

Il doppio senso della scienza (G. Ghirlanda – Brera 30/11/2016)

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J. Bell Burnell racconta la storia della scoperta delle pulsar (con commento sul Nobel) <http://www.bigeart.org/vol1no1/burnell.htm>

Archivio digitalizzato dei lavori di Einstein:
<http://einsteinpapers.press.princeton.edu>

“Einstein, Eddington and the Eclipse” (Peter Coles)
<http://arxiv.org/pdf/astro-ph/0102462v1.pdf>

“Einstein” per i quaderni delle scienze (dicembre 1998) Le Scienze

Le Scienze, Numero Speciale “Einstein” Novembre 2015.

“Einstein, dalla relativita’ alle onde gravitazionali”, Grandangolo (in edicola con il Corriere) inizio Novembre.



IL DOPPIO SENSO della SCIENZA

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Ricerca astronomica/fisica



SCOPERTA

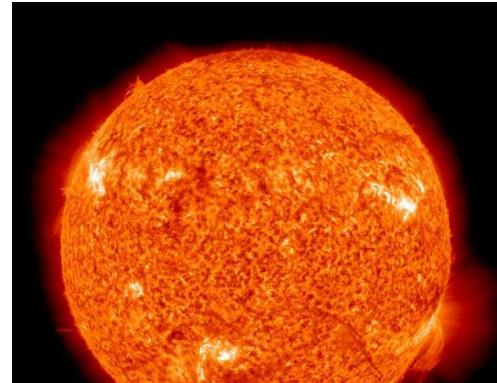
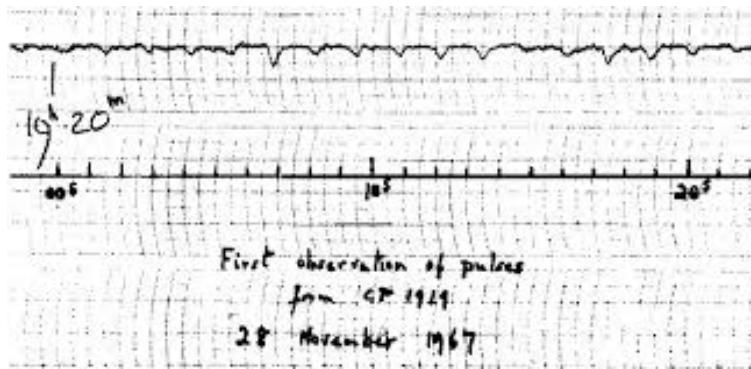


$$\frac{\partial \theta}{\partial \theta} M T(\xi) = \frac{\partial}{\partial \theta} \int_{\mathbb{R}_+} T(x) f(x, \theta) dx = \int_{\mathbb{R}_+} \frac{\partial}{\partial \theta} f(x, \theta) dx,$$
$$\frac{\partial}{\partial a} \ln f_{a, \sigma^2}(\xi_1) = \frac{(\xi_1 - a)}{\sigma^2} f_{a, \sigma^2}(\xi_1) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\xi_1 - a)^2}{2\sigma^2}},$$
$$\int_{\mathbb{R}_+} T(x) \cdot \frac{\partial}{\partial \theta} f(x, \theta) dx = M \left(T(\xi), \frac{\partial}{\partial \theta} \ln f(\xi, \theta) \right) \int_{\mathbb{R}_+} T(x) f(x, \theta) dx,$$
$$\int_{\mathbb{R}_+} T(x) \cdot \left(\frac{\partial}{\partial \theta} \ln f(x, \theta) \right) f(x, \theta) dx = \int_{\mathbb{R}_+} T(x) \begin{pmatrix} \frac{\partial}{\partial \theta} \ln f(x, \theta) \\ f(x, \theta) \end{pmatrix} dx,$$
$$\frac{\partial}{\partial \theta} M T(\xi) = \frac{\partial}{\partial \theta} \int_{\mathbb{R}_+} T(x) f(x, \theta) dx = \int_{\mathbb{R}_+} \frac{\partial}{\partial \theta} f(x, \theta) dx,$$
$$= \exp \left(\frac{(\xi_1 - a)^2}{2\sigma^2} \right) \frac{\partial}{\partial a} \ln f_{a, \sigma^2}(\xi_1).$$



Little Green Men

$$\frac{\partial \theta}{\partial \alpha} \ln f_{\alpha,\sigma}(z_0) = \frac{1}{\sigma^2} \int_{-\infty}^{z_0} \frac{f(z)}{f(z_0)} \frac{dz}{z-z_0}$$
$$\frac{\partial \theta}{\partial \alpha} \ln f_{\alpha,\sigma}(z_0) = \frac{(z_0 - z)}{\sigma^2} \int_{-\infty}^{z_0} \frac{f(z)}{f(z_0)} \frac{dz}{z-z_0}$$
$$\int T(x) \frac{\partial}{\partial \theta} f_{\alpha,\sigma}(x) dx = \lambda \left(\frac{1}{\sigma^2} \int_{-\infty}^{z_0} \frac{f(z)}{f(z_0)} \frac{dz}{z-z_0} \right)$$
$$\int T(x) \left(\frac{1}{\sigma^2} \int_{-\infty}^{z_0} \frac{f(z)}{f(z_0)} \frac{dz}{z-z_0} \right) dx = \lambda \left(\frac{1}{\sigma^2} \int_{-\infty}^{z_0} \frac{f(z)}{f(z_0)} \frac{dz}{z-z_0} \right)$$
$$\frac{\partial}{\partial \theta} M(T(x)) = \frac{2}{\sigma^2} \int_{-\infty}^{z_0} \frac{|T(x)|^2}{f(x)^2} dx - \frac{2}{\sigma^2} \int_{-\infty}^{z_0} \frac{|T(x)|^2}{f(z_0)^2} dz$$



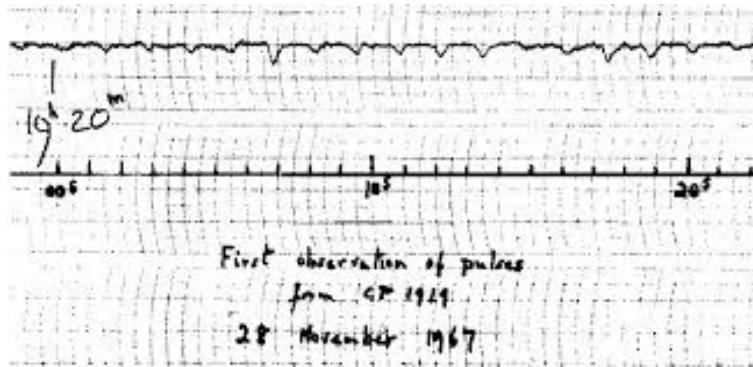
1.4 g cm⁻³
[130 g cm⁻³]



Pulsar



$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{\partial}{\partial \theta} \int_{\Omega} f(\theta) \phi_{\theta}(x) dx + \int_{\Omega} \frac{\partial f(\theta)}{\partial \theta} \phi_{\theta}(x) dx$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\omega}(t_0) = \frac{(t_0 - t_0)}{\sigma^2} \int_{\Omega} \phi_{\alpha}(x) dx - \frac{1}{\sigma^2} \int_{\Omega} \phi_{\alpha}^2(x) dx$$
$$\int_{\Omega} f(x) \frac{\partial}{\partial \theta} f(x) dx = \int_{\Omega} f(x) \frac{\partial}{\partial \theta} \phi_{\theta}(x) dx + \int_{\Omega} f(x) \frac{\partial \phi_{\theta}}{\partial \theta} dx$$
$$\int_{\Omega} f(x) \left(\frac{\partial}{\partial \theta} \phi_{\theta}(x) \right) f(x) dx = \int_{\Omega} f(x) \left(\frac{\partial}{\partial \theta} \phi_{\theta}(x) \right) \phi_{\theta}(x) dx + \int_{\Omega} f(x) \phi_{\theta}(x) \frac{\partial \phi_{\theta}}{\partial \theta} dx$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{\partial}{\partial \theta} \int_{\Omega} f(\theta) \phi_{\theta}(x) dx + \int_{\Omega} \frac{\partial f(\theta)}{\partial \theta} \phi_{\theta}(x) dx$$

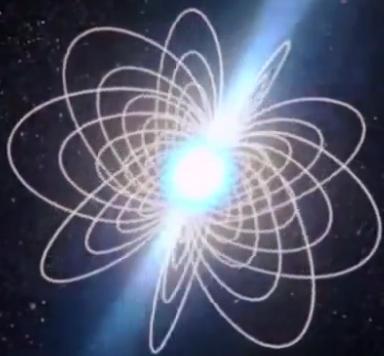


Neutron Star

R=30 Km

M~1.5 Masse solari

B=10¹² Gauss



H HISTORY.COM

Le pulsazioni
osservate ... oggetti
velocemente rotanti
che emettono un
fascio di luce
(orientato “di
sbieco”)



Pulsar: rotazione

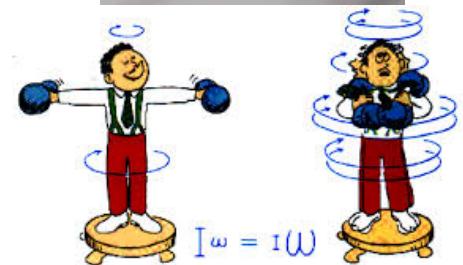
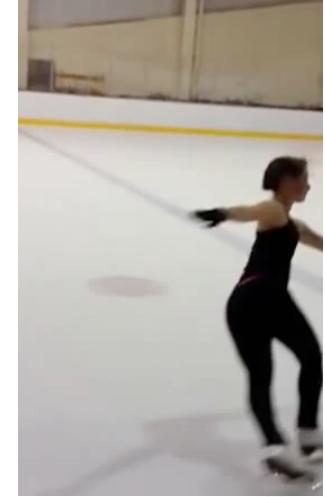
$$\frac{\partial \theta}{\partial t} \delta T(t) = \frac{\partial}{\partial \theta} \int_{\Omega} f(\theta) \delta T(t) d\theta = \int_{\Omega} f'(\theta) \delta T(t) d\theta$$
$$\frac{\partial}{\partial \theta} \ln f_{\theta, \text{rot}}(t_0) = \frac{(t_0 - t)}{\sigma^2} \int_{\Omega} f'(\theta) \delta T(t) d\theta$$
$$\int_{\Omega} f(t) \frac{\partial}{\partial t} f_{\theta, \text{rot}}(t_0) dt = \lambda \left(\frac{t_0 - t}{\sigma^2} \right) \int_{\Omega} f'(\theta) \delta T(t) d\theta$$
$$\int_{\Omega} f(t) \left(\frac{\partial}{\partial t} f_{\theta, \text{rot}}(t_0) \right) f_{\theta, \text{rot}}(t_0) dt = \lambda \left(\frac{t_0 - t}{\sigma^2} \right)^2 \int_{\Omega} f'(\theta) \delta T(t) d\theta$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\sigma^2} \int_{\Omega} f(t)^2 f_{\theta, \text{rot}}(t_0)^2 \left(\frac{t_0 - t}{\sigma^2} \right)^2 \int_{\Omega} f'(\theta) \delta T(t) d\theta$$

PERCHE' RUOTANO COSI' VELOCEMENTE?



