concerning the fundamental mode frequency and its harmonic and first overtone frequency and

Table 2. Parameters of the synthetic light curve of CO Aur

i	Term	ν_i	A_i	ϕ_i
1	f_1	0.5610	0.241	4.096
2	$2f_1$	1.1219	0.031	0.840
3	$3f_1$	1.6829	0.010	4.780
4	f_2	0.6995	0.060	2.321
5	$2f_2$	1.3991	0.006	0.075
6	f_1+f_2	1.2605	0.009	4.561
7	f_2-f_1	0.1385	0.008	1.811

$T_0 = 2445381.045$
$\Delta B_0 = 0.043$

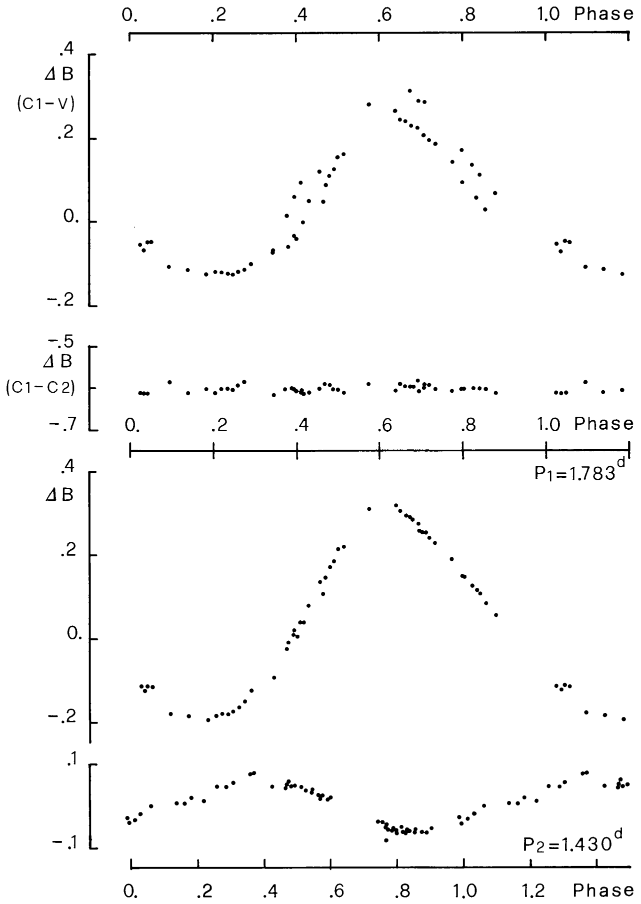

Fig. 1. From top to bottom: the ΔB magnitudes of (C1 − V) phased with P_1; the ΔB magnitudes of (C1–C2) phased with P_1; the ΔB magnitudes of (C1 − V) rectified for the presence of the secondary wave and the first order coupling terms and phased with P_1; the ΔB magnitudes of (C1 − V) rectified for the presence of the primary wave and the first order coupling terms and phased with P_2

Table 3. Fourier parameters for CO Aur, TU Cas, and U TrA

		CO Aur $T_0 = 2437677.876$ $A_0 = 7.708$		TU Cas $T_0 = 2435210.902$ $A_0 = 7.720$		U TrA $T_0 = 2436770.896$ $A_0 = 0.194$	
i	j	A_{ij}	φ_{ij}	A_{ij}	φ_{ij}	A_{ij}	φ_{ij}
1	0	0.170	3.664	0.299	5.370	0.251	1.074
		12	52	9	21	7	21
0	1	0.056	4.511	0.174	3.055	0.102	1.743
		12	139	12	44	8	49
2	0	0.030	5.161	0.125	2.283	0.088	6.204
		11	261	8	46	8	66
1	1	0.030	5.376	0.086	6.236	0.069	0.612
		13	286	10	78	7	76
−1	1	0.006*	0.873*	0.058	1.486	0.032	1.765
		12	1345	12	87	8	189
0	2	0.005*	1.961*	0.076	5.000	0.014	1.499
		13	1711	8	80	7	480
3	0	0.018	4.889	0.052	5.433	0.038	5.397
		12	445	7	99	8	141
0	3	–	–	0.013	3.666	0.005*	1.094*
				6	335	7	1022