$$\mathcal{L} = I \Omega$$

Momento angolare SI CONSERVA





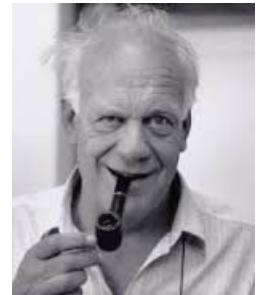
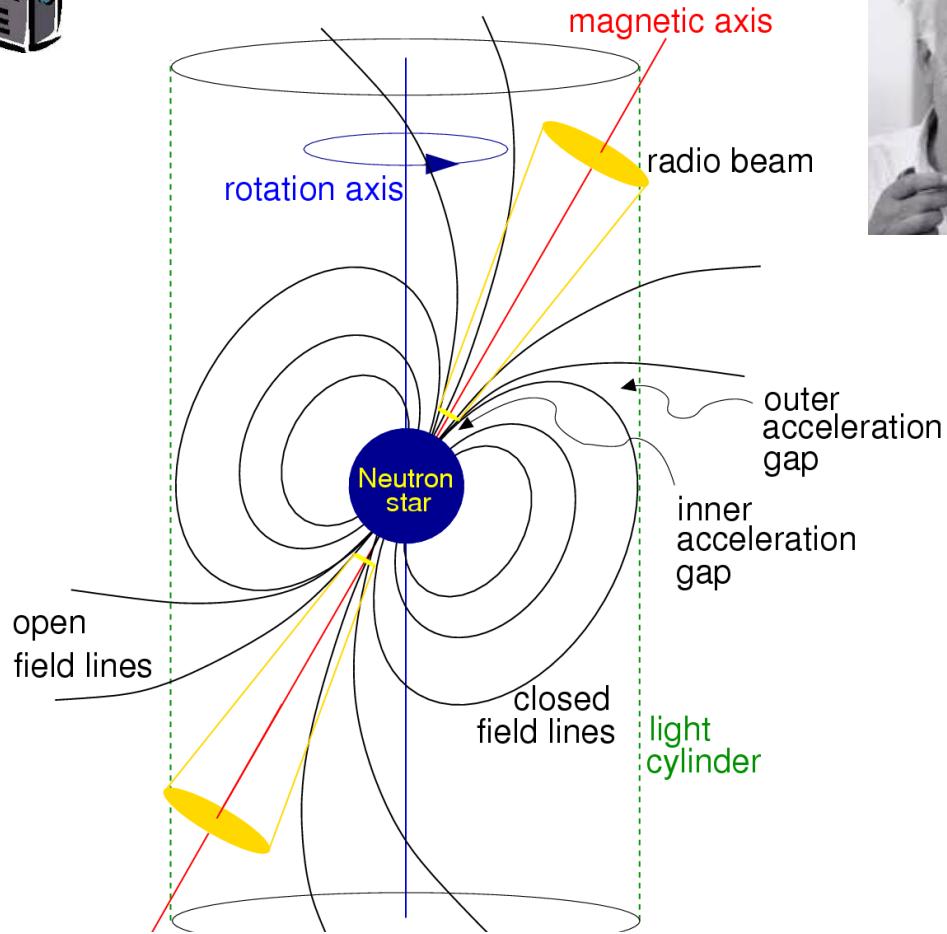
Pulsar: emissione (I)

$$\frac{\partial \theta}{\partial t} \ln T(\zeta) = -\frac{2}{\sigma^2} \int_{\nu_0}^{\nu} f(\nu') d\nu' + \frac{2}{\sigma^2} \int_{\nu_0}^{\nu} \frac{f'(\nu')}{f(\nu')} d\nu'$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\sigma^2}(\zeta_0) = \frac{(2-\alpha)}{\sigma^2} \int_{\nu_0}^{\nu_0} \frac{f'(\nu')}{f(\nu')} d\nu'$$
$$\int T(x) \frac{d}{dx} f(x) dx = \lambda \left(\int T(x) dx \right)^2$$
$$\int T(x) \left(\frac{d}{dx} f(x) \right) dx = \lambda \left(\int T(x) dx \right)^2$$
$$\frac{\partial}{\partial \theta} M(T(\zeta)) = \frac{2}{\sigma^2} \int_{\nu_0}^{\nu} f(\nu') d\nu' + \frac{2}{\sigma^2} \int_{\nu_0}^{\nu} \frac{f'(\nu')}{f(\nu')} d\nu'$$

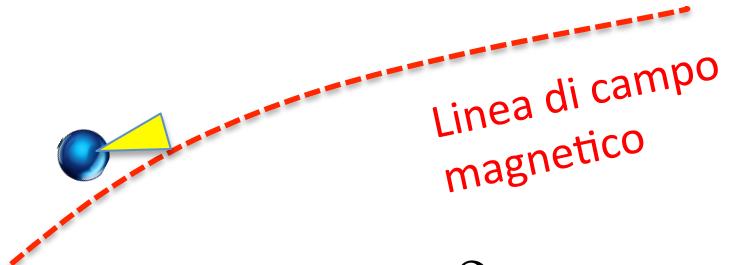
Che cosa produce la luce che vediamo?



CLASSE 1: Rotation Powered Pulsars



Una carica accelerata produce luce



$$L = \frac{2}{3} \frac{m_e r_e}{c} a^2$$



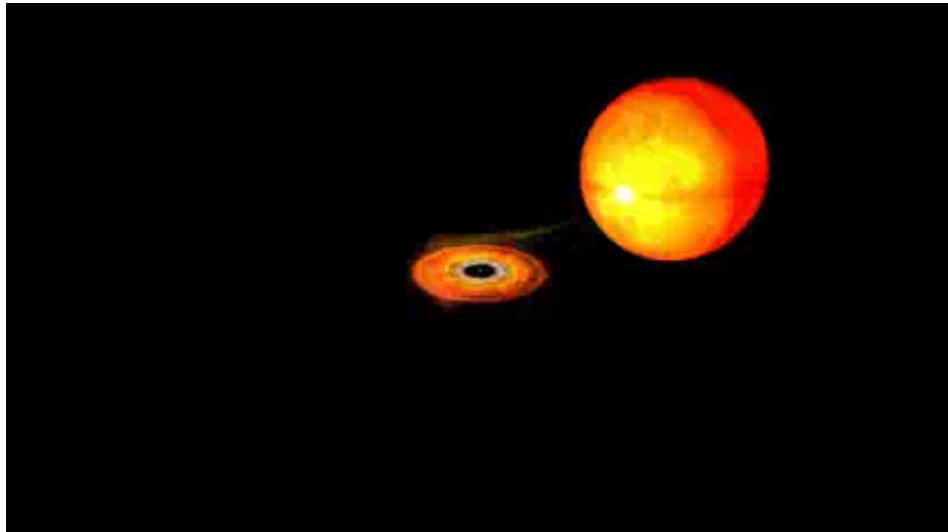
Pulsar: emissione (II)

$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{\partial}{\partial t} \int_{\Omega} f(\theta) \phi_{\theta}(t) \phi_{\theta}(t)^T \frac{1}{2} \nabla^2 \phi_{\theta}(t) \nabla^2 \phi_{\theta}(t)^T dt$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\sigma^2}(z_i) = \frac{(z_i - \mu)}{\sigma^2} \int_{\Omega} \phi_{\theta}(t) \phi_{\theta}(t)^T \frac{1}{2} \nabla^2 \phi_{\theta}(t) \nabla^2 \phi_{\theta}(t)^T dt$$
$$\int_{\Omega} f(z) \frac{\partial}{\partial \theta} f(z) \phi_{\theta}(z) \times \left(\frac{\partial z}{\partial \theta} \right) \phi_{\theta}(z)^T dz = \int_{\Omega} f(z) \frac{\partial}{\partial \theta} f(z) \phi_{\theta}(z) \phi_{\theta}(z)^T dz$$
$$\int_{\Omega} f(z) \left(\frac{\partial}{\partial \theta} \phi_{\theta}(z) \right) \phi_{\theta}(z)^T dz = \int_{\Omega} f(z) \phi_{\theta}(z) \phi_{\theta}(z)^T dz$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{\partial}{\partial \theta} \int_{\Omega} f(\theta) \phi_{\theta}(t) \phi_{\theta}(t)^T \frac{1}{2} \nabla^2 \phi_{\theta}(t) \nabla^2 \phi_{\theta}(t)^T dt$$

Che cosa produce la luce che vediamo?

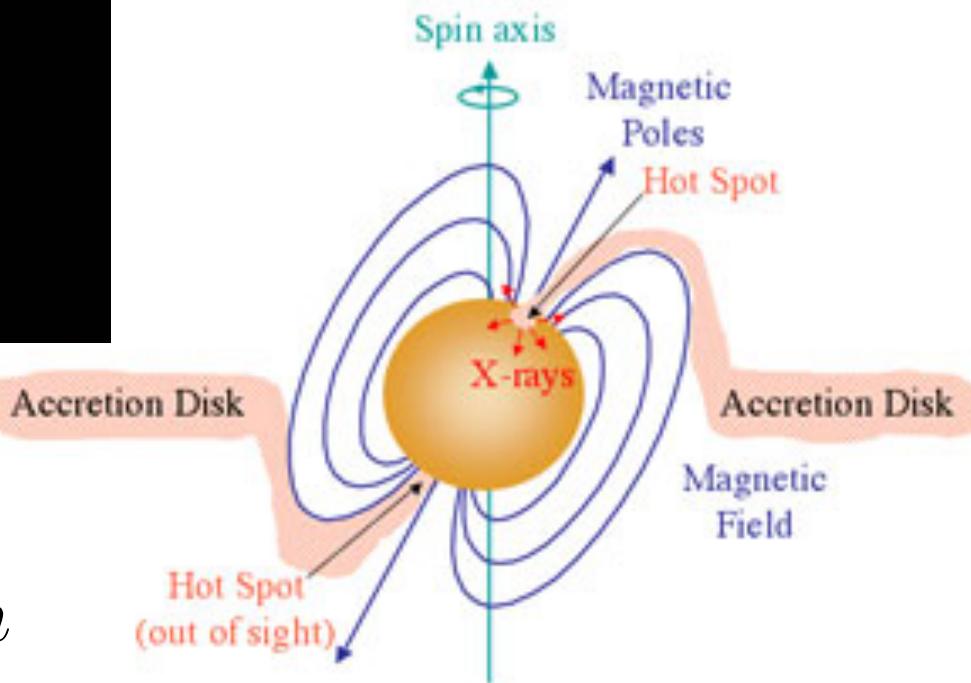


CLASSE 2: Accretion Powered Pulsars



$$U = \frac{G M}{R} m$$

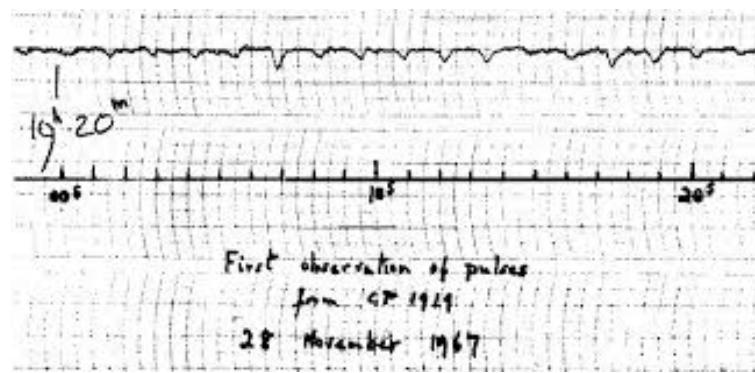
$$L = \frac{G M}{R} \dot{m}$$





Pulsar... come e' finita?

$$\frac{\partial \theta}{\partial t} M(T) = \frac{d}{dt} \int_{\Omega} \rho(t) \left(\frac{\partial \theta}{\partial t} \right)^2 d\Omega$$
$$\frac{\partial}{\partial \alpha} \ln f_{\text{prior}}(\alpha) = \frac{(a - \bar{a})}{\sigma^2} \int_{\Omega} \rho(t) \frac{\partial \theta}{\partial t} d\Omega$$
$$\int T(x) \frac{\partial}{\partial x} f(x) dx = \int_{\Omega} \rho(t) \frac{\partial \theta}{\partial t} d\Omega$$
$$\int T(x) \left(\frac{\partial}{\partial x} f(x) \right) f(x) dx = \int_{\Omega} \rho(t) \left(\frac{\partial \theta}{\partial t} \right)^2 d\Omega$$
$$\frac{\partial}{\partial \theta} M(T) = \frac{2}{\sigma^2} \int_{\Omega} \rho(t) \left(\frac{\partial \theta}{\partial t} \right)^2 d\Omega$$

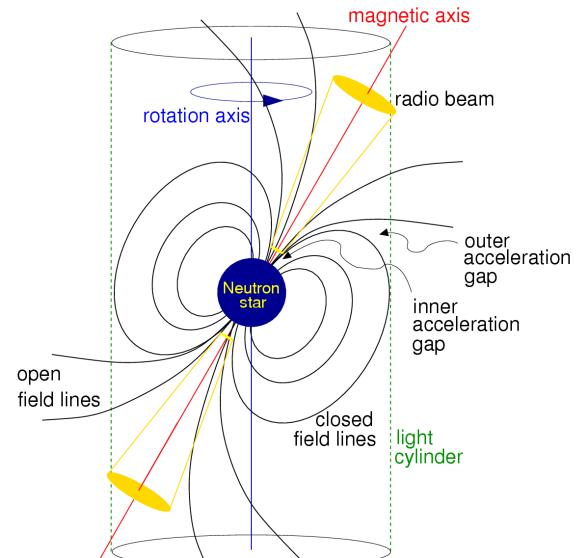


Observation of a Rapidly Pulsating Radio Source

A. HEWISH, S. J. BELL, J. D. H. PILKINGTON, P. F. SCOTT & R. A. COLLINS

Mullard Radio Astronomy Observatory, Cavendish Laboratory,
University of Cambridge

Unusual signals from pulsating radio sources have been recorded at the Mullard Radio Astronomy Observatory. The radiation seems to come from local objects within the galaxy, and may be associated with oscillations of white dwarf or neutron stars.



"... Finally, I am not myself upset about it - after all, I am in good company, am I not!"



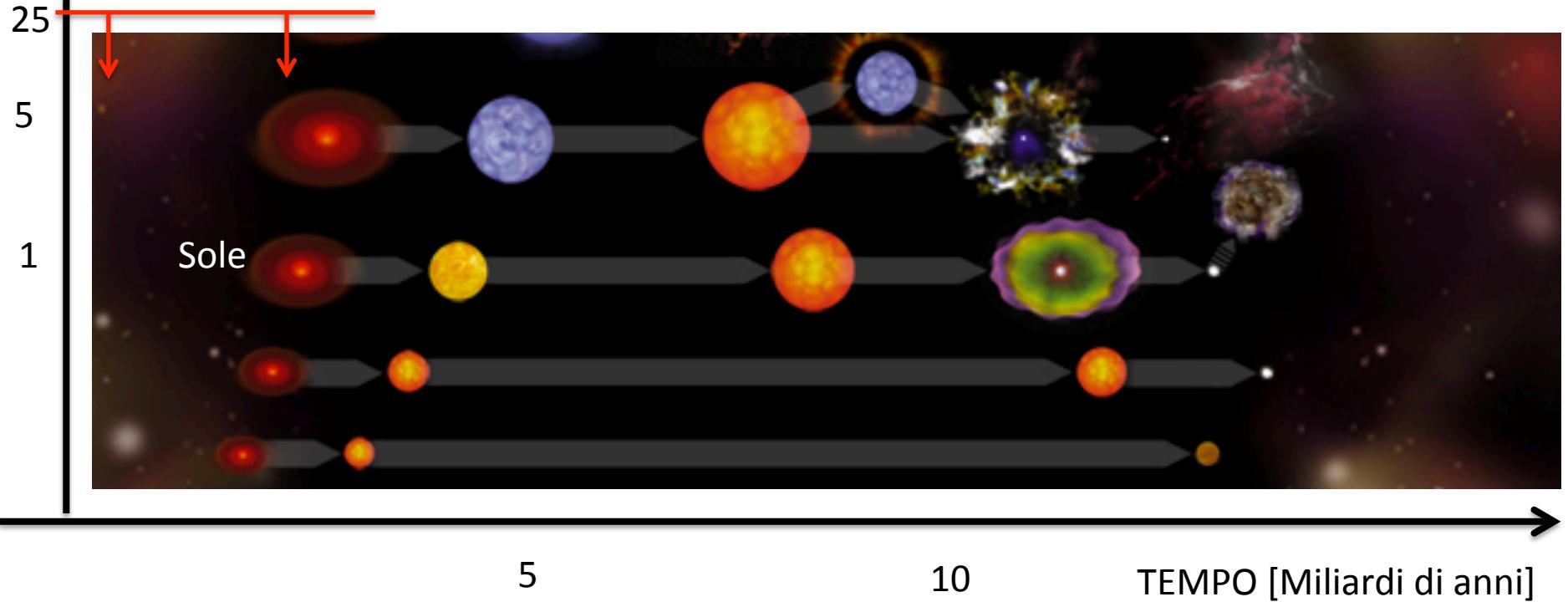
Quando finisce il carburante

$$\frac{\partial \theta}{\partial \alpha} \ln f_{\alpha,\sigma}(z_0) = \frac{\partial}{\partial \alpha} \int_{-\infty}^{z_0} p(v) \ln f_{\alpha,\sigma}(v) dv = \int_{-\infty}^{z_0} p(v) \frac{\partial}{\partial \alpha} \ln f_{\alpha,\sigma}(v) dv =$$
$$\int_{-\infty}^{z_0} p(v) \frac{\partial}{\partial \alpha} f_{\alpha,\sigma}(v) / f_{\alpha,\sigma}(v) dv = \int_{-\infty}^{z_0} p(v) f'_{\alpha,\sigma}(v) / f_{\alpha,\sigma}(v) dv =$$
$$\int_{-\infty}^{z_0} p(v) \left(\frac{f'_\alpha(v)}{f_\alpha(v)} + \frac{f_\alpha'(v)}{f_\alpha(v)} \right) dv = \int_{-\infty}^{z_0} p(v) \left(\frac{f'_\alpha(v)}{f_\alpha(v)} + \frac{1}{\sigma^2} \right) dv =$$
$$\frac{\partial}{\partial \theta} M(T(\theta)) = \frac{\partial}{\partial \theta} \int_{-\infty}^{z_0} p(v) f_{\alpha,\sigma}(v) dv = \int_{-\infty}^{z_0} p(v) \frac{\partial}{\partial \theta} f_{\alpha,\sigma}(v) dv =$$

Baade W, Zwicky F (1934) On super-novae. *Proc Natl Acad Sci USA* **20**(5):254–259

Massa/ M_{sole}

“With all reserve we advance the view that a super-nova represents the transition of an ordinary star into a neutron star, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density.”

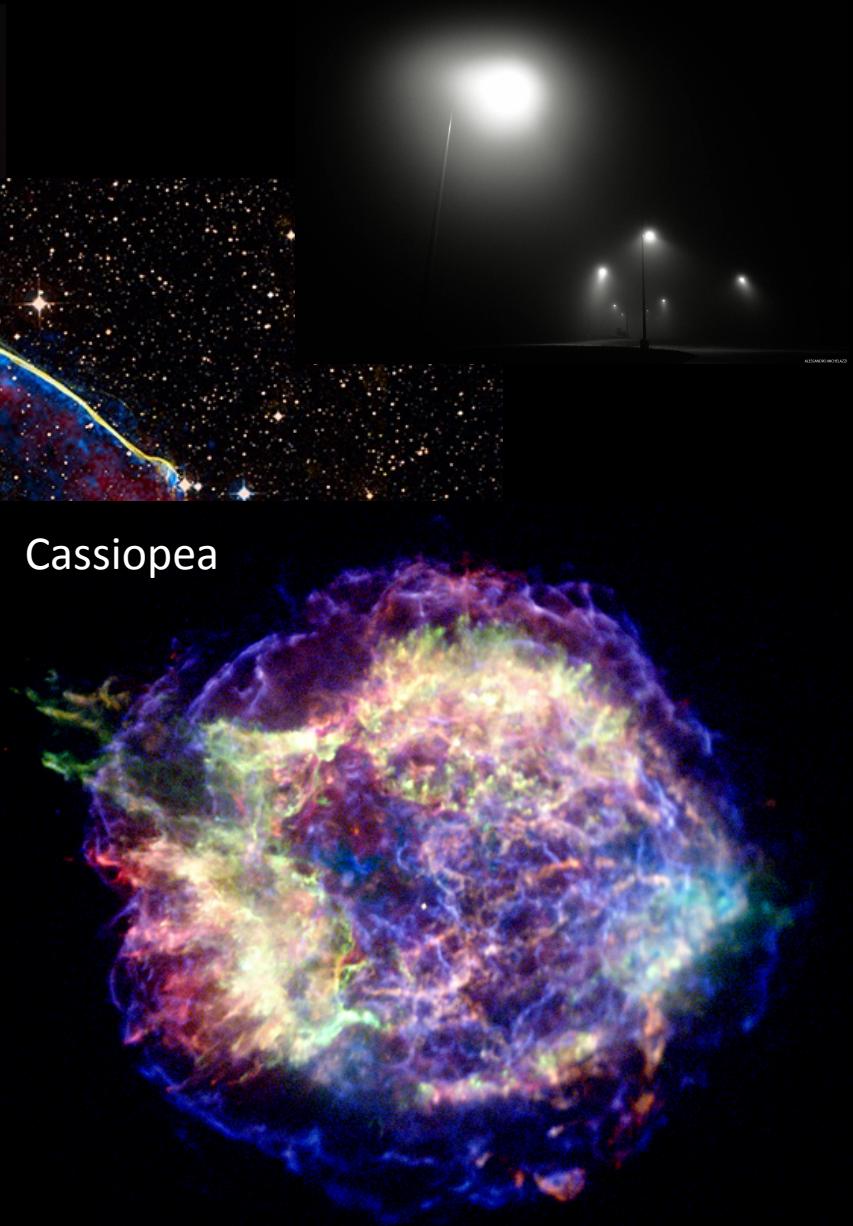




“Polvere” di stelle

$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{\partial}{\partial t} \int_{\Omega} f(\theta)(t) \phi_{\theta}(t) dx = \int_{\Omega} f'(\theta)(t) \phi_{\theta}(t) dx + \int_{\Omega} f(\theta)(t) \phi'_{\theta}(t) dx \\ = \int_{\Omega} f'(x)(t) \phi_{\theta}(x,t) dx + \lambda \left(\int_{\Omega} \frac{f(x)}{\phi_{\theta}(x,t)} dx - \int_{\Omega} \frac{f(x)}{\phi_{\theta}(x,t)} \phi'_{\theta}(x,t) dx \right) \\ = \int_{\Omega} f'(x)(t) \phi_{\theta}(x,t) dx + \lambda \left(\int_{\Omega} f(x) \phi_{\theta}(x,t) dx - \int_{\Omega} f(x) \phi'_{\theta}(x,t) dx \right) \\ = \int_{\Omega} f'(x)(t) \phi_{\theta}(x,t) dx + \lambda \left(\int_{\Omega} f(x) \phi_{\theta}(x,t) dx - \int_{\Omega} f(x) \phi_{\theta}(x,t) \phi'_{\theta}(x,t) dx \right) \\ = \int_{\Omega} f'(x)(t) \phi_{\theta}(x,t) dx + \lambda \left(\int_{\Omega} f(x) \phi_{\theta}(x,t) dx - \int_{\Omega} f(x) \phi_{\theta}(x,t) \phi'_{\theta}(x,t) dx \right) \\ = \int_{\Omega} f'(x)(t) \phi_{\theta}(x,t) dx + \lambda \left(\int_{\Omega} f(x) \phi_{\theta}(x,t) dx - \int_{\Omega} f(x) \phi_{\theta}(x,t) \phi'_{\theta}(x,t) dx \right)$$

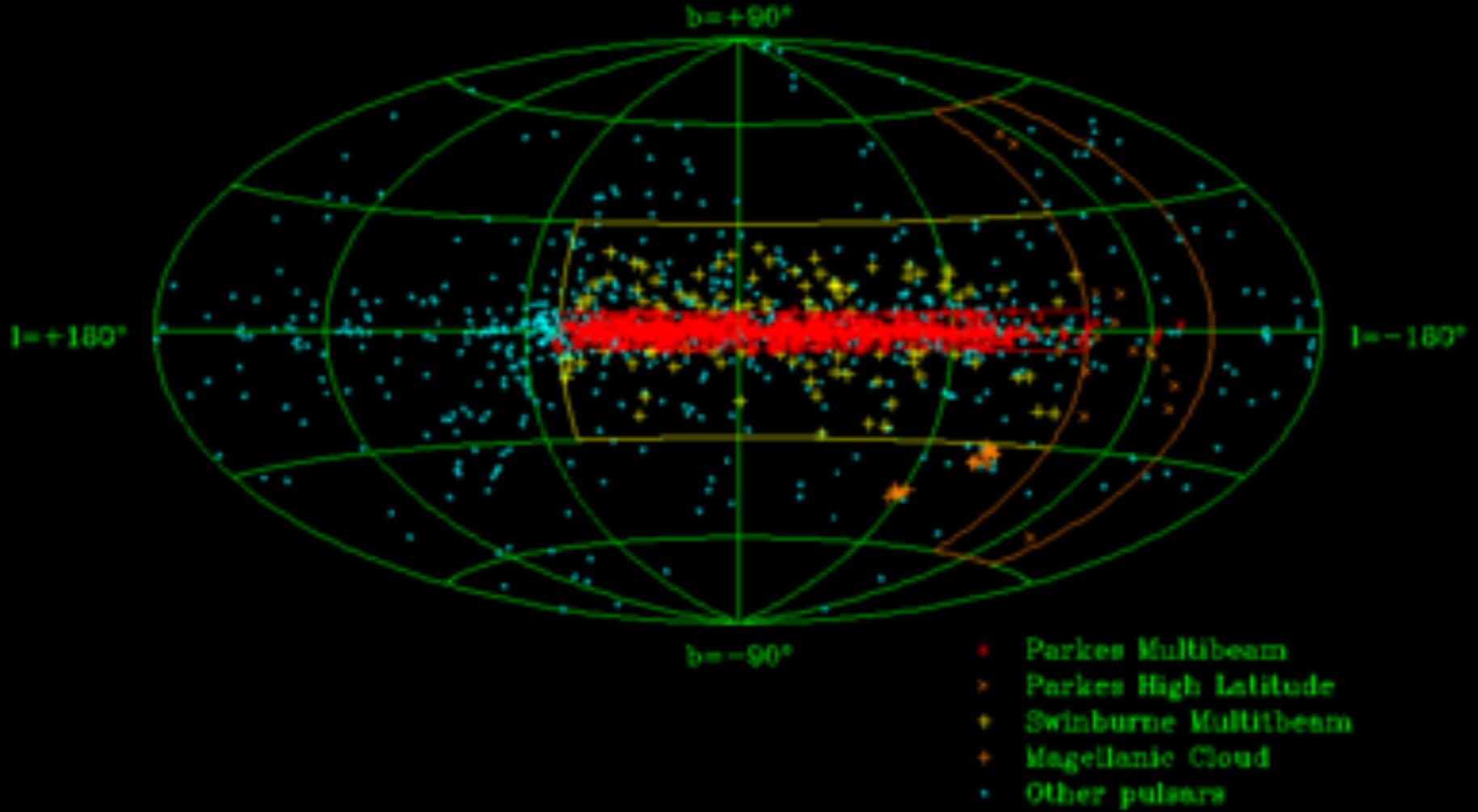
CRAB NEBULA





Pulsar (quante e dove)