Note. The asterisks indicate data which are not significant. The integer numbers in A_{ij} and φ_{ij} columns are the errors (thousandths of magnitude and radian, respectively)

1984A&A...133...52A

Table 4. Phase differences ϕ_{21} for double mode Cepheids and the RR Lyrae stars AQ Leo. P_0 and P_1 indicate the fundamental and first overtone mode periods

	P_0	ϕ_{21}	P_1	ϕ_{21}	Source
CO Aur	$(P_1)1^{\mathrm{d}}.78$	4.10	$(P_2)1^{\mathrm{d}}.42$	4.70	
TU Cas	2.14	4.11	1.52	5.17	
U TrA	2.57	4.05	1.82	4.30	
VX Pup	3.01	4.18	2.14	4.33	S
AP Vel	3.13	4.30	2.20	4.11	S
BK Cen	3.17	4.35	2.22	(4.63)	S
UZ Cen	3.33	4.18	2.35	4.48	S
Y Car	3.64	4.16	2.56	4.60	S
AX Vel	3.67	(5.20)	2.59	4.45	S
GZ Car	4.16	4.19	2.93	(4.76)	S
AQ Leo	0.55	3.76	0.41	4.77	J

S: Stobie and Balona (1979)
J: Jerzykiewicz et al. (1982)

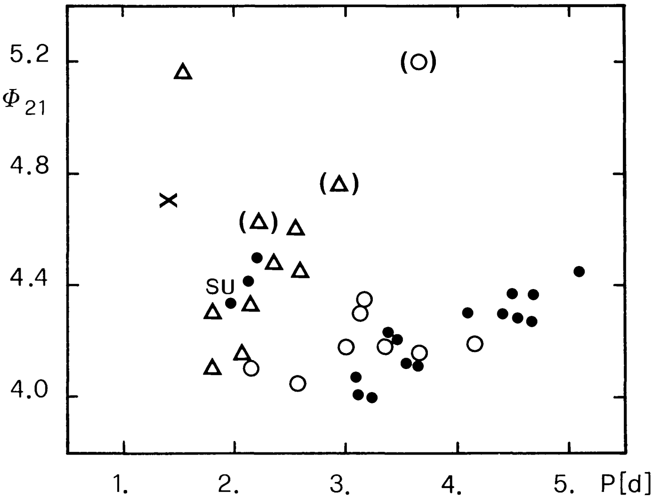

Fig. 2. The phase difference ϕ_{21} versus period for the single mode Cepheids (filled circles), the fundamental mode (open circles) and the first overtone (triangles) of double mode Cepheids; the uncertain data are between brackets. The cross indicates the second overtone of CO Aur (however, see text), and SU the star SU Cas

its harmonic were taken into account (for CO Aur, the first and the second overtone frequency, respectively); obviously, the coupling terms data were ignored, and, in any case, this is a limit to the validity of the verification. The phase differences for double mode Cepheids are reported in Table 4. Some of the data (those between brackets) are uncertain owing to the small amplitude of the harmonic, which is comparable to the respective error; in particular, the phase difference for the second overtone of CO Aur may not be significant because the respective curve seems to be a sinusoid. For the uncertain data, the mean error in ϕ_{21} is 0.72, while for the other cases, the mean error is 0.23.

Figure 2 shows ϕ_{21} versus the period for double mode Cepheids and single mode, short period Cepheids (Simon and Lee, 1981). One may see the different grouping of data for the first overtone and the fundamental mode, and one may note in particular the position of the single mode Cepheid SU Cas; this star, according to Gieren (1982), is pulsating in the first overtone.