$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{2}{\sigma^2} \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t - \frac{1}{\sigma^2} \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\sigma}(t_0) = \frac{(t_0 - t)}{\sigma^2} \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t$$
$$\int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t = \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t$$
$$\int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t = \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\sigma^2} \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t - \frac{1}{\sigma^2} \int_{\mathbb{R}^d} f(\nu_t)(\nu_t) d\nu_t$$





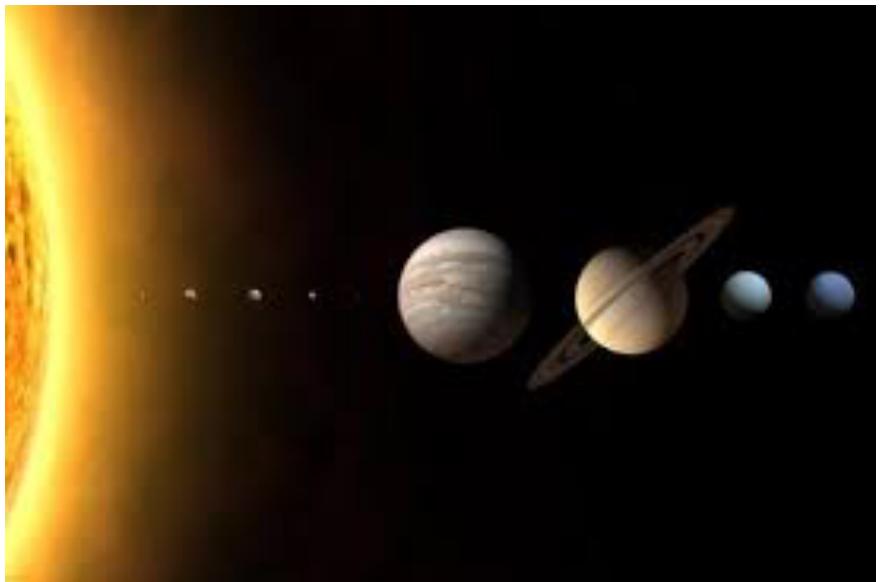
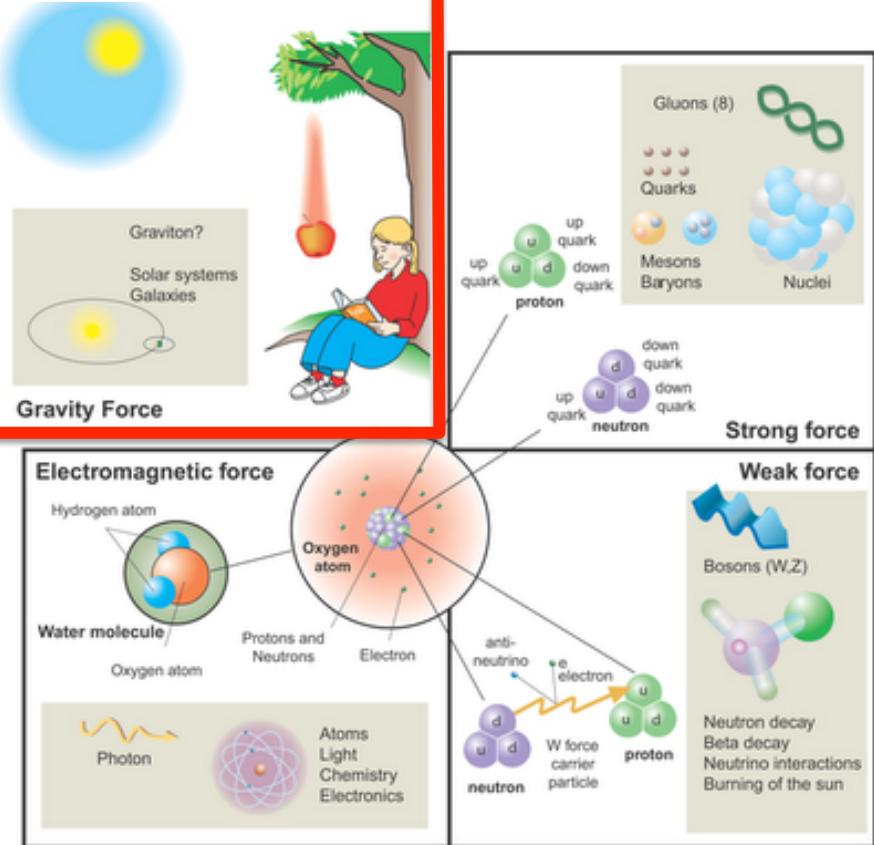
La teoria: Newton

$$\frac{\partial \theta}{\partial t} \text{RT}(t) = \frac{\partial}{\partial t} \int_{\Omega} f(t)(\theta(t)) \frac{\partial u}{\partial x} dx + \int_{\Omega} f(t) \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} dx \\ \frac{\partial}{\partial t} \ln f_{\alpha,\beta,\gamma}(t_0) = \frac{(t_0-t)}{\sigma^2} \int_{\Omega} f(t_0) \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} dx \\ \int T(t) \frac{\partial}{\partial t} f_{\alpha,\beta,\gamma}(t_0) dt = \left(\int_{\Omega} f(t_0) \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} dx \right)^2 \\ \int T(t) \left(\frac{\partial}{\partial t} f_{\alpha,\beta,\gamma}(t_0) \right) dt = \left(\int_{\Omega} f(t_0) \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} dx \right)^2 \\ \frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\sigma^2} \int_{\Omega} f(t) \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} \left(\int_{\Omega} f(t) \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} dx \right)^2$$



- 1) Principio di inerzia
- 2) $F = ma$
- 3) Azione e reazione

$$F = G \frac{M_1 M_2}{R^2}$$





La teoria: verso la relativita' generale

$$\frac{\partial \theta}{\partial t} \Delta T(t) = \frac{\partial}{\partial t} \int_{-\infty}^t f(v_i)(v_i) dv_i = \int_{-\infty}^t f'(v_i)(v_i) dv_i$$
$$\frac{\partial}{\partial \alpha} \ln \int_{-\infty}^t f(v_i)(v_i) dv_i = \frac{(t-a)}{a^2} \int_{-\infty}^t f'(v_i)(v_i) dv_i$$
$$\int_{-\infty}^t f(v_i) \frac{d}{dt} f(v_i)(v_i) dv_i = \int_{-\infty}^t f'(v_i)(v_i) dv_i$$
$$\int_{-\infty}^t f(v_i) \left(\frac{d}{dt} f(v_i)(v_i) + f(v_i) f'(v_i)(v_i) \right) dv_i = \int_{-\infty}^t f'(v_i)(v_i) dv_i$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{a^2} \int_{-\infty}^t f(v_i)(v_i) dv_i - \frac{2}{a^2} \int_{-\infty}^t f'(v_i)(v_i) dv_i$$

Marzo – Giugno 1905 ANNUS MIRABILIS

1. “Il primo ha per oggetto la radiazione e le proprietà energetiche della luce ed è decisamente rivoluzionario (La luce è fatta di particelle fotoni ...)
2. Il secondo è una determinazione delle dimensioni effettive degli atomi
3. Il terzo dimostra che corpi delle dimensioni di 1/1000 di mm in sospensione nei liquidi sono soggetti a un moto casuale generato dall’agitazione termica (moto browniano)
4. Il quarto è soltanto un abbozzo iniziale ed è un’elettrodinamica dei corpi in movimento che fa un ricorso ad una modifica della teoria dello spazio tempo (relatività)”



14/03/1879



La teoria: verso la relatività generale

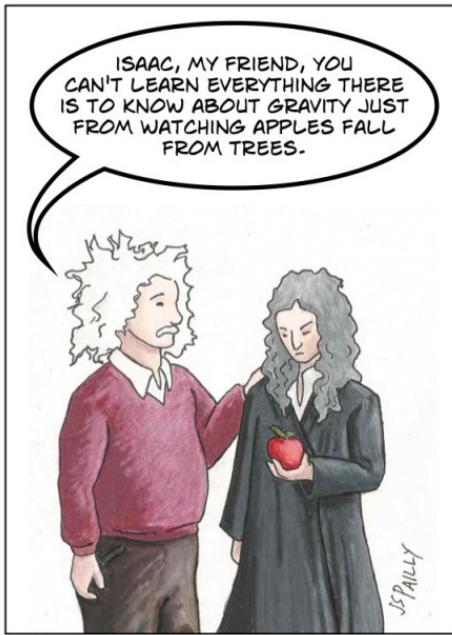
$$\frac{\partial \theta}{\partial t} \delta T(t) = -\frac{\partial}{\partial t} \int_{\Gamma_0}^t \int_{\Omega} \left(\frac{\partial u}{\partial t} \right)_t \left(\frac{\partial u}{\partial t} \right)_t \frac{1}{2} \left(\frac{\partial u}{\partial t} \right)^2 dt$$

$$\frac{\partial}{\partial t} \ln f_{\mu,\nu,\sigma}(t_0) = \frac{(g_{\mu\nu})_{\sigma}}{c^2} \int_{\Gamma_0}^t \int_{\Omega} \frac{1}{2} \left(\frac{\partial u}{\partial t} \right)^2 dt$$

$$\int_{\Gamma_0}^t \int_{\Omega} \left(\frac{\partial u}{\partial t} \right)_t \left(\frac{\partial u}{\partial t} \right)_t \times \left(\frac{\partial u}{\partial t} \right)_{\sigma} \frac{1}{2} \left(\frac{\partial u}{\partial t} \right)^2 dt$$

$$\int_{\Gamma_0}^t \int_{\Omega} \left(\frac{\partial u}{\partial t} \right)_t \left(\frac{\partial u}{\partial t} \right)_t \left(\frac{\partial u}{\partial t} \right)_{\sigma} \frac{1}{2} \left(\frac{\partial u}{\partial t} \right)^2 dt$$

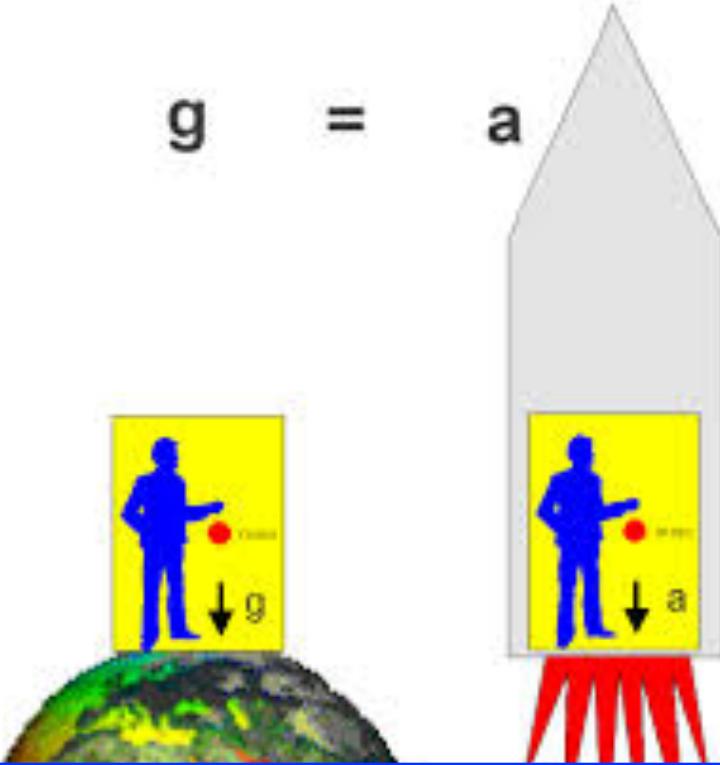
$$\frac{\partial}{\partial \theta} \delta T(t) = -\frac{2}{c^2} \int_{\Gamma_0}^t \int_{\Omega} \left(\frac{\partial u}{\partial t} \right)_t \left(\frac{\partial u}{\partial t} \right)_{\sigma} \frac{1}{2} \left(\frac{\partial u}{\partial t} \right)^2 dt$$



1907... “il pensiero piu’ felice della mia vita” = “se una persona cade liberamente non sentirà il proprio peso”

Esperimento Mentale [Gedankenexperiment]

$$g = a$$



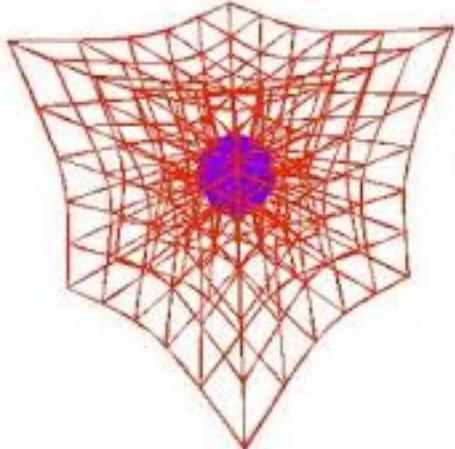
Principio di Equivalenza: gli effetti locali della gravità (**g**) e dell’accelerazione (**a**) sono equivalenti. Quindi **gravità** e **accelerazione** sono manifestazione dello stesso fenomeno



La teoria: massa = curvatura spaziotempo

$$\frac{\partial \theta}{\partial t} \delta T(t) = \frac{\partial}{\partial t} \int_{\Omega} f(\theta) \phi_{\alpha} \phi_{\beta} \frac{\partial \theta}{\partial t} \delta T(t) \phi_{\alpha} \phi_{\beta}$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\beta,\gamma}(t_0) = \frac{(t_0 - t)}{t^2} \int_{\Omega} \frac{\partial \theta}{\partial t} \delta T(t) \phi_{\alpha} \phi_{\beta} \phi_{\gamma}$$
$$\int_{\Omega} f(t) \frac{\partial}{\partial t} \delta T(t) \phi_{\alpha} \phi_{\beta} \phi_{\gamma} = \left(\int_{\Omega} \frac{\partial \theta}{\partial t} \delta T(t) \phi_{\alpha} \phi_{\beta} \phi_{\gamma} \right)^2$$
$$\int_{\Omega} f(t) \left(\frac{\partial \theta}{\partial t} \delta T(t) \phi_{\alpha} \phi_{\beta} \phi_{\gamma} \right)^2$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{t^2} \int_{\Omega} f(t) \phi_{\alpha} \phi_{\beta} \phi_{\gamma} \int_{\Omega} \frac{\partial \theta}{\partial t} \delta T(t) \phi_{\alpha} \phi_{\beta} \phi_{\gamma}$$

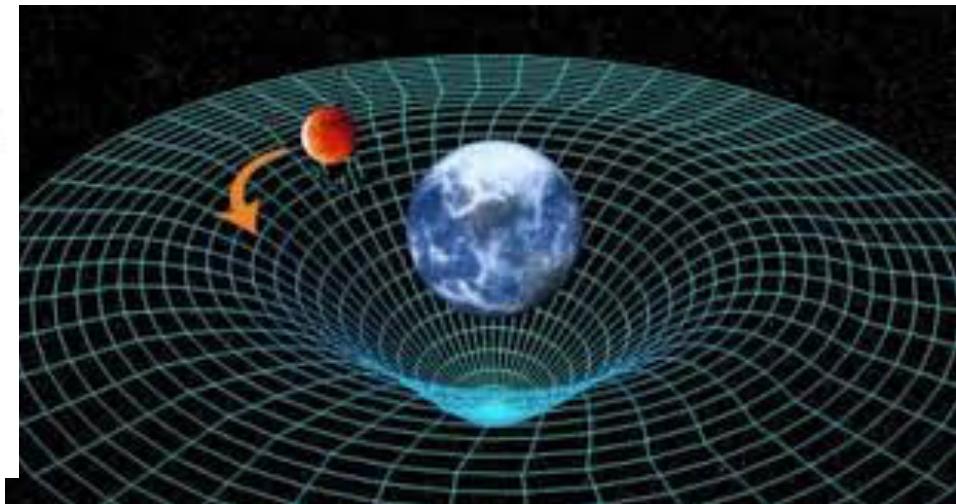
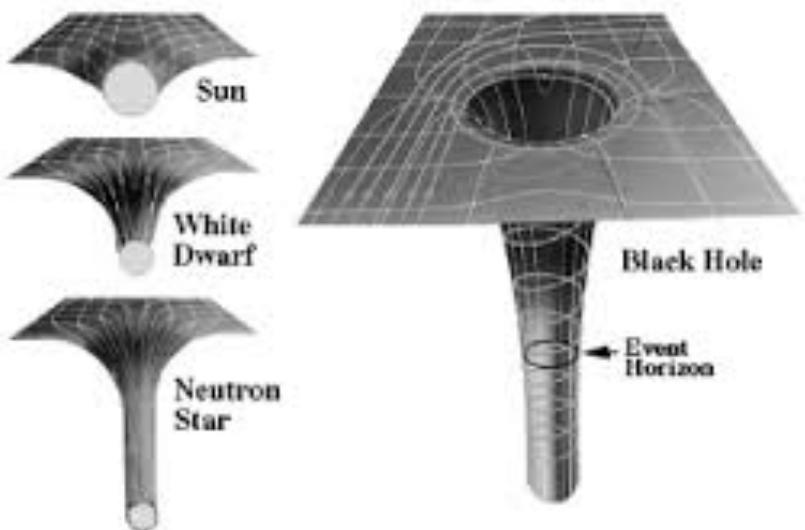
1907 ... 1915 verso la relativita' generale



Massa

Curvatura dello
spazio tempo

Movimento
dei corpi





La teoria: equazione di campo di Einstein

$$\frac{\partial \theta}{\partial t} \delta T(t) = \frac{\partial}{\partial t} \int_{\Omega} f(v) |v|^{p-2} v \, dx + \int_{\Omega} f'(v) |v|^{p-2} v \, dx \\ \frac{\partial}{\partial \alpha} \ln f_{\alpha,\omega}(t_0) - \frac{(q-\alpha)}{\sigma^2} \int_{\Omega} f(v) |v|^{p-2} v \, dx \\ \int_{\Omega} f(x) \frac{\partial}{\partial \theta} f_{\alpha,\omega}(x) \, dx + \lambda \left(\int_{\Omega} \frac{f(x)}{|v|^{p-2}} \, dx \right)^2 \\ \int_{\Omega} f(x) \left(\frac{\partial}{\partial \theta} f_{\alpha,\omega}(x) \right) f_{\alpha,\omega}(x) \, dx \\ \frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\sigma^2} \int_{\Omega} f(v) |v|^{p-2} v \, dx + \frac{2}{\sigma^2} \int_{\Omega} f(v) |v|^{p-2} v \, dx$$

1907 ... 1915 verso la relativita' generale

25 Novembre 1915
(4° lezione Accademia
Prussiana delle Scienze)

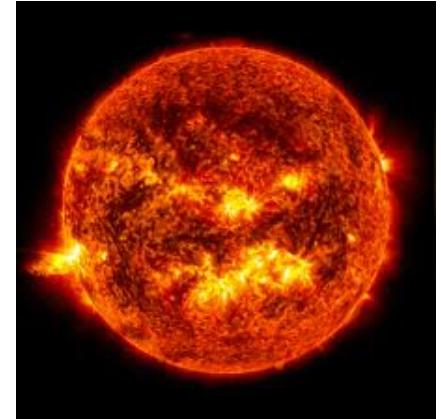
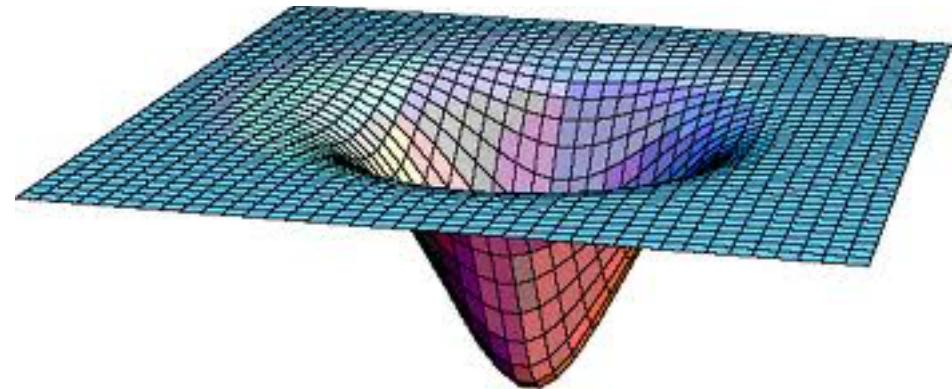
"Le equazioni di campo della gravitazione"

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$$

Massa

Curvatura dello
spazio tempo

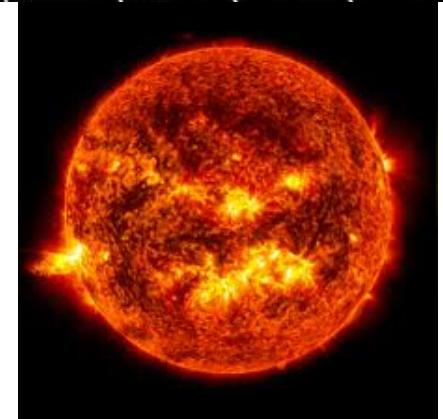
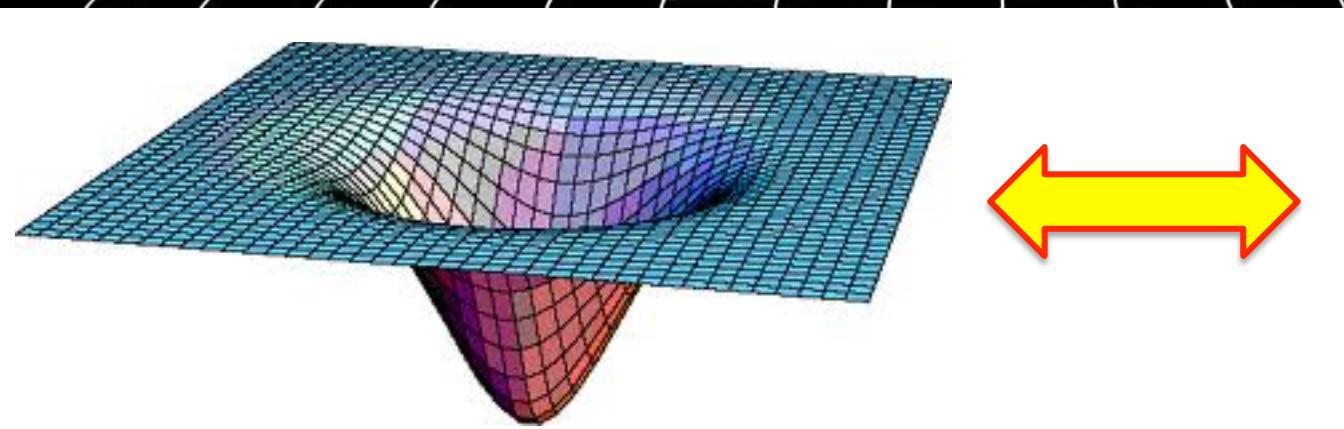
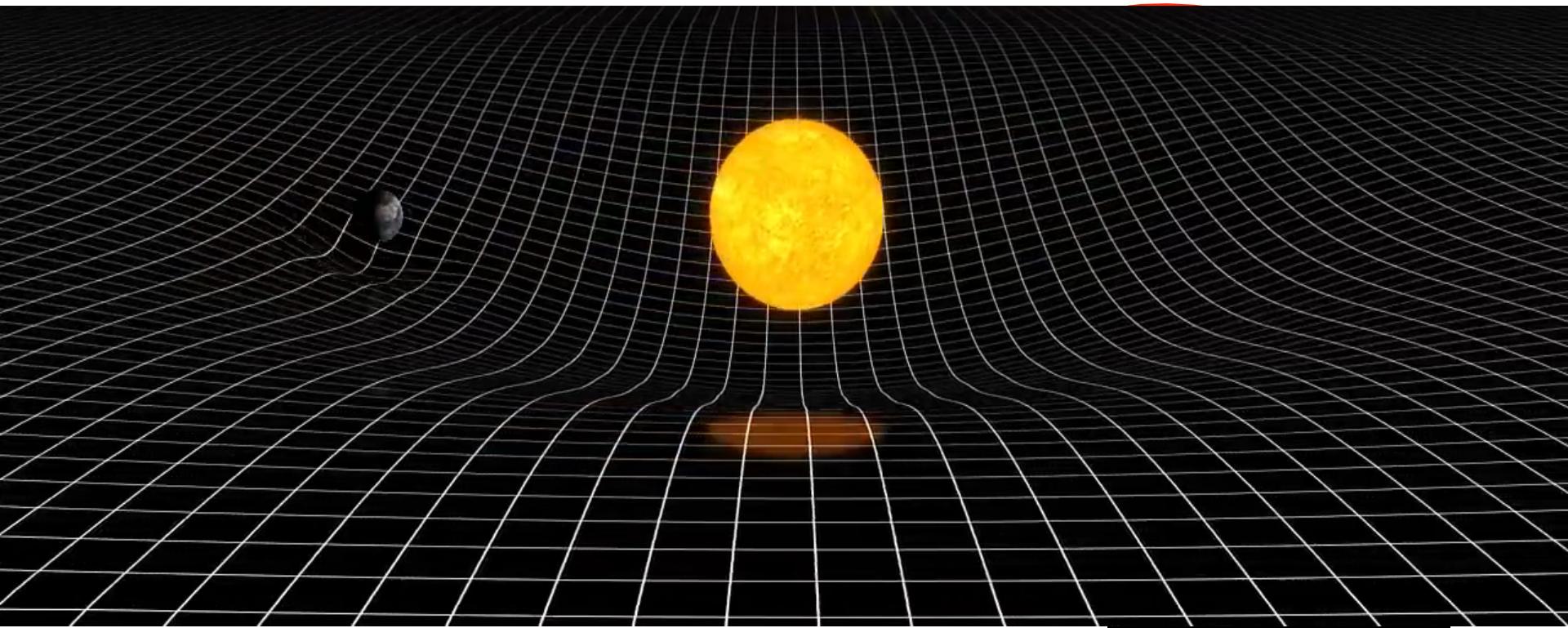
Movimento
dei corpi