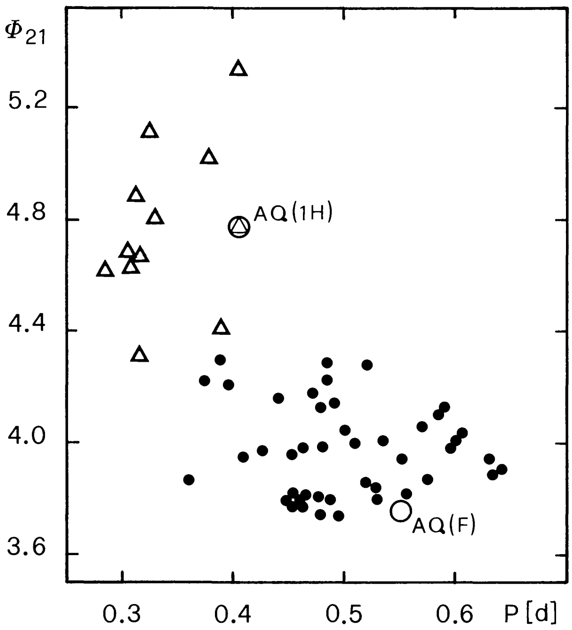

Fig. 3. The phase difference ϕ_{21} versus period for RRab stars (filled circles), RRc stars (triangles), the fundamental mode (F) of AQ Leo (open circle) and the first overtone ($1H$) of AQ Leo (triangle into a circle)

A similar result (see Fig. 3) has been obtained for the double mode RR Lyrae star AQ Leo and the field RR Lyrae stars (Simon and Teays, 1982). AQ Leo is pulsating in the fundamental and first overtone modes and the analysis of its light curve has been published recently by Jerzykiewicz et al. (1982).

Conclusion

New photoelectric observations confirm that CO Aur is a short period, double mode Cepheid. The period ratio indicates that it is pulsating in the first and second overtones.

The result of the analysis of the light curves of double mode Cepheids, compared with that of the single mode, short period Cepheids, seems to confirm the importance of the phase difference ϕ_{21} for the determination of the pulsational mode of the latter variables. A similar result has been obtained for the double mode RR Lyr star AQ Leo and the single mode RRab and RRc stars.

In conclusion, these results seem to indicate that the shape of the variation curve of a short period Cepheid (or RR Lyrae star), defined mathematically be means of Fourier parameters, is different according to the pulsational mode, while there is a similarity (or a regular trend) of the parameter ϕ_{21} for stars pulsating in the same mode.

References

Antonello, E., Mantegazza, L.: 1982, *Astron. Astrophys. Suppl.* **49**, 709
Faulkner, D.J.: 1977a, *Astrophys. J.* **216**, 49
Faulkner, D.J.: 1977b, *Astrophys. J.* **218**, 209

Gieren, W.: 1982, *Publ. Astron. Soc. Pacific* **94**, 960

Jansen, A.G.: 1962, *Bull. Astron. Inst. Neth.* **16**, 141

Jerzykiewicz, M., Schult, R.H., Wenzel, W.: 1982, *Acta Astron.* **32**, 357

Mantegazza, L.: 1983, *Astron. Astrophys.* **118**, 321

Scargle, J.D.: 1982, *Astrophys. J.* **263**, 835

Simon, N.R. Davis, C.G.: 1983, *Astrophys. J.* **266**, 787

Simon, N.R., Lee, A.S.: 1981, *Astrophys. J.* **248**, 291

Simon, N.R., Teays, T.J.: 1982, *Astrophys. J.* **261**, 586

Smak, J.: 1964, *Publ. Astron. Soc. Pacific* **76**, 40

Stobie, R.S., Balona, L.A.: 1979, *Monthly Notices Roy Astron. Soc.* **189**, 627

Vaniček, P.: 1971, *Astrophys. Space. Sci.* **12**, 10

Worley, C.E., Eggen, O.J.: 1957, *Astron. J.* **62**, 104