La teoria: GR

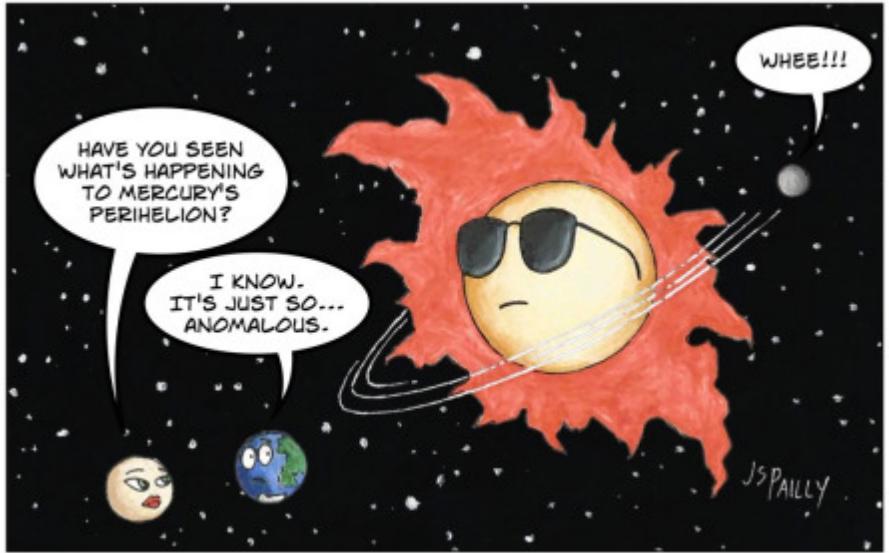
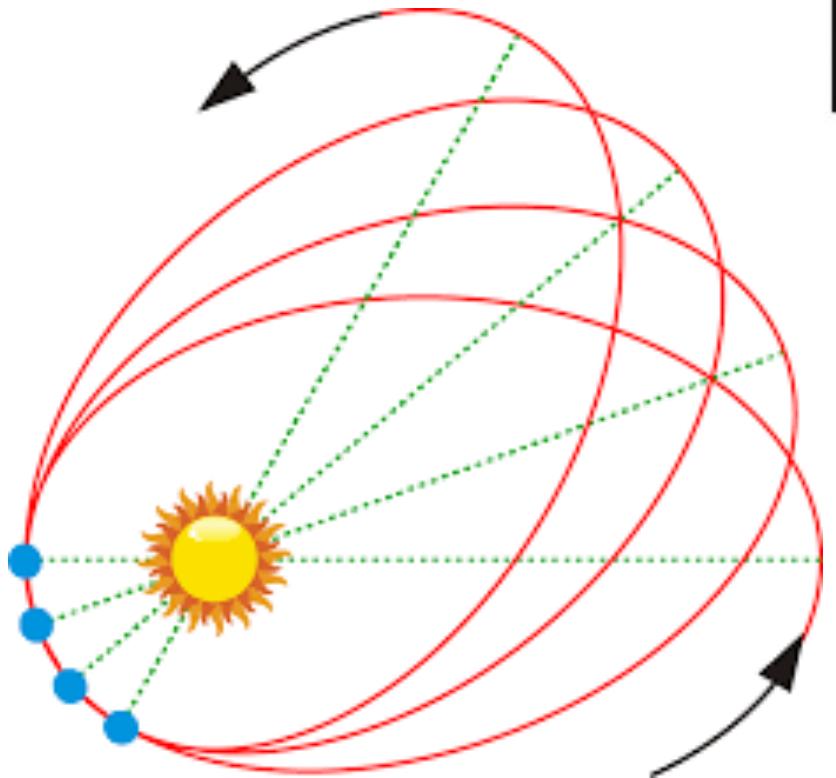
$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{\partial}{\partial t} \int_{\Omega} f(v) |v|_{H^1(\Omega)}^{-2} v \cdot \nabla v \, dx \\ \frac{\partial}{\partial t} \ln f_{\alpha,\omega}(t_0) = \frac{(q-2)}{c_0^2} \int_{\Omega} |v|_{H^1(\Omega)}^{-2} v \cdot \nabla v \, dx \\ \int_{\Omega} T(x) \frac{\partial}{\partial t} f_{\alpha,\omega}(x,t) + \lambda \left(\int_{\Omega} \frac{1}{|v|_{H^1(\Omega)}^2} \right)^2 \, dx \\ \int_{\Omega} T(x) \left(\frac{\partial}{\partial t} u_{\alpha,\omega}(x,t) \right) f_{\alpha,\omega}(x,t) \, dx \\ \frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\theta} \int_{\Omega} f(t) |v|_{H^1(\Omega)}^{-2} \int_{\Omega} \frac{1}{|v|_{H^1(\Omega)}^2} \, dx$$



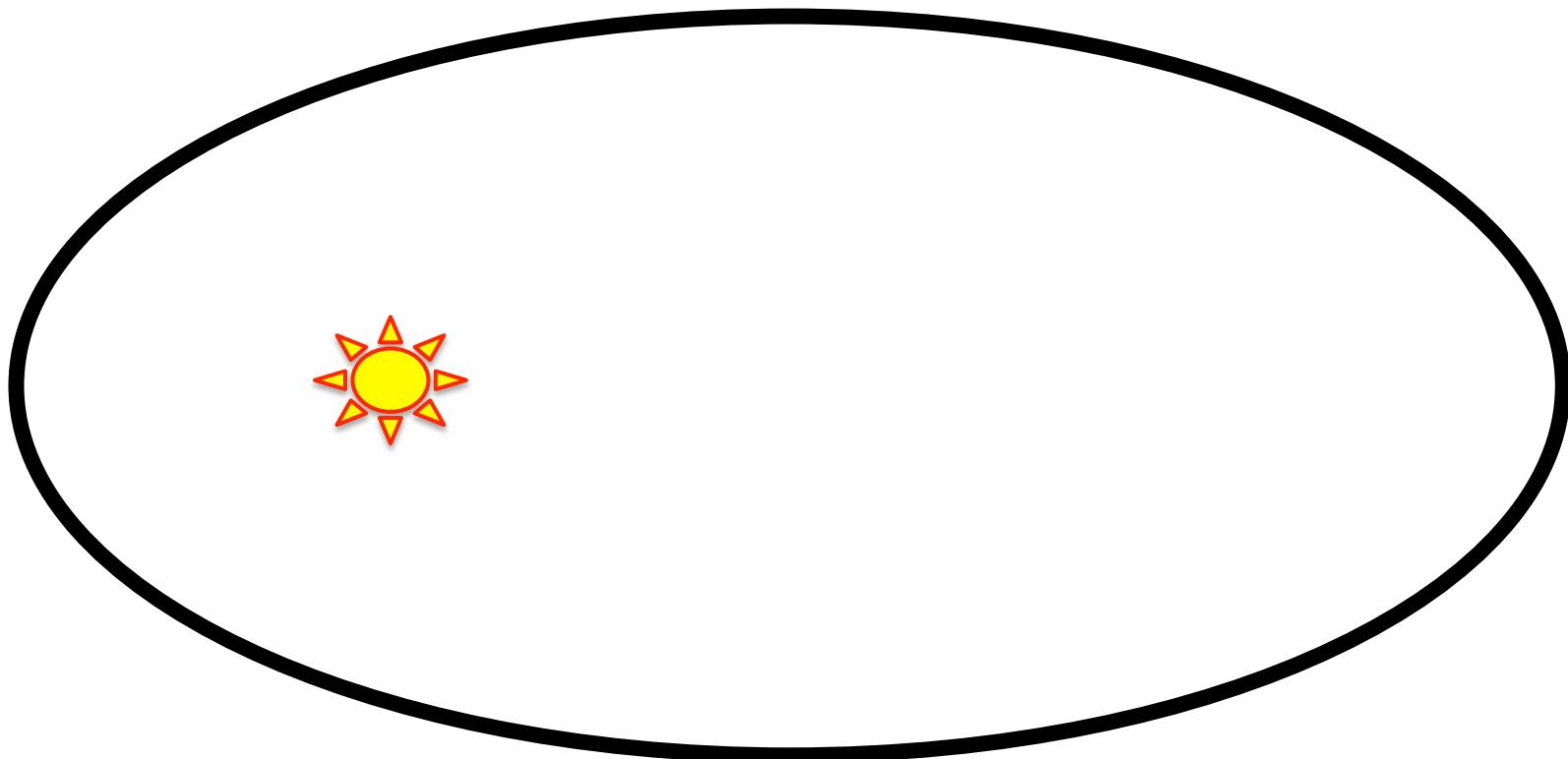


Le prove classiche (1)

$$\frac{\partial \theta}{\partial t} M(T) = -\frac{2}{\partial \theta} \int_{\theta_0}^{\theta} f(v) dv + \frac{C}{\theta} \int_{\theta_0}^{\theta} \frac{f(v)}{v} dv$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\omega}(t_0) = \frac{(t_0 - \alpha)}{\sigma^2} \int_{\theta_0}^{t_0} \frac{f(v)}{v} dv + \frac{C}{\theta_0^2}$$
$$\int T(x) \frac{\partial}{\partial \theta} f(x) dx = \int (T(x) - \frac{C}{x}) f(x) dx$$
$$\int T(x) \left(\frac{2}{\partial \theta} \int_{\theta_0}^{\theta} f(v) dv + \frac{C}{\theta} \int_{\theta_0}^{\theta} \frac{f(v)}{v} dv \right) dx$$
$$\frac{\partial}{\partial \theta} M(T) = \frac{2}{\partial \theta} \int_{\theta_0}^{\theta} f(v) dv + \frac{C}{\theta} \int_{\theta_0}^{\theta} \frac{f(v)}{v} dv$$



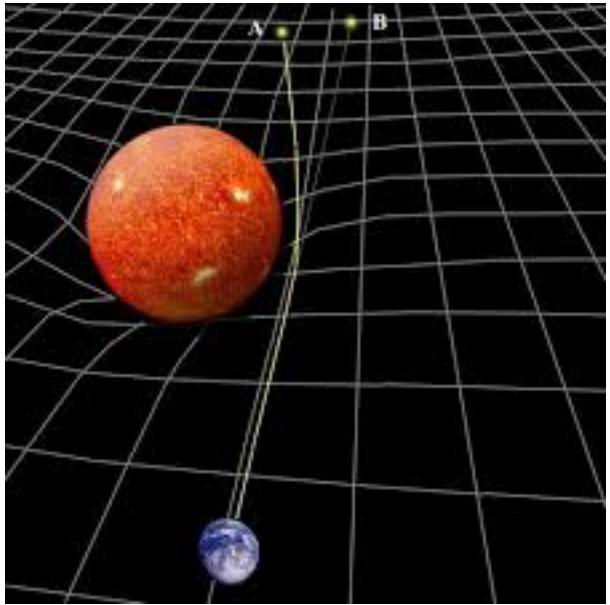
Il doppio senso della scienza (G.Ghirlanda – Brera 30.11.2016)



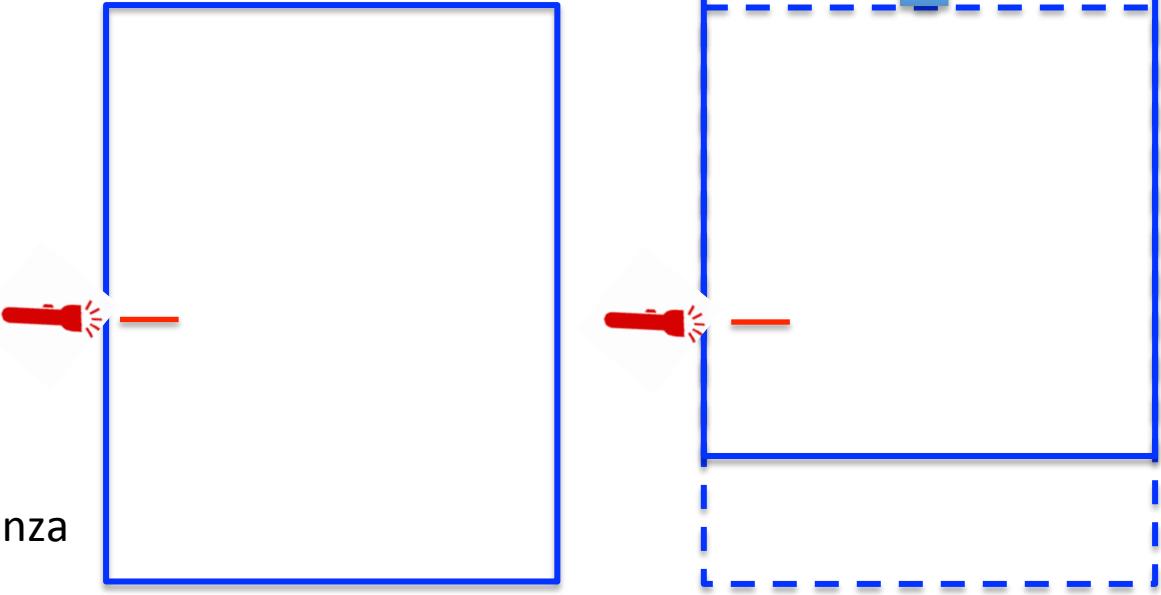


Le prove classiche (2)

$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{2}{\sigma^2} \int_{\Omega} f(v) |v|_{L^2(\Omega)}^2 \left(\int_{\Omega} f(v) v \right)^{-1} \int_{\Omega} f(v) v \, dx$$
$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\sigma^2}(z_0) = \frac{(z_0 - \bar{z})}{\sigma^2} \int_{\Omega} f(v) v \, dx + \frac{1}{\sigma^2} \int_{\Omega} f(v) v^2 \, dx$$
$$\int_{\Omega} f(v) \frac{\partial}{\partial \theta} f(v) v \, dx + \lambda \left(\int_{\Omega} f(v) v \right)^{-1} \int_{\Omega} f(v) v^2 \, dx$$
$$\int_{\Omega} f(v) \left(\frac{2}{\sigma^2} \int_{\Omega} f(v) |v|_{L^2(\Omega)}^2 \right) f(v) v \, dx + \frac{1}{\sigma^2} \int_{\Omega} f(v) v^2 \, dx$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\sigma^2} \int_{\Omega} f(v) |v|_{L^2(\Omega)}^2 \left(\int_{\Omega} f(v) v \right)^{-1} \int_{\Omega} f(v) v^2 \, dx$$



La curvatura della luce



Gedankenexperiment
+ principio di equivalenza

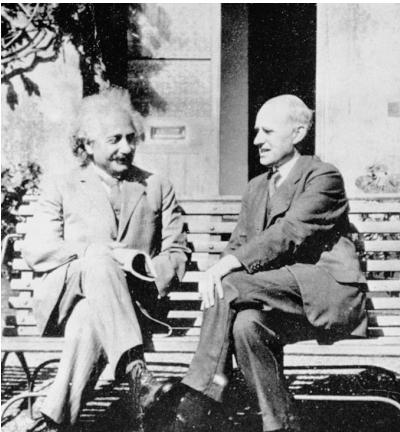
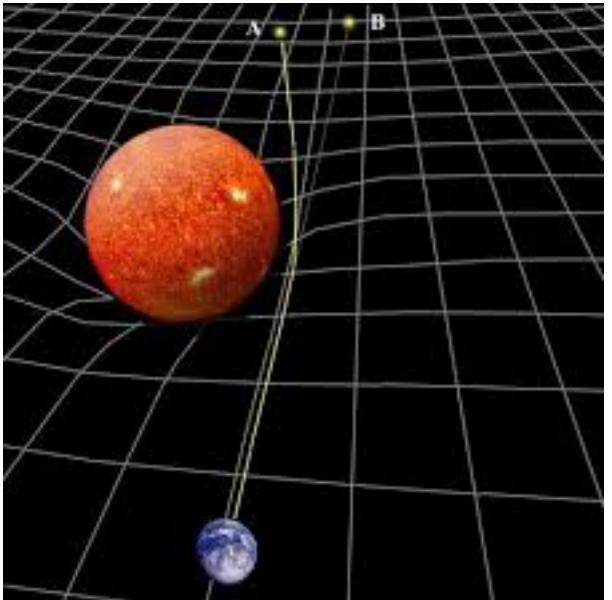


“Do not Bodies act upon Light at a distance and by their action bend its Rays;
and is not this action ... strongest at the least distance?”
[Opticks, I. Newton]

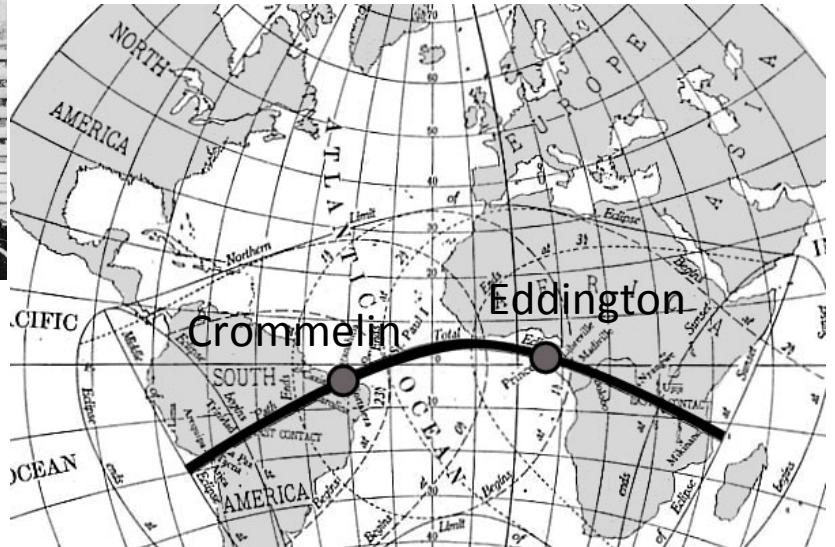


Le prove classiche (2)

$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{\partial}{\partial t} \int_{\Omega} f(v) |v|^{p-2} v \, dx$$
$$\frac{\partial}{\partial t} \ln f_{\alpha,\beta,\gamma}(t_0) = \frac{(q-p)}{p} \int_{\Omega} \frac{|v|^{p-2}}{|v|^{q-2}} v^2 \, dx$$
$$\int T(x) \frac{\partial}{\partial t} f_{\alpha,\beta,\gamma}(t_0) + \chi(t_0) \frac{\partial}{\partial t} \int_{\Omega} f(v) |v|^{p-2} v \, dx$$
$$\int T(t) \left(\frac{\partial}{\partial t} f_{\alpha,\beta,\gamma}(t) + \chi(t) \frac{\partial}{\partial t} \int_{\Omega} f(v) |v|^{p-2} v \, dx \right)$$
$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{p} \int_{\Omega} f(v) |v|^{p-2} v^2 \, dx$$



29 Maggio 1919
Eclissi di sole ... con dietro le Hyadi



£100 attrezzatura £1000 spedizione

Crommelin: "Eclipse splendid"
Eddington: "Through Cloud. Hopefully"



Le prove classiche (2)

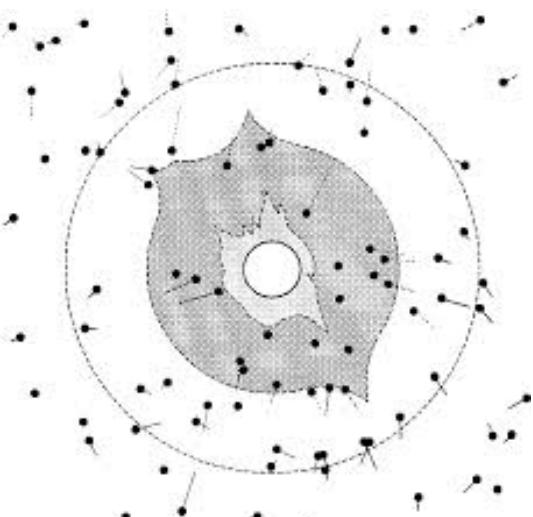
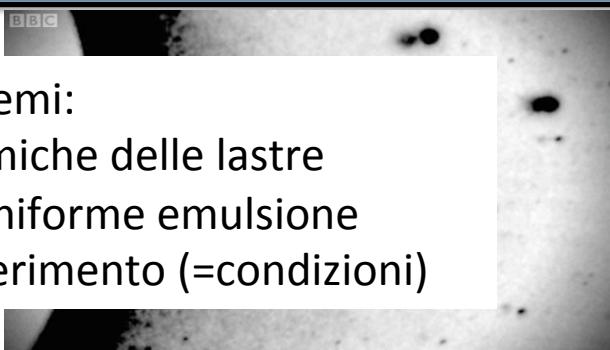
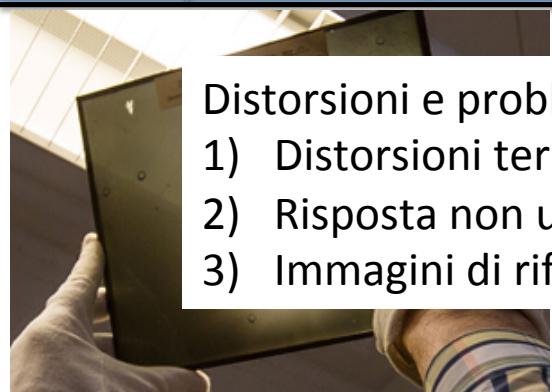
$$\frac{\partial \theta}{\partial t} M(T(t)) = \frac{\partial}{\partial t} \int_{\Omega} f(v) |v|^{p-2} v \, dx + \int_{\Omega} g(x) v \, dx$$

$$\frac{\partial}{\partial \alpha} \ln f_{\alpha,\omega}(t_0) = \frac{(t_0 - t)}{\sigma^2} \int_{\Omega} g(x) v \, dx$$

$$\int_{\Omega} f(x) \frac{\partial}{\partial t} M(T(t)) v \, dx + \int_{\Omega} g(x) v \, dx = \int_{\Omega} f(x) v \, dx$$

$$\int_{\Omega} f(x) \left(\frac{\partial}{\partial t} M(T(t)) v + g(x) v \right) \, dx = \int_{\Omega} f(x) v \, dx$$

$$\frac{\partial}{\partial \theta} M(T(t)) = \frac{2}{\sigma^2} \int_{\Omega} f(v) |v|^{p-2} v^2 \, dx + \int_{\Omega} g(x)^2 \, dx$$



7 stars $\rightarrow 1.98+0.16''$



5 stars $\rightarrow 1.61+0.40''$



"We owe it to that great man (Newton) to proceed very carefully in modifying or retouching his Law of Gravitation" (L. Silberstein)



"This is the most important result obtained in connection with the theory of gravitation since Newton's days" (J. J. Thomson)



Le prove classiche (2)

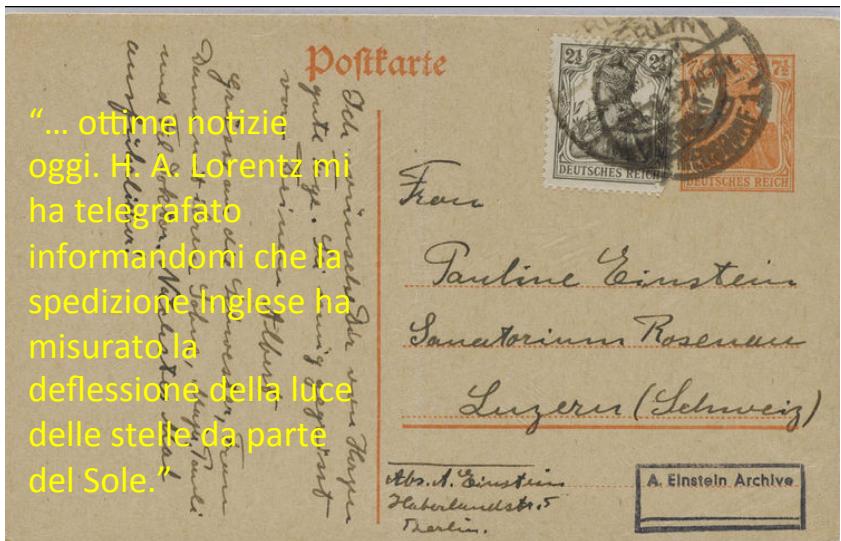
$$\frac{\partial \theta}{\partial \alpha} T(S) = \frac{\partial}{\partial \alpha} \int_{S_0}^S f(\alpha) d\alpha = \int_{S_0}^S \frac{\partial f(\alpha)}{\partial \alpha} d\alpha = \int_{S_0}^S f'(S) d\alpha = f(S) - f(S_0)$$

$$\frac{\partial}{\partial \alpha} \ln f_{\text{pert}}(S_0) = \frac{(S-S_0)}{S_0^2}$$

$$\int T(S) = \frac{S}{S_0} \ln f(S_0) + C = \frac{S}{S_0} \ln f(S_0) + \frac{C}{S_0}$$

$$\int T(S) = \frac{S}{S_0} \ln f(S_0) + C = \frac{S}{S_0} \ln f(S_0) + \frac{C}{S_0}$$

$$\frac{\partial}{\partial \theta} M(T) = \frac{2}{S_0^2} \int_{S_0}^S \frac{(S-S_0)}{S^2} dS = \frac{2}{S_0^2} \left[\frac{(S-S_0)^2}{2} \right] \Big|_{S_0}^S = \frac{2}{S_0^2} \cdot \frac{(S-S_0)^2}{2} = \frac{(S-S_0)^2}{S_0^2}$$



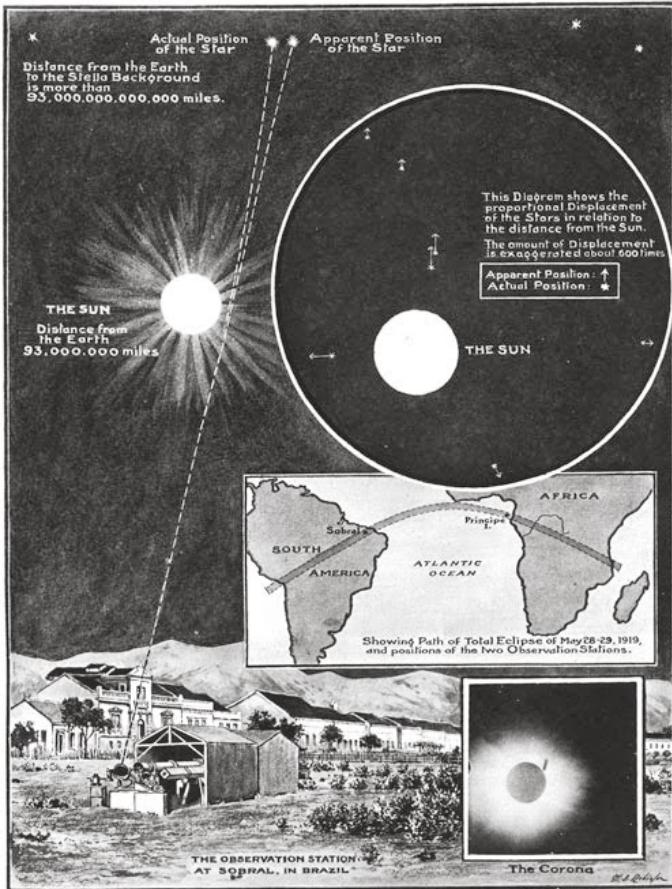
LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

New York Times (10/11/1919)



Illustrated London News (1919)



La deflessione della luce oggi

$$\frac{\partial \theta}{\partial \alpha} \Omega T(\zeta) = \frac{\partial}{\partial \alpha} \int_{\Gamma} \rho(\zeta') \frac{1}{|\zeta - \zeta'|} d\zeta' = \int_{\Gamma} \frac{\partial \rho(\zeta')}{\partial \alpha} \frac{1}{|\zeta - \zeta'|} d\zeta'$$
$$\frac{\partial}{\partial \alpha} \ln f_{\text{prior}}(\zeta_0) = \frac{(z_0 - z)}{z^2}$$
$$\int_{\Gamma} T(x) \frac{\partial}{\partial \alpha} \rho(\zeta') d\zeta' = \int_{\Gamma} \rho(\zeta') \frac{\partial}{\partial \alpha} T(x) d\zeta'$$
$$\int_{\Gamma} T(x) \left(\frac{\partial}{\partial \alpha} \rho(\zeta') \right) \rho(\zeta') d\zeta' = \int_{\Gamma} \rho(\zeta') \left(\frac{\partial}{\partial \alpha} \rho(\zeta') \right) T(x) d\zeta'$$
$$\frac{\partial}{\partial \alpha} M(T(x)) = \frac{\partial}{\partial \theta} \int_{\Gamma} \rho(\zeta') \rho(\zeta') d\zeta' = \int_{\Gamma} \rho(\zeta') \frac{\partial}{\partial \alpha} \rho(\zeta') d\zeta'$$



Vedere oltre il buco nero

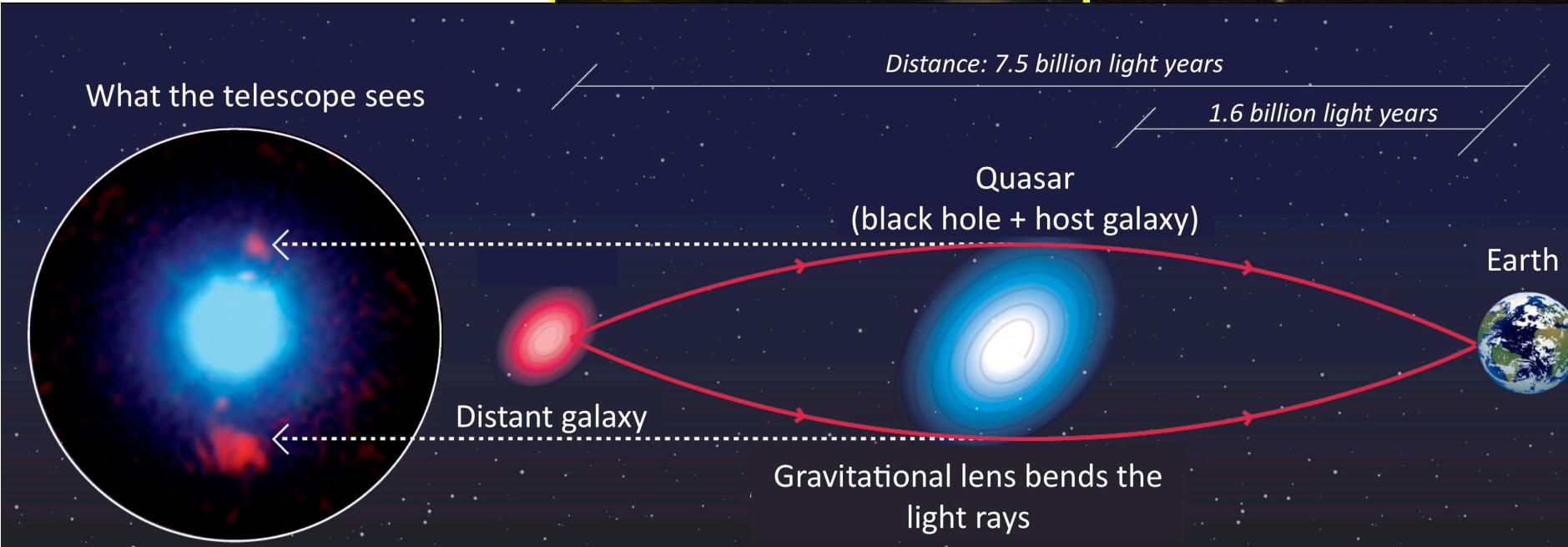
Il telescopio gravitazionale



La supernova si fa in quattro



Galassie con lo strascico





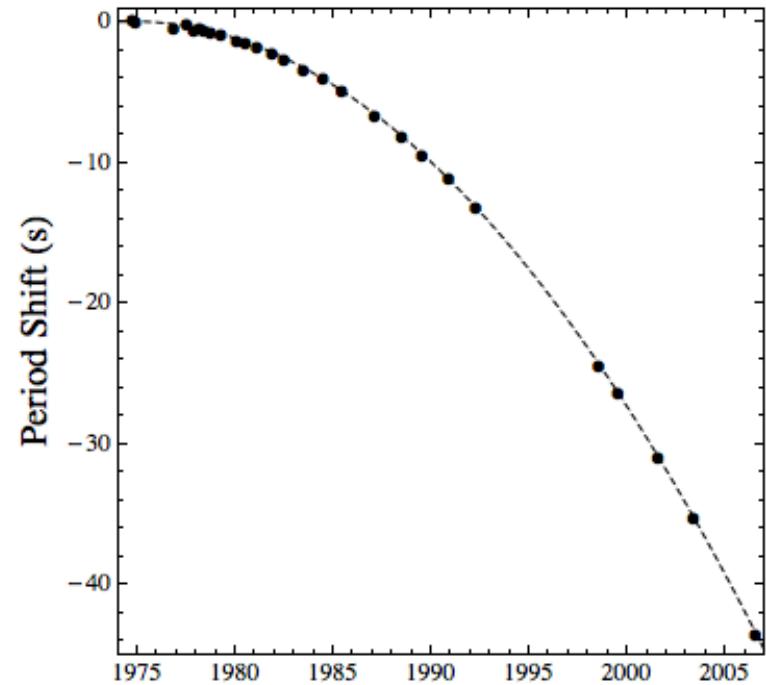
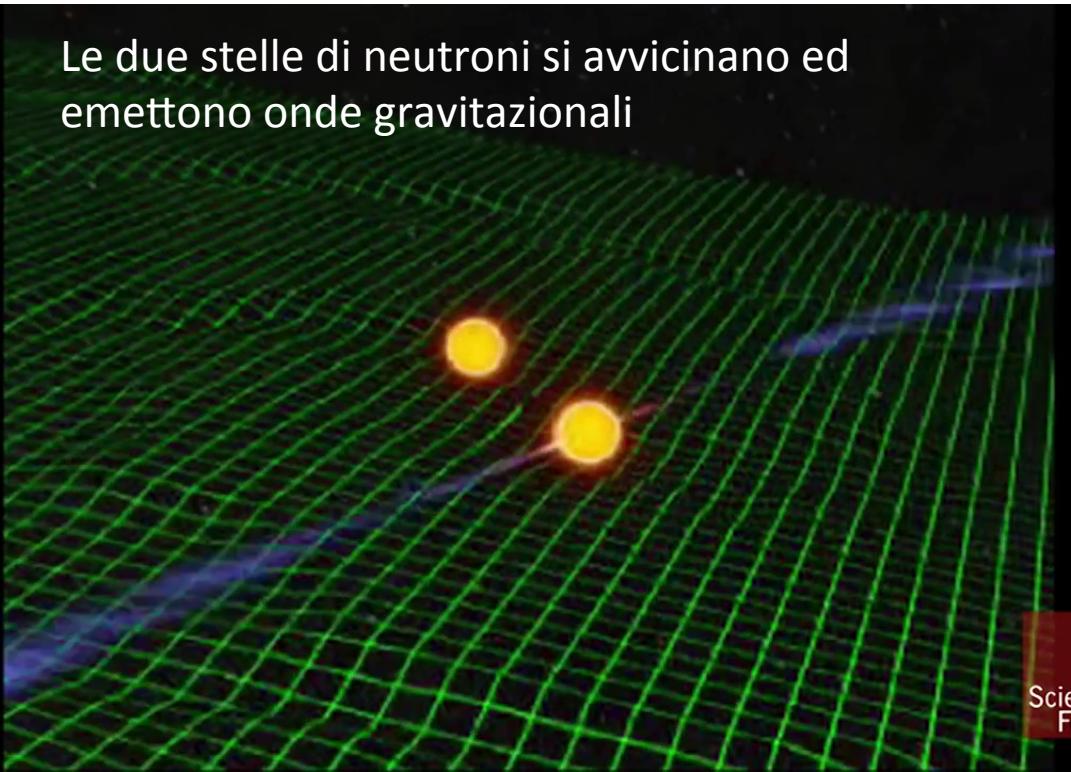
Le prove moderne: test in campo forte

$$\frac{\partial \theta}{\partial \alpha} \ln f_{\alpha, \sigma^2}(z_i) = \frac{1}{\sigma^2} \int_{-\infty}^{z_i} \frac{f(z)}{\sqrt{2\pi}} dz - \frac{1}{\sigma^2} \int_{-\infty}^{z_i} \frac{f(z)}{\sqrt{2\pi}} dz$$
$$\frac{\partial \theta}{\partial \alpha} \ln f_{\alpha, \sigma^2}(z_i) = \frac{(z_i - \bar{z})}{\sigma^2} - \frac{1}{\sigma^2} \int_{-\infty}^{z_i} \frac{f(z)}{\sqrt{2\pi}} dz$$
$$\int T(x) \frac{\partial}{\partial \theta} f_{\alpha, \sigma^2}(x) dx = \lambda \left(\int_{-\infty}^{\infty} \frac{f(x)}{\sqrt{2\pi}} dx \right)^2 - \lambda \left(\int_{-\infty}^{\infty} \frac{f(x)}{\sqrt{2\pi}} dx \right)^2$$
$$\int T(x) \frac{\partial}{\partial \theta} f_{\alpha, \sigma^2}(x) dx = \lambda \left(\int_{-\infty}^{\infty} \frac{f(x)}{\sqrt{2\pi}} dx \right)^2 - \lambda \left(\int_{-\infty}^{\infty} \frac{f(x)}{\sqrt{2\pi}} dx \right)^2$$
$$\frac{\partial}{\partial \theta} M(T) = \frac{2}{\sigma^2} \int_{-\infty}^{\infty} |T(x)|^2 f_{\alpha, \sigma^2}(x) dx - \frac{2}{\sigma^2} \int_{-\infty}^{\infty} |T(x)|^2 f_{\alpha, \sigma^2}(x) dx$$

Hulse & Taylor (NP 1993)



Le due stelle di neutroni si avvicinano ed emettono onde gravitazionali



Le misure (pallini)
sono perfettamente
sulla linea che e'
quando previsto
dalla teoria della
relativita' generale